



STAR SPANGLED BANNER



How are the stars arranged on the American flag? Are there five rows of ten stars? Ten rows of five stars? Maybe it's two rows of twenty-five, or a single row of fifty!

In this lesson we'll use **factors** to explore the mathematics behind the American flag. *Oh, say, can you see...patterns in the stars??*

Students Will...

- Find all factor pairs for a whole number between 1-100, and use factor pairs to arrange stars on American flag
- Recognize that a whole number is a multiple of its factors
- Determine whether a given number is prime or composite
- Discuss six main star patterns in American history and use them to redesign flags

Common Core Standards

Grade 4 OA.4

Materials

- Student Handout
- Presentation
- LCD projector & speakers
- Pennies, beans, etc.: something that students can easily arrange (optional)

Approximate Time

50-60 minutes

Before Beginning...

It would be helpful if students are already familiar with factor pairs (e.g. $24 = 1 \times 24 = 2 \times 12 = 3 \times 8 = 4 \times 6$), but not necessary. The lesson is sufficiently concrete and intuitive that they should be able to figure it out fairly easily.

Plot Summary & Teaching Tips

Preview

5 min.

If you have an American flag in your classroom, hide it. If you have an American flag tattoo on your forehead, wear a hat. We'll start by asking students something they think they know but probably don't: *how are the stars arranged on the American flag?*

Have them sketch their best guess before revealing the answer. Ask students to describe how the stars are actually arranged (alternating rows of 6-5-6-5-6...), and why they think is this.



Follow-Up Questions

- Have there always been fifty states? If not, what is the fewest there have been? (Ans: zero!)
- What do the stars and stripes represent?
- Who designed the original American flag and when? How many stars were on it?
- Do you think there will ever be a 51st state? If so, what will it be?

Act One

20-25 min.

Now that we know what the flag actually looks like, we'll redesign it. For this section we'll assume that stars must be in rows and that **each row must include the same number of stars** (i.e. that the current 6-5-6-5-6 arrangement is off limits), which will allow us to explore factors.

50 stars	1 x 50	2 x 25	5 x 10	or	Stars in Row	1	2	5	10	25	50
	50 x 1	25 x 2	10 x 5		Stars in Column	50	25	10	5	2	1

Students should recognize that there are six ways to arrange fifty stars, and come up with a systematic way to identify them all. Next they'll sketch the corresponding flags. As they do, let's really explore what these factor pairs mean in a real-world context. For instance, does $10 \times 5 = 5 \times 10$? On one hand, of course. On the other, "10 rows & 5 columns" looks different than "5 rows & 10 columns:" one is tall while the other is wide.

After doing this for the 50-star flag, students will create possible patterns for the flag at other points in American history. For instance, what might the flag have looked like after Maine (23rd state) joined the union? Since 23 is a prime number, students should realize that there are only two possible configurations: one long row or one tall column. On the other hand, Missouri (24) and Arkansas (25) offer more possibilities, and this is a good opportunity to discuss the differences between prime numbers, composite numbers and even perfect squares.

Note: Giving students pennies, beans, etc. to represent stars may be particularly helpful for kinesthetic learners who prefer to arrange and rearrange stars manually instead of drawing them.

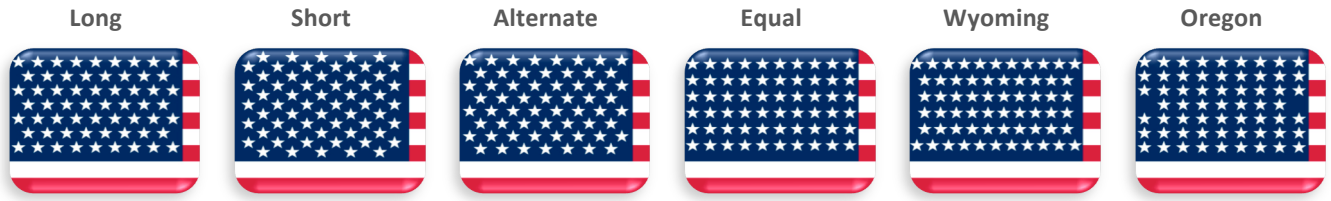
Follow-Up Questions

- How many stars did the first American flag have and why?
- Based on the animation of the U.S. states by date of statehood, how did the country grow? ("Go west, young man!")
- Which states—by number—would be the most difficult to design for and which would be the easiest?

Act Two

20-25 min.

In this section we'll drop the "all rows equal" requirement, and will look at how flags have actually been designed. According to Emory University professor Skip Garibaldi, there have been six main star patterns throughout American history:



Students will find that for the 50-star flag, only the Long, Equal & Wyoming patterns work. (For a challenge, consider asking them why the others don't.) At the same time, there's one number of stars that won't work for any of the patterns. At first blush, students may have a hard time figuring out which state this corresponds to, and may resort to a guess-and-check method. Ideally we want students to apply logical thinking, and consider having them work in pairs. After a few minutes, you might ask, "Are there any numbers that will *definitely* work for at least one of the patterns?"

Ultimately, students should realize that the Equal pattern will work for every even number, leaving us with the odds. In fact, it will work for *every* composite number, leaving us with the primes: 2, 3, 5, 7, 11, 13, 17, 19, 23, 29, 31, 37, 41, 43, 47. At this point students can either manually narrow the list down, or you might assign each prime number to a group and have the class work on it as a team.

Follow-Up Questions

- Of the six patterns—long, short, alternate, equal, Wyoming & Oregon—which do you think looks the nicest? The worst?
- Why do you think they called them the "Wyoming" and "Oregon" patterns? (Also: Who's buried in Grant's tomb?)
- What did the American flag look like after Iowa joined the union?

Just One More

5 min.

Before we go, let's return to our roots: the original American flag. Students read the 1777 Continental Congress resolution calling for "thirteen stars" and think about how Betsy Ross might have designed it. They will likely agree that 1x13 (or 13x1) would make for a pretty ugly flag, which should prove a nice way to bring the lesson full circle...literally!

Follow-Up Questions

- If you were Betsy Ross, would you have used a circle, or some other arrangement?