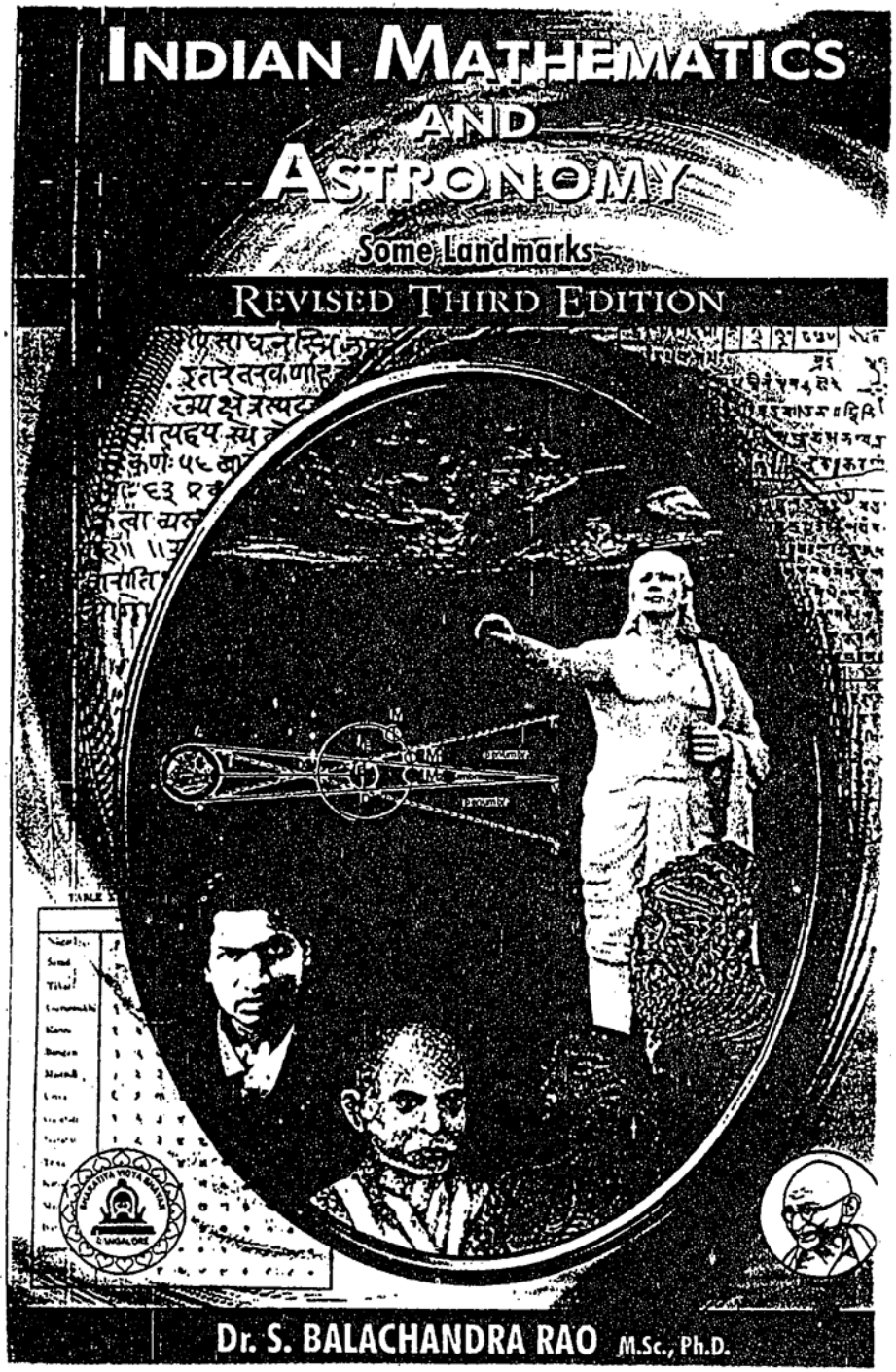


(4) "The two parallel faces of a figure resembling a *paṇava* (a drum-shaped musical instrument) are each 8 units, the central width 2 units, and the length between the faces is 16 units. Say what is the area of this figure resembling a *paṇava*."

– *ĀBh. Bhā.* II, ; *Ud.* 4

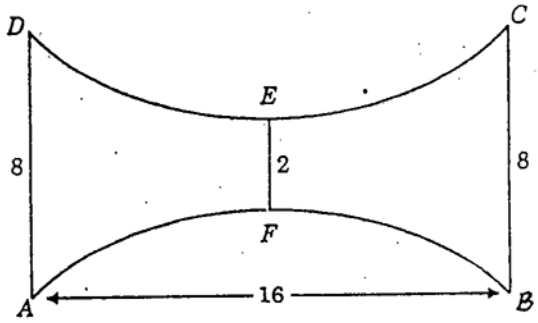


Fig. 4.2 Area of *Paṇava*-shaped figure

The figure *AFBCED* (Fig. 4.2), resembling the musical instrument *paṇava* (*ḍhakkā, ḍamaru* a double-drum) is approximately combination of two indentical trapeziums *AFED* and *BFEC.* Bhāskara I uses the formula

$$\text{Area} = \frac{1}{2}\left[\frac{AD + BC}{2} + EF\right] AB$$

In the figure, $AD = BC = 8$, $EF = 2$, $AB = 16$ therefore

$$\text{Area} = \frac{1}{2}\left[\frac{8+8}{2} + 2\right] 16 = 80 \text{ sq.units}$$

This example reappears in a later commentary of Raghunātharāja of Karnataka.

MODIFIED FORMULA OF BHASKARA I OF ANCIENT INDIA AND COMPUTATION OF PROPOSED π VALUE

"The importance of Bhaskara I lies in his being the greatest and the earliest exponent of Aryabhata I and his school of astronomy". He belongs to seventh Century."

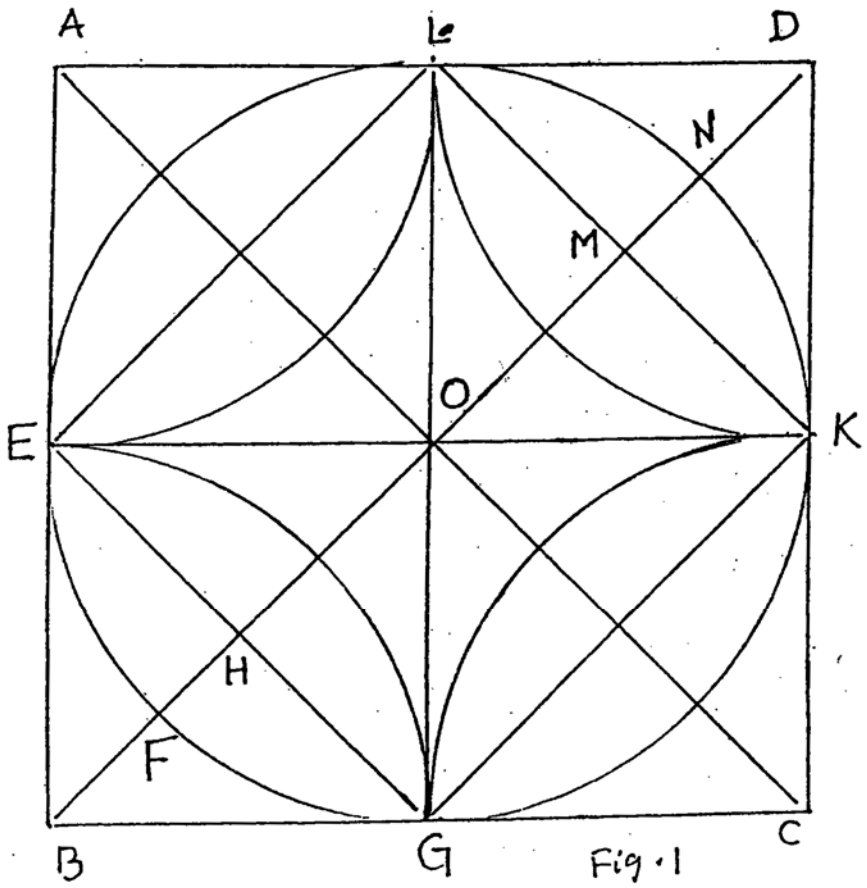


Fig. 1

Square = ABCD

BC = Side = FN

= diameter = 1

$AC = \sqrt{2}$, $BO = \frac{\sqrt{2}}{2}$

$BH = MD = \frac{\sqrt{2}}{4} = OH$

$FH = OF - OH = \frac{1}{2} - \frac{\sqrt{2}}{4} = \frac{2-\sqrt{2}}{4}$

$OF = ON = \frac{1}{2}$ = radius

$MN = ON - OM = \frac{2-\sqrt{2}}{4}$

$EG = LK = \frac{\sqrt{2}}{2}$

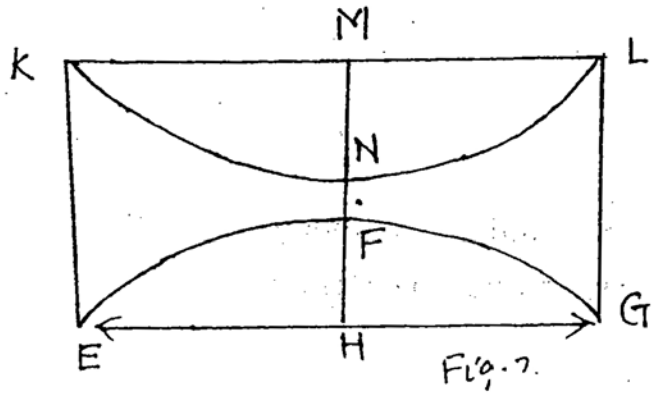


Fig. 2.

$KE = LG = MH = \frac{\sqrt{2}}{4}$

$MN = FH = \frac{2-\sqrt{2}}{4}$

NF = MH – (MH + FH)

$= \frac{\sqrt{2}}{4} - \frac{2-\sqrt{2}}{4} = \frac{3\sqrt{2}+4}{4}$

$$KE = LG = MH = \frac{\sqrt{2}}{4};\ KL = EG = \frac{\sqrt{2}}{2};\ NF = \frac{3\sqrt{2}-4}{4}$$

1. Area of KEGL rectangle $= \frac{\sqrt{2}}{4} \times \frac{\sqrt{2}}{2} = \frac{1}{4}$

2. The figure EFGLNK resembling the musical instrument – double drum – is approximately combination of two identical trapeziums. Bhaskara-I uses the following formula for its area :

3. $\underset{\text{(approximate)}}{\text{Area}} = \frac{1}{2}\left[\frac{KE+GL}{2} + NF\right]EG$

4. Exact area $= \left\{\frac{1}{2}\left[\frac{KE+GL}{2} + NF\right]EG\right\} \frac{1}{8\left(\frac{2-\sqrt{2}}{2+\sqrt{2}}\right)}$

$$= \left\{\frac{1}{2}\left[\frac{\frac{\sqrt{2}}{4}+\frac{\sqrt{2}}{4}}{2} + \frac{3\sqrt{2}-4}{4}\right]\frac{\sqrt{2}}{2}\right\} \frac{1}{8\left(\frac{2-\sqrt{2}}{2+\sqrt{2}}\right)} = \frac{2+\sqrt{2}}{32}$$

5. Four equal segments are = EFH = FHG = KMN = LMN

6. Area of each segment in Fig.1 in terms of $\pi = \frac{\pi-2}{32}$

7. Area of each segment as per the modified formula of Bhaskar-I

$\frac{1}{4}$ (area of rectangle – area of EFGLNK)

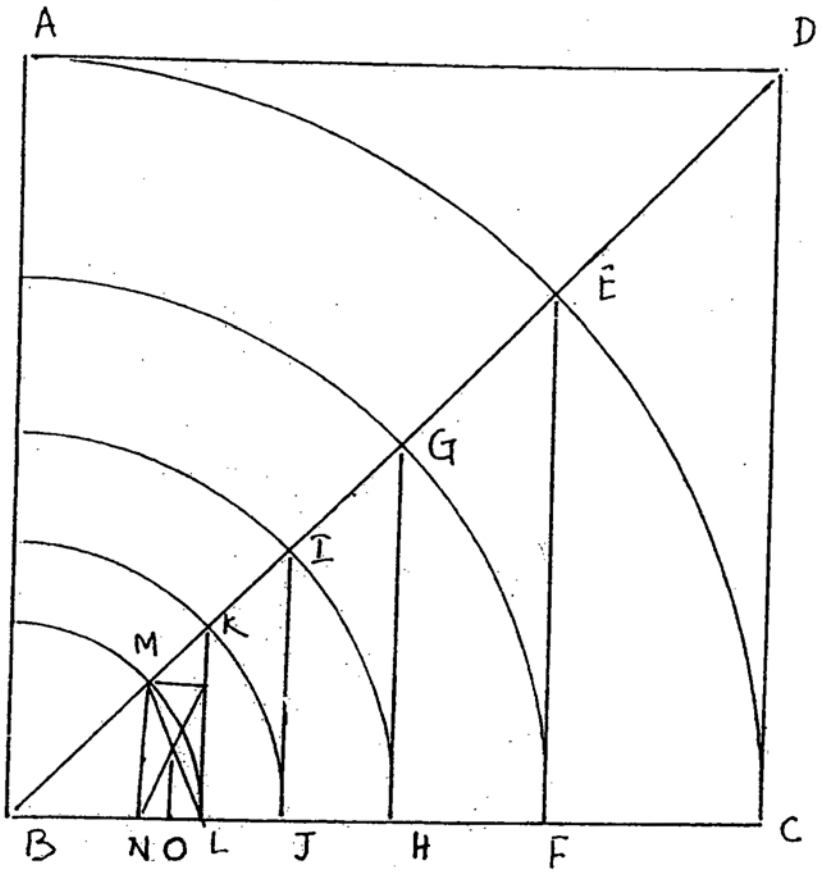
$$= \frac{1}{4}\left\{\frac{1}{4} - \left(\frac{2+\sqrt{2}}{32}\right)\right\}$$

8. S.No.7 = S.No.6 $= \frac{1}{4}\left\{\frac{1}{4} - \left(\frac{2+\sqrt{2}}{32}\right)\right\} = \frac{\pi-2}{32}$

9. $\pi = \frac{1}{4}\left(14-\sqrt{2}\right)$

10. Source: Indian Mathematics and Astronomy. By Dr.S.Balachandra Rao, 3rd ed. 2004. Bhavan's Gandhi Centre of Science – BANGALORE, India.

ARCUAL LENGTH FROM D.S. PILLAY'S GEOMETRICAL CONSTRUCTION



Square = ABCD

BC = 1, BD = $\sqrt{2}$

FC = GE = $\frac{2-\sqrt{2}}{2}$

HF = IG = $\frac{\sqrt{2}-1}{2}$

JH = KI = $\frac{2-\sqrt{2}}{4}$

LJ = MK = $\frac{\sqrt{2}-1}{4}$

NL = $\frac{2-\sqrt{2}}{8}$

Bisect NL to get NO = OL

= $\frac{2-\sqrt{2}}{16}$

OC = Arcual Length EC

OC = OL + LJ + JH + HF + FC

$$= \frac{2-\sqrt{2}}{16} + \frac{\sqrt{2}-1}{4} + \frac{2-\sqrt{2}}{4} + \frac{\sqrt{2}-1}{2} + \frac{2-\sqrt{2}}{2}$$

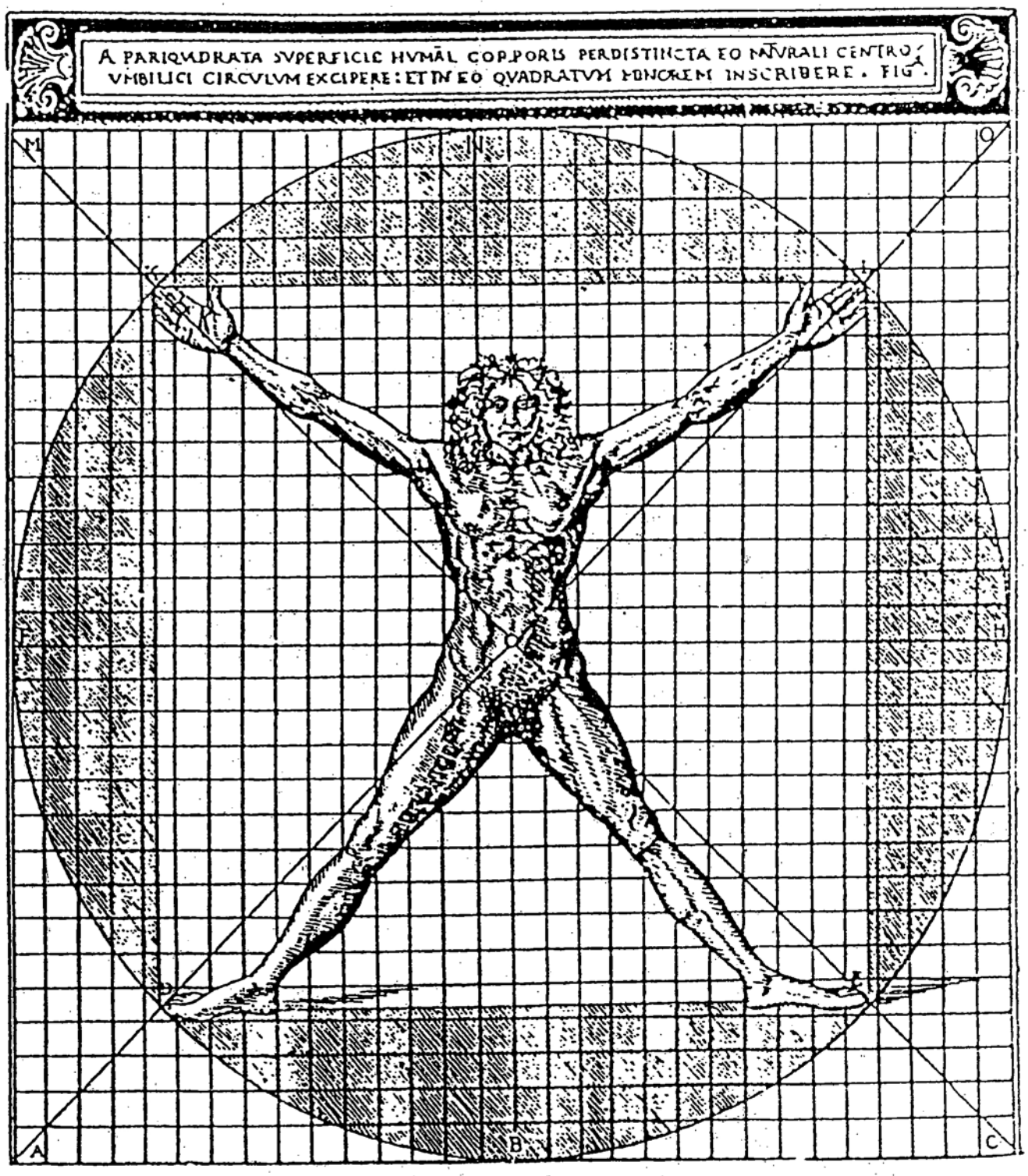
$$= \frac{14-\sqrt{2}}{16}$$ = Length of OC and arcual length of EC

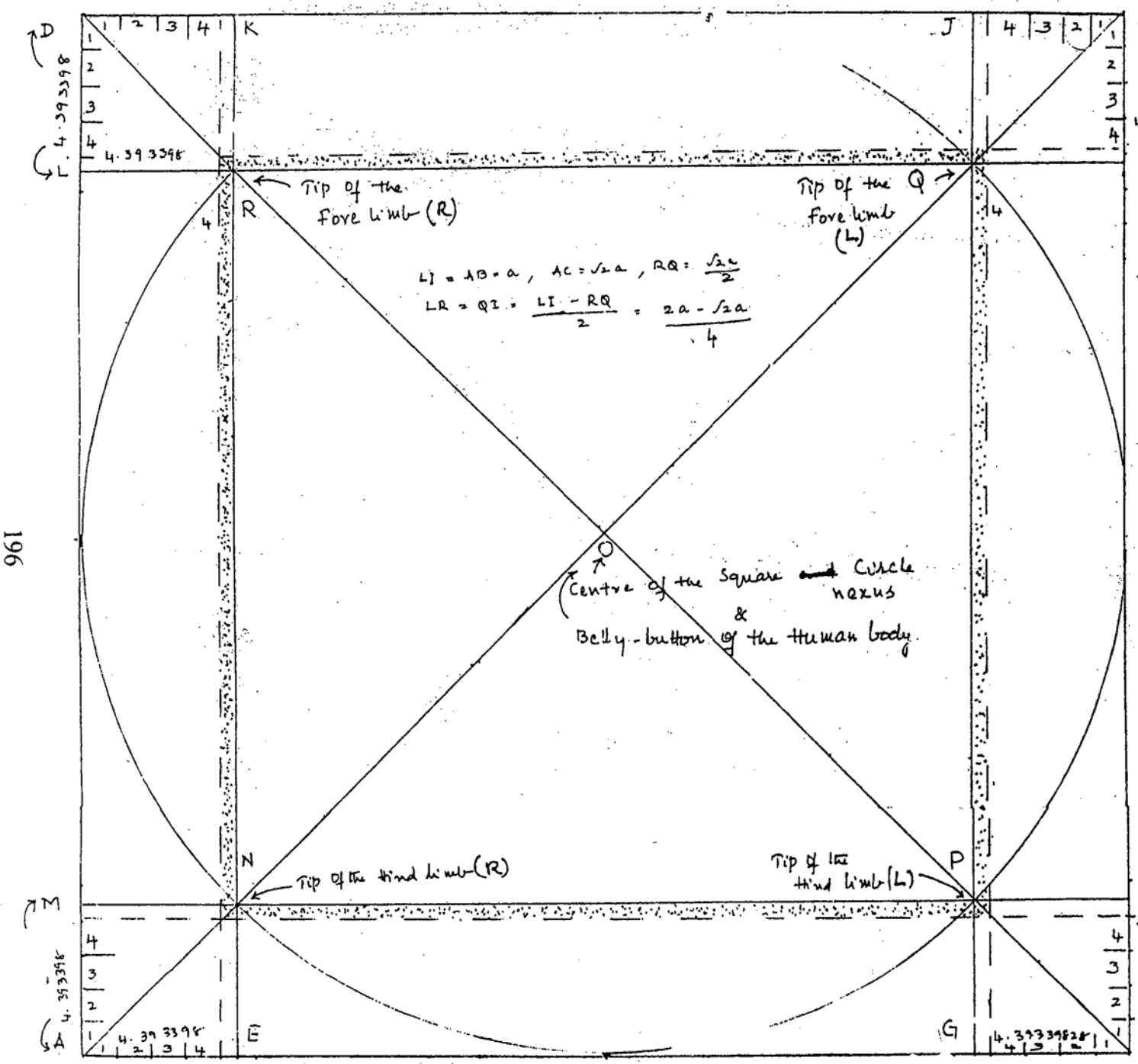
The author is grateful to Mr. D.S. Pillay,

(D.No.22/149, Kattamanchi Road, Chittoor - 517001) for his

ingenious geometrical construction.

THE MACROCOSM AND THE MICROCOSM





CENTRE OF SQUARE-CIRCLE NEXUS IS THE NAVEL OF HUMAN BODY

1. ABCD = SQUARE = 30 CMS. = A
2. AB = SIDE = NQ = DIAMETER = 3C as in the Figure of body.
3. INSCRIBE A CIRCLE
4. AE = GB = BH = IC = CJ = KD = DL = MA
5. BGEAMLDKJCI = Circumference
6. HUMAN BODY IS BILATERAL IN SYMMETRY
7. TIPS OF FORE LIMBS are R and Q
8. TIPS OF HIND LIMBS are N and P
9. 'O' is the naval of the body
10. Extremities of body make a SQUARE NPRQ
11. All Celestial bodies are SPHERES
12. Mathematical accuracy in MICRO - and MACROCOSMS IS CLEAR
13. Ref: The BOOK OF PHI By JAIN, 777, Left Bank Road Mullumbimby Creek NSW, Australia

4.39339828

The author is grateful to Mr. JAIN.

R. SARVA JAGANNADHA REDDY

STRAIGHTENED AREAS OF ARCUAL SEGMENTS

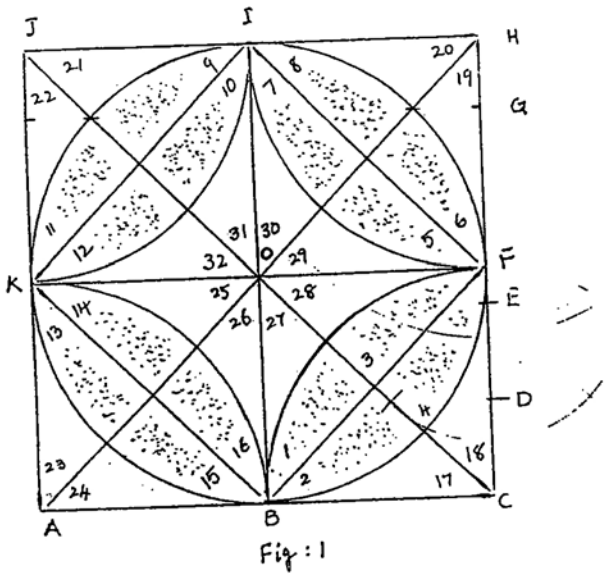
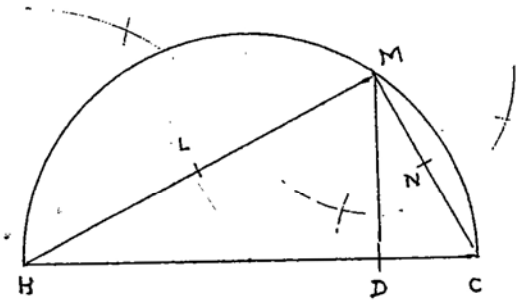


Fig : 1



Draw a square. Inscribe a circle. Side = diameter = 1. Divide the square into 32 segments as shown in the figure. Square is segmentically segmented.

Inside 8 (b)

Outside 8 (b)

Middle 16 (a).

Inside = Outside

Area of a = $\frac{(6-\sqrt{2})}{128} = \frac{\pi-2}{32}$

Area of b = $\frac{(2+\sqrt{2})}{128} = \frac{4-\pi}{32}$

Square = 16a + 16b = 1

Circle = 16a + 8b = πr^2

$= \frac{(14-\sqrt{2})}{16}$

$\pi = \frac{(14-\sqrt{2})}{4}$

Fig.1: HG = $\frac{(2-\sqrt{2})}{4}$, GC = $\frac{(2+\sqrt{2})}{4}$ Bisect GC twice.

GE = EC; ED = DC; GE = $\frac{(2+\sqrt{2})}{8}$ ED = $\frac{(2+\sqrt{2})}{16}$;

HD = $\frac{(14-\sqrt{2})}{16} = \frac{\pi}{4}$

Fig.2: HC = 1; HD = $\frac{(14-\sqrt{2})}{16}$; DC = $\frac{(2+\sqrt{2})}{16}$

Altitude Theorem : HM = $\frac{\sqrt{14-\sqrt{2}}}{4}$ = 0.886911299....

CM = $\frac{\sqrt{2+\sqrt{2}}}{4}$ = 0.461939766....

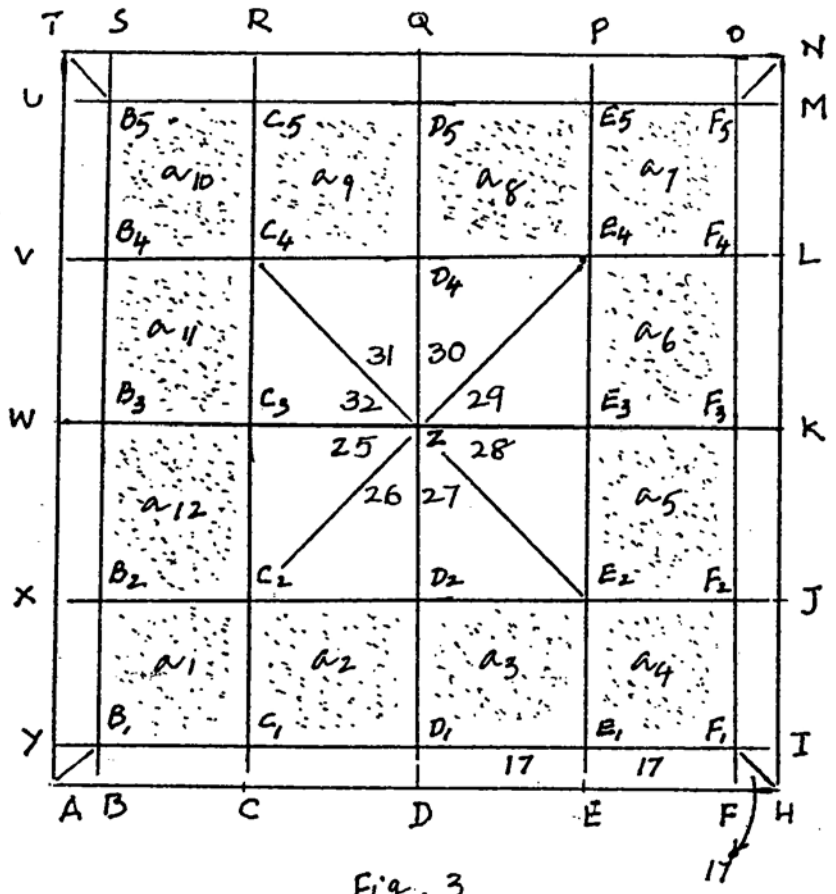


Fig. 3

Draw a square AHNT. Side = 1. D, K, Q and W are mid points. Join DQ and KW. Z is the centre.

INSIDE

Square $C_2\ E_2\ E_4\ C_4$

Side = MC of Fig. 2 = 0.461939766....

$$= \frac{\sqrt{2+\sqrt{2}}}{4}$$

Area of square = 0.213388347.... = $\frac{2+\sqrt{2}}{16}$

Areas of 25 to 32 of Fig.1 = $8\left(\frac{2+\sqrt{2}}{128}\right)$ = 0.213388347.....

MIDDLE REGION

Square $B_1\ F_1\ F_5\ B_5$

Side = $B_1 F_1$ = HM of Fig.2 = 0.886911299.... = $\frac{\sqrt{14-\sqrt{2}}}{4}$

It consists of 4 squares and 8 rectangles

A) **Squares** = $a_1 = a_4 = a_7 = a_{10}$

Side = B_1C_1 = 0.212485766.... = $\frac{\sqrt{14-\sqrt{2}}-\sqrt{2+\sqrt{2}}}{8}$

Area = 0.0451502...

Total area = 4 x 0.0451502... = 0.180600803...

B) **Rectangles** = $a_2 = a_3 = a_5 = a_6 = a_8 = a_9 = a_{11} = a_{12}$

Side I = $C_1D_1 = D_1E_1 = E_2E_3 = E_3E_4 = E_4D_4$

= $D_4C_4 = C_4C_3 = C_3C_2$

= 0.230969883... = $\frac{\sqrt{2+\sqrt{2}}}{8}$

Side II = C_1C_2 = 0.212485766...

Area = Side I x Side II = 0.230969883... x 0.212485766...

= 0.049077812...

Area of 8 rectangles = 8 x 0.049077812... = 0.3926225...

Total area of 4+8 = 0.180600803... + 0.3926225...

= 0.573223303...

Areas of **a** segments : 1 to 16 of Fig.1

$=16\left(\frac{6-\sqrt{2}}{128}\right)$ = 0.573223304...

INSIDE + MIDDLE = CIRCLE

$$0.213388347... + 0.573223303... = 0.786611651... = \frac{14-\sqrt{2}}{16}$$

OUTSIDE

In Fig.1 there are 8 corner entities 17 to 24.

In Fig.3 there are similarly 8 entities

Each entity has two different types of rectangles and one triangle. For example No.17 is worked out below:

A) **Rectangle** DEE_1D_1

Side DQ = 1

$$D_1D_5 = 0.886911299... = \frac{\sqrt{14-\sqrt{2}}}{4}$$

$$DQ - D_1D_5 = 1 - 0.886911299... = 0.113088701...$$

$$Side_1 = DD_1 = D_5Q = \frac{0.113088701...}{2} = 0.05654435...$$

$$= \frac{4-\sqrt{14-\sqrt{2}}}{8}$$

$$Side_2 = DE = 0.230969883... = \frac{\sqrt{2+\sqrt{2}}}{8}$$

Area $= DD_1 \times DE$

$= 0.05654435... \times 0.230969883... = 0.013060041...$

B) **Rectangle** EFF_1E_1

Side $EE_1 = DD_1 = 0.05654435...$

Side $EF = 0.212485766... = \dfrac{\sqrt{14-\sqrt{2}}-\sqrt{2+\sqrt{2}}}{8}$

Area = EE_1 X EF = 0.05654435... x 0.212485766...

= 0.012014869...

C) **Triangle** F_1FH

$F_1F = DD_1 = 0.05654435...$, $F_1F = FH$

Area = $\dfrac{F_1F \times FH}{2}$ = 0.001598631...

Areas of (A) + (B) + (C) = 0.013060041...+ 0.012014869...

+ 0.001598631... = 0.026673541...

Area of 17 of Fig.1 = $\dfrac{2+\sqrt{2}}{128}$ = 0.026673543...

Total area of 8 corner entities 8 x 0.026673543... = 0.213388344...

Areas of 17 to 24 of Fig.1 = $8\left(\dfrac{2+\sqrt{2}}{128}\right)$ = 0.213388347...

Inside = 0.213388347...

Middle = 0.573223303...

Outside = 0.213388344...

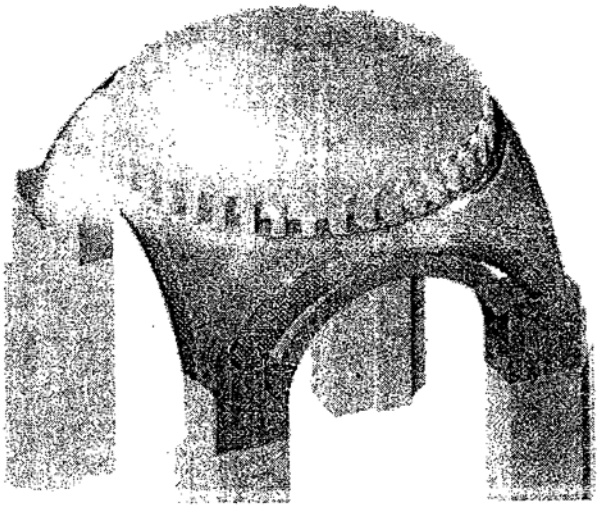
0.999999994... = Area of the Square of AHNT

Side = AH = AB + BC + CD + DE + EF + FH = 1

$$\frac{4-\sqrt{14-\sqrt{2}}}{8}+\frac{\sqrt{14-\sqrt{2}}-\sqrt{2+\sqrt{2}}}{8}+\frac{\sqrt{2+\sqrt{2}}}{8}+\frac{\sqrt{2+\sqrt{2}}}{8}+\frac{\sqrt{14-\sqrt{2}}-\sqrt{2+\sqrt{2}}}{8}+\frac{4-\sqrt{14-\sqrt{2}}}{8}=1$$

Gernot Hoffmann

Circle a Square



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1. Introduction

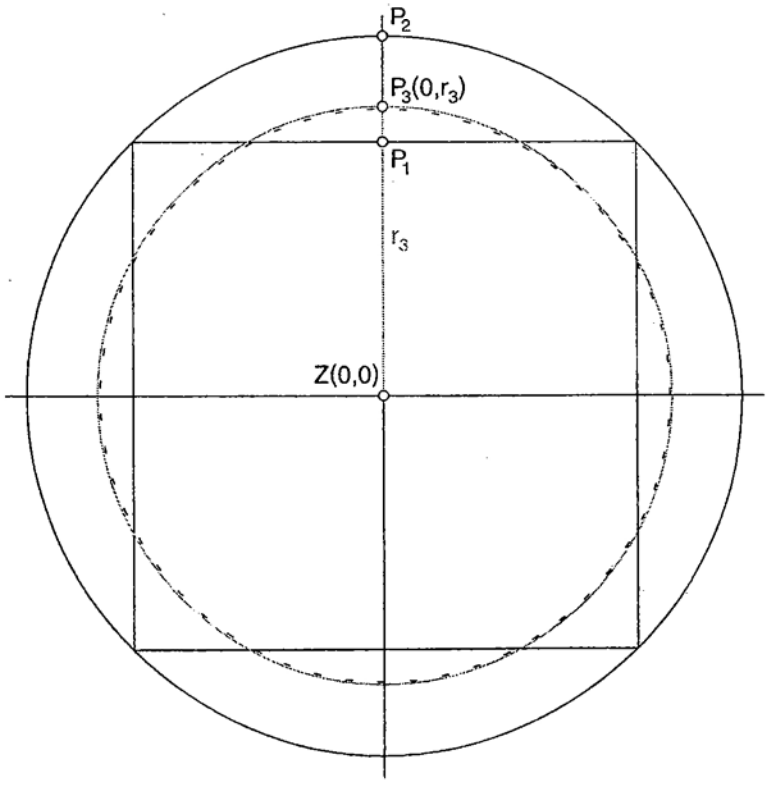
'Circle a square' is the inverse problem to the classic task 'square a circle': find the radius of a circle which has the same area as a square.

This doc shows four methods. Method A and method B as published by Volker Hoffmann on p.39 of this exhibition catalogue [1]. Method A is very old, it is hardly possible to date the first publication.
Unfortunately, the accuracies of the approximations were indicated wrong by a factor of hundred (percentage). The new method B itself is rather accurate though, as proved by the mathematical derivation.
Method C is not as simple as method B and less accurate.
Method D is the most accurate solution, by courtesy of Mr.R.S.J.Reddy [4].

2. Method A

The square has the edge length two units.

1. Draw the black circle about the center Z
2. Divide P_1,P_2 by factor 3, using compass and ruler
3. Find P_3
4. The red circle about Z through P_3 with radius r_3 is the circled square
   The black dashed circle shows the accurate solution



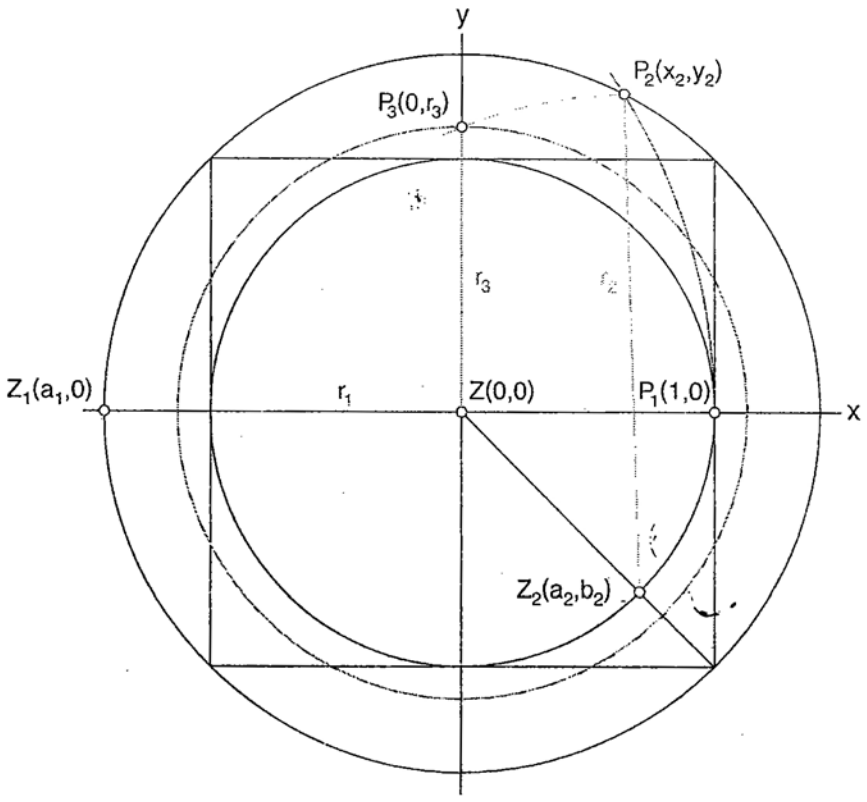
Square-Area / Circle-Area = 0.98304016, equivalent to an accuracy of 1.7%

3. Method B

This is the method by Nikolaos Theocharis and Volker Hoffmann.

The square has the edge length two units.

1. Draw the two black circles about the center Z
2. Draw the blue circle about Z_1 through P_1
3. Find the intersection P_2
4. Draw the green circle about Z_2 through P_2
5. Find the intersection P_3 at x=0
6. The red circle about Z through P_3 with radius r_3 is the circled square
   The black dashed circle shows the accurate solution



Square-Area/Circle-Area = 1.00212785, equivalent to an accuracy of 0.21%

4. Method B / Mathematics

The calculation is straightforward. The intersection of two overlapping circles delivers two solutions. The proper one was chosen.

Edge length of the square: 2 units

$$a_1 = -\sqrt{2}$$
$$r_1 = +\sqrt{2}+1$$

Black and blue circle:

$$x^2 + y^2 = a_1^2$$
$$(x - a_1)^2 + y^2 = r_1^2$$

Subtract first equation from second and solve for x_2, y_2:

$$x_2 = \frac{2a_1^2 - r_1^2}{2a_1}$$
$$y_2 = \sqrt{a_1^2 - x_2^2}$$

Green circle:

$$a_2 = +\sqrt{2}/2$$
$$b_2 = -\sqrt{2}/2$$

$$(x - a_2)^2 + (y - b_2)^2 = r_2^2$$
$$r_2^2 = (x_2 - a_2)^2 + (y_2 - b_2)^2$$

$$x_3 = 0$$
$$y_3 = r_3$$

$$(-a_2)^2 + (r_3 - b_2)^2 = r_2^2$$

Red circle:

$$r_3 = b_2 + \sqrt{r_2^2 - a_2^2}$$

Ratio of areas:

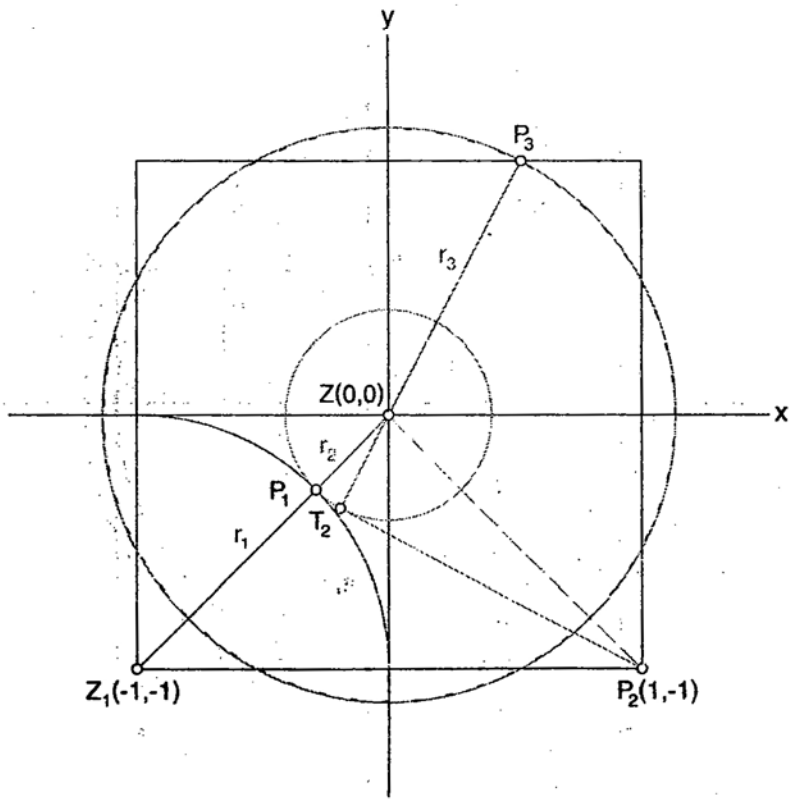
$$\text{Square / Circle} = 4/(\pi r_3^2)$$

5. Method C

This method by Tan Tai Nguyen [3] is an approximation as well. The author seems to believe that his solution is accurate.
Neither for 'circling a square' nor for 'squaring a circle' an accurate solution by ruler and compass exists.

The square has the edge length two units.

1. Draw the blue circle about Z_1
2. Draw the diagonal and find P_1
3. Draw the green circle about Z through P_1
4. Find the tangent point T_2 from P_2 for the green circle (*Thales* circle over P_2,Z)
5. Draw the line through T_2 and Z and find the intersection P_3
6. The red circle about Z through P_3 with radius r_3 is the circled square
   The black dashed circle shows the accurate solution
   Note: the radius r_3 is the longer part of T_2,P_2, intersected at x=0, as well



Square-Area/Circle-Area = 0.99318851, equivalent to an accuracy of 0.68%

6. Method D

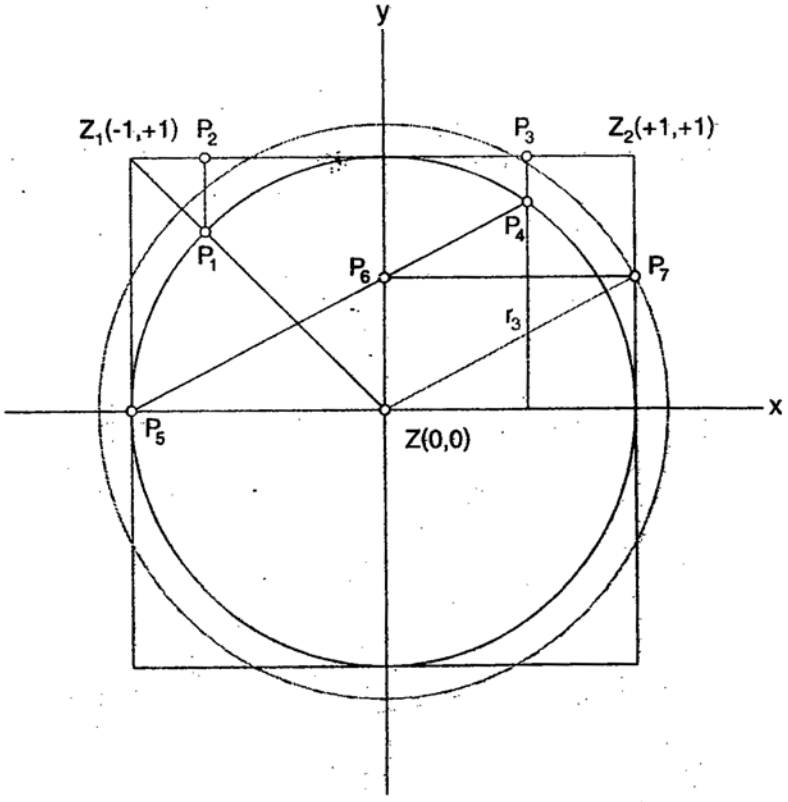
This method by R.S.J.Reddy [4] is an approximation as well. The author wants to replace the common value for π by an algebraic irrational expression π

$\pi = 3.14159265$
$\pi' = 3.14644661 = (14-\sqrt{2})/4$
$r_3 = \sqrt{16/(14-\sqrt{2})}$

This is a good approximation and delivers the best solution so far. The drawing below is a little modified, but essentially based on the original author's ideas as published: a cute method how to establish π' by elementary geometrical constructions.

The square has the edge length two units.

1. Draw the black circle about Z
2. Draw the diagonal and find P_1
3. Find P_2 in vertical direction
4. Divide Z_2P_2 into four parts and find P_3 at the first quarter
5. Go down to P_4 on the black circle
6. Draw the line through P_4 to P_5
7. Find the intersection P_6
8. Project P_6 to P_7. The delivers the radius, which is again called r_3.



Square-Area/Circle-Area = 1.001545, equivalent to an accuracy of 0.15%

7. Method D / Mathematics

This proof (by G.Hoffmann) is not elegant but hopefully correct. For better readability the graphic is shown again.

$x_1 = -\sqrt{2}/2$
$x_2 = x_1$
$x_3 = 1-(1+\sqrt{2}/2)/4 = (6-\sqrt{2})/8$
$x_4 = x_3$
$1+x_4 = (14-\sqrt{2})/8$

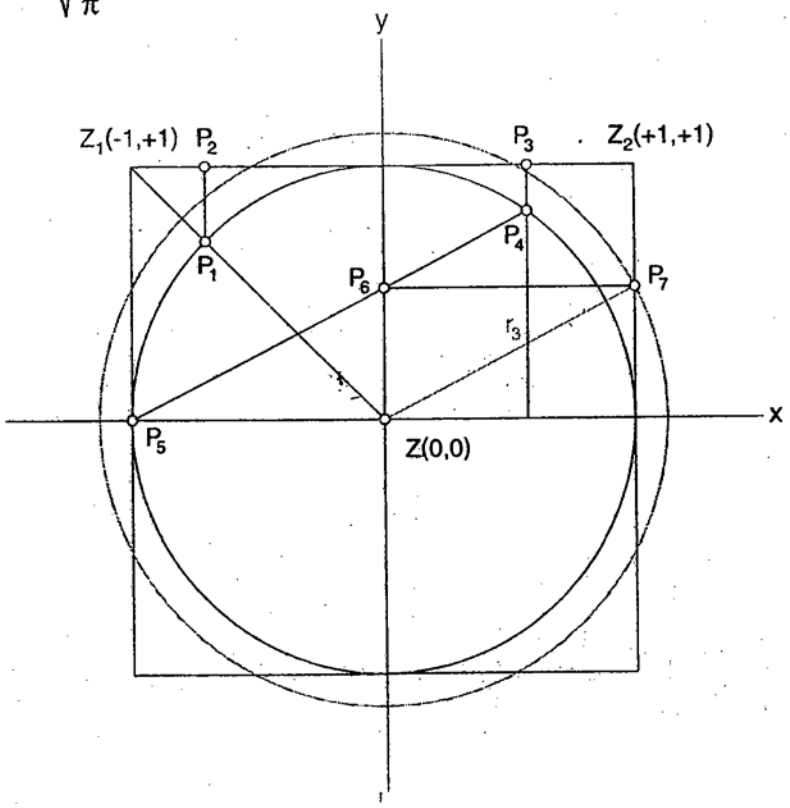
Proportion
$y_4 / y_6 = (1+x_4)/1$
$y_6 = y_4/(1+x_4)$

Circle
$y_4^2 = 1-x_4^2$

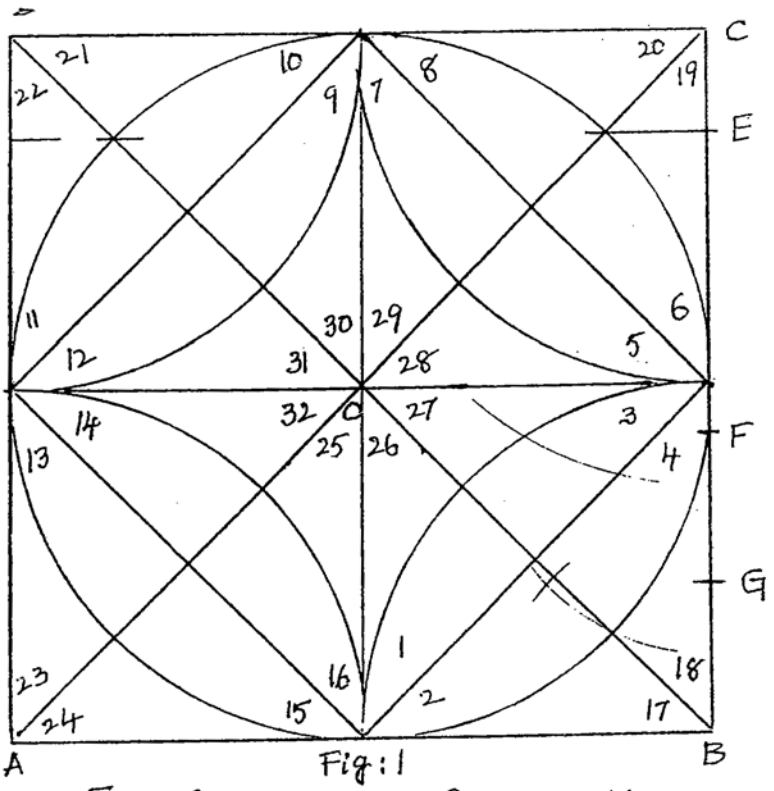
Pythagoras
$r_3^2 = 1+y_6^2 = 1+y_4^2/(1+x_4)^2$
$= 1+(1-x_4^2)/(1+x_4)^2$
$= 2/(1+x_4)$

$$r_3 = \sqrt{\frac{16}{14-\sqrt{2}}} = 2\sqrt{\frac{1}{\pi'}}$$

True radius
$$r = 2\sqrt{\frac{1}{\pi}}$$



RECTANGULARIZED ARCUAL SEGMENTS AND EXACTNESS BETWEEN TWO CONSTRUCTIONS IN AREA & PERIMETER



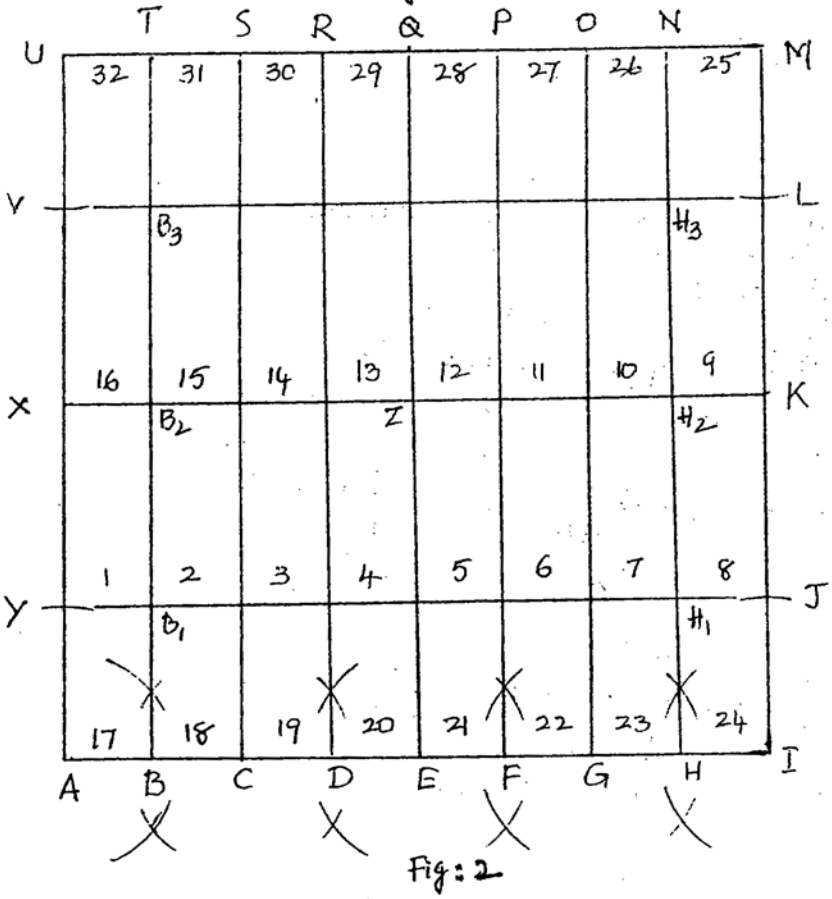


Fig.1: Square ABCD. Inscribe a circle.

Side = diameter = 1

Divide the square into 32 segments as shown in the Figure1.

$$CE = \frac{2-\sqrt{2}}{4}$$

$$EB = \frac{2+\sqrt{2}}{4}$$

Bifurcate EB twice

$$FG = GB = \frac{2+\sqrt{2}}{16}$$

$$CG = \frac{14-\sqrt{2}}{16} = \frac{\pi}{4}$$

Figure.2:

Square: AIMU. Side = 1. Divide AI and UM into 8 equal parts. Each part = $\frac{1}{8}$

IJ = ML of Fig.2 = BG of Fig.1 = $\frac{2+\sqrt{2}}{16}$

Bifurcate JL. JK = KL = $\frac{6-\sqrt{2}}{16}$

Square AIMU is divided into 32 rectangles. There are two types of rectangles in the square.

AB = BC = CD = DE = EF = FG = GH = HI = MN = NO = OP = PQ = QR = RS = ST = TU = $\frac{1}{8}$

Rectangle Type- I

$HIJH_1 = H_3LMN = (8 + 8 = 16)$

Side I = HI = $H_3L = \frac{1}{8}$

Side II = IJ = LM = $\frac{2+\sqrt{2}}{16}$

Area of each rectangle = $\frac{1}{8}\left(\frac{2+\sqrt{2}}{16}\right) = \frac{2+\sqrt{2}}{128}$ = 17 to 32 (16) of Fig.1.

Total area of 16 rectangles = $\frac{2+\sqrt{2}}{8}$

Rectangle Type- II

$H_1IKH_2 = H_2KLH_3 = (8 + 8 = 16)$

Side I = $H_1J = H_2K = \frac{1}{8}$

Side II = JK = KL = $\frac{6-\sqrt{2}}{16}$

Area of each rectangle = $\frac{1}{8}\left(\frac{6-\sqrt{2}}{16}\right) = \frac{6-\sqrt{2}}{128}$ = 1 to 16 of Fig.1.

Total area of 16 rectangles = $\frac{6-\sqrt{2}}{8}$

Area of square AIMU = Type I & II = $\frac{2+\sqrt{2}}{8} + \frac{6-\sqrt{2}}{8} = 1$

Larger rectangle

Area of the larger rectangle YJMU

Side I =·1 = YJ = MU = 1

Side II = JK + KL + LM = $\frac{6-\sqrt{2}}{16}+\frac{6-\sqrt{2}}{16}+\frac{2+\sqrt{2}}{16}=\frac{14-\sqrt{2}}{16}$

Area = Side I & Side II = $\frac{14-\sqrt{2}}{16}$

Area of the inscribed circle of Fig.1. = $\frac{14-\sqrt{2}}{16}$

Smaller rectangle

Area of the smaller rectangle YJLV

Side I = YJ = LV = 1

Side II = JL = YV = $\frac{6-\sqrt{2}}{8}$

Area = Side I & Side II = $\frac{6-\sqrt{2}}{8}$

Area of 1 to 16 of arcual segments of Fig.1 = $\frac{6-\sqrt{2}}{8}$

Perimeter of YJLV

Perimeter = YJ + JK + KL + LV + VX + XY =

$$1+\frac{6-\sqrt{2}}{16}+\frac{6-\sqrt{2}}{16}+1+\frac{6-\sqrt{2}}{16}+\frac{6-\sqrt{2}}{16}=\frac{14-\sqrt{2}}{4}$$

Circumference of the inscribed circle = $\frac{14-\sqrt{2}}{4}$

Total area of 1 to 16 arcual segments of Fig.1 and the total area of 1 to 16 rectangles of Fig.2 are equal. Similarly, the outer edge of peripheral arcual segments 2, 4, 6, 8, 10, 11 13 & 15 and the perimeter of the smaller rectangle YJLV are exactly equal. Same area and same perimeter.

Circle from Square and Vice Versa

$$\frac{\text{Perimeter of Square}}{\text{Circumference of Circle}} = \frac{4a}{\pi d} \qquad \text{where } a = d$$

Classical π Value = 3.1415926...

Proposed π Value = 3.1464466... $= \frac{1}{4}\left(14-\sqrt{2}\right)$

We are yet to get a geometrical length equal to 3.1415926...

From a simple geometrical construction we can get 1/4th of circumference or $\frac{\pi}{4}$ where diameter is equal to 1.

$$\frac{4a}{\pi a} = \frac{4}{\frac{1}{4}\left(14-\sqrt{2}\right)} = \frac{16}{14-\sqrt{2}} = 1.2712753...$$

If we can get a length equal to 1.2712753 we can get a square whose side is 1 and $\frac{\pi}{4} = \frac{14-\sqrt{2}}{16}$ equal to one fourth of perimeter of square and one forth of circumference of circle respectively.

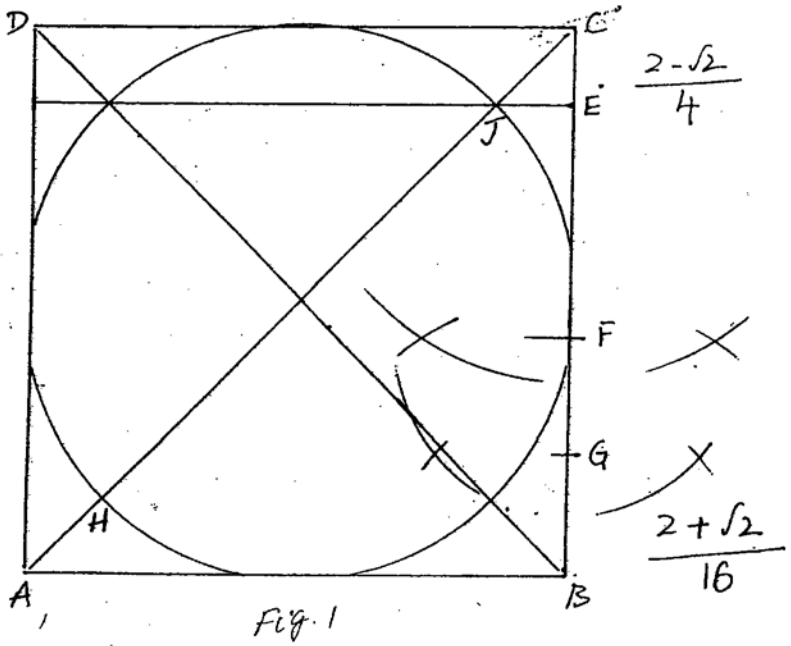


Fig. 1

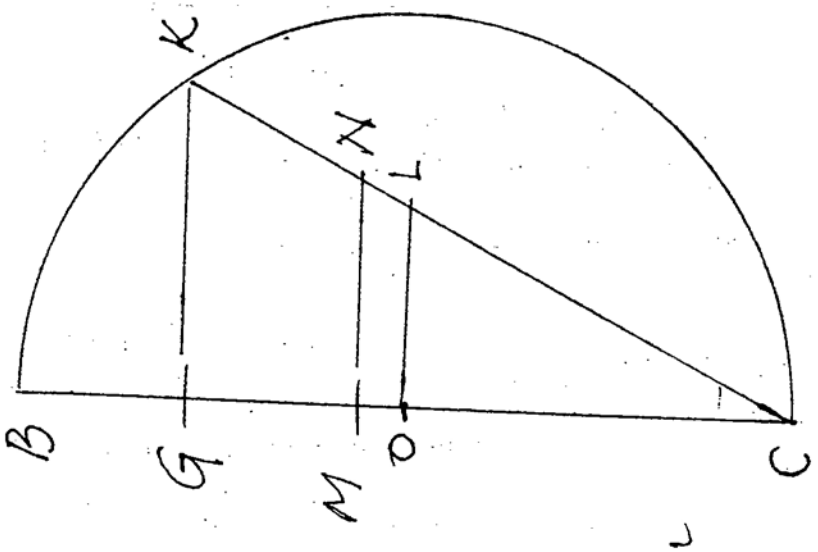


Fig. 2

AB = Side = HJ = Diameter = 1

$CE = \frac{2-\sqrt{2}}{4}$ BE bisected twice. $BG = \frac{2+\sqrt{2}}{16}$

$CG = \frac{14-\sqrt{2}}{16}$ = 0.7866116... = 1/4th of Circumference = $\frac{\pi}{4}$

BC = 1

BO = OC = 0.5

$CG = \frac{14-\sqrt{2}}{16}$

$BG = \frac{2+\sqrt{2}}{16}$

Altitude theorem

$GK = \sqrt{\frac{26+12\sqrt{2}}{256}}$

Δ KGC

Pythagorean theorem

$$KC = \frac{\sqrt{14-\sqrt{2}}}{4} = 0.886911299...$$

Perpendicular at 0 on BC meets L. Similar triangles to get CL

$$CL = \sqrt{\frac{4}{14-\sqrt{2}}} = 0.563754233...$$

CL = CM

Perpendicular at M on BC meets N. Similar triangles to get CN

$$CN = \frac{8}{14-\sqrt{2}} = 0.635637672...$$

$$2 \text{ X } CN = \frac{4a}{\pi d} = \frac{16}{14-\sqrt{2}} = 1.2712753...$$

Ratio of square and circle

Square- II

$$\text{Side} = \frac{16}{14-\sqrt{2}}$$

Repeat the entire process of Figure 1 to get a geometrical length equal to 1

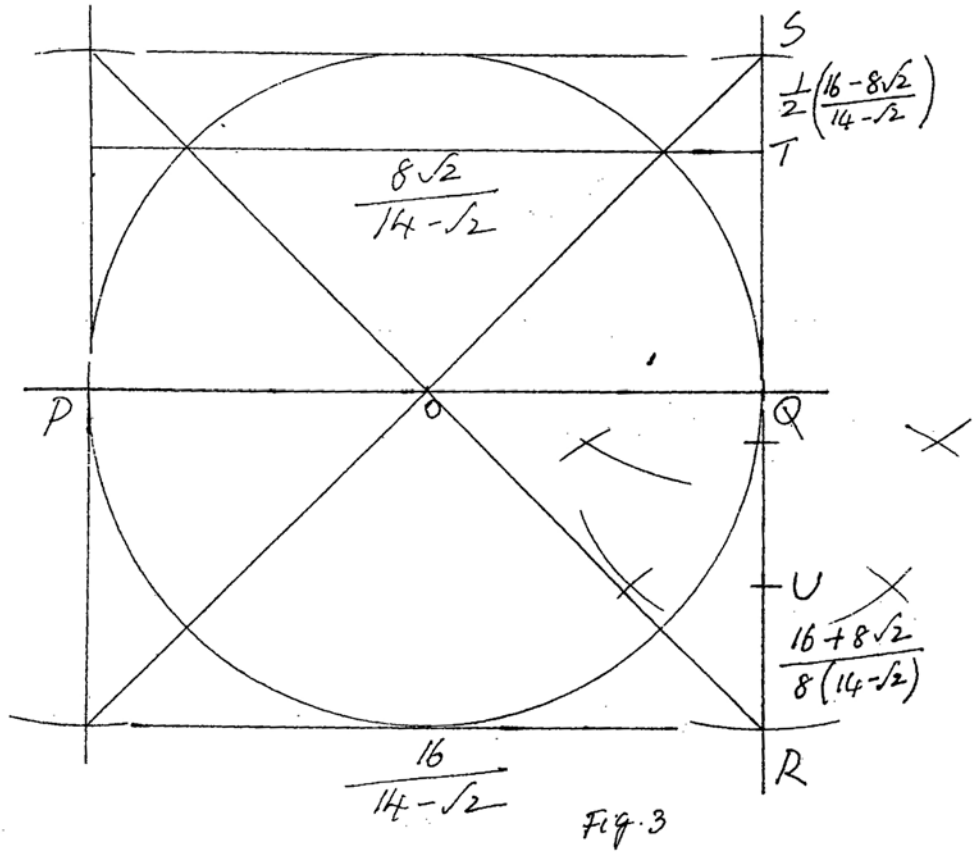


Fig. 3

Draw a square

$$PQ = 2 \times CN = \frac{16}{14-\sqrt{2}} = 1.2712753...$$

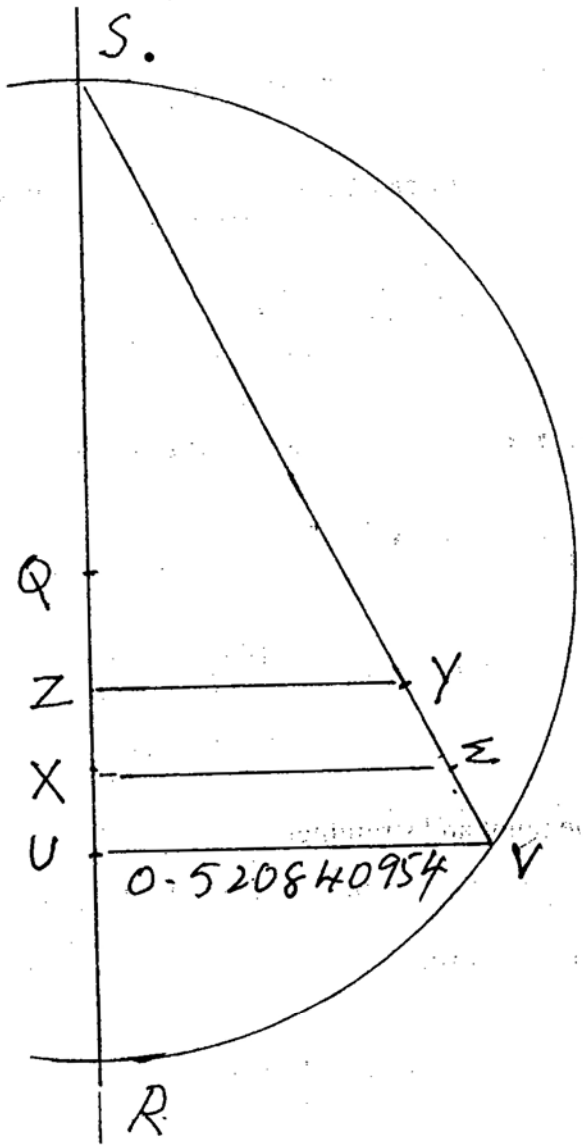
Draw two diagonals. Inscribe a circle

$$ST = \frac{1}{2}\left(\frac{16-8\sqrt{2}}{14-\sqrt{2}}\right) = 0.186173963...$$

Bisect TR twice

$$UR = \frac{16+8\sqrt{2}}{8\left(14-\sqrt{2}\right)} = 0.2712753...$$

SU = 1



$$SR = \frac{16}{14-\sqrt{2}}$$

$$SQ = QR = \frac{8}{14-\sqrt{2}}$$

$$SU = 1$$

$$UR = \frac{16+8\sqrt{2}}{8\left(14-\sqrt{2}\right)}$$

Altitude theorem to get UV

$$UV = \sqrt{\frac{16+8\sqrt{2}}{8\left(14-\sqrt{2}\right)}}$$

Δ VUS

Pythagorean theorem

$$VS = \sqrt{\frac{16}{14-\sqrt{2}}}$$

$$= 1.127508448...$$

SU = SW = 1

Join WX

Apply similar triangles to get SX

$$SX = \frac{\sqrt{14-\sqrt{2}}}{4} = 0.886911314...$$

SX = SY

Join YZ

Apply similar triangles to get SZ

$$0.7866116... = SZ = \frac{14-\sqrt{2}}{16} = \frac{\pi}{4} = \frac{1}{4}$$ Circumference of circle

Results:

The following are the most sought after

1. **Circle of unit area** CL = $\sqrt{\frac{4}{14-\sqrt{2}}}$ = radius

$$\text{Area} = \pi r^2 = \frac{14-\sqrt{2}}{4}\left(\sqrt{\frac{4}{14-\sqrt{2}}}\right)^2 = 1$$

2. **Circumference = Perimeter of square**

$$CN = \frac{8}{14-\sqrt{2}} = \text{radius}$$

$$\text{Circumference} = 2\pi r = 2\times\frac{14-\sqrt{2}}{4}\times\frac{8}{14-\sqrt{2}} = 4$$

(Circling a square)

3. **Circle of Unit area**

Half of VS of Fig. 4 = CL of Fig.2

4. **Squaring a circle**

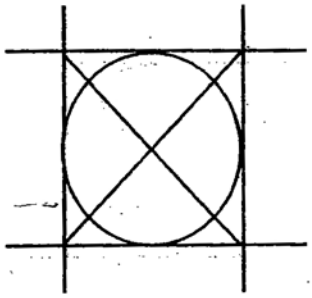
$$SX = \frac{\sqrt{14-\sqrt{2}}}{4} = \text{Side} = KC \text{ of Fig.2}$$

$$\text{Area of square} = SX^2 = \left(\frac{\sqrt{14-\sqrt{2}}}{4}\right)^2 = \frac{14-\sqrt{2}}{16}$$

π VALUE FROM THE CIRCUMFERENCE OF A CIRCLE

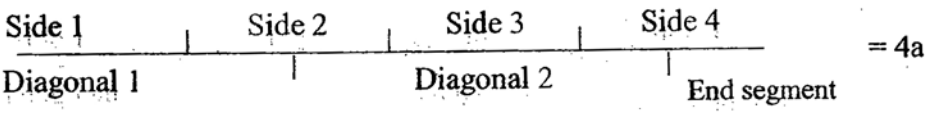
In Siva method the total area of the circle is calculated first and π value derived out of the total area. In this method the length of the circumference of a circle is arrived at first and π value derived from the value of circumference.

Method



Draw a circle with radius $\frac{a}{2}$ where 'a' is a diameter. Draw four equidistant tangents on the circle. They intersect at four points creating a square whose side is also 'a'. Connect four corners of the square with two diagonals. The length of each diagonal is $\sqrt{2}a$. The perimeter of the square is 4a.

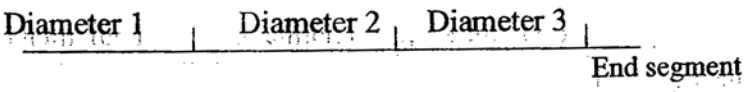
I. Cut at one point and straighten the perimeter of the square.



Perimeter – sum of the 2 diagonals = End segment of the perimeter (esp)

$$esp = 4a - 2\sqrt{2}\,a$$

II. Cut at one point and straighten the circumference of the circle



Till yesterday, the length of the end segment of the circumference (esc) was considered as equal to 0.1415926..... obtained from the Classical method and a host of infinite series calculated with the help of super computers. However, it is still an approximation. Hence, a new approach to get an exact value of the end segment of the circumference is tried here.

Let, the length of the circumference = x

Circumference–3 diameters = end segment of the circumference (esc) = $x - 3a$

III. We have seen above that the square is an outcome of four equidistant tangents on the circumference. Hence the diameter and side are one and the same i.e., 'a' and radius = $\frac{a}{2}$

r1 | r2 | r3 | r4 | r5 | r6 | r7 | r8 | = 4a

The length of the perimeter of the square (4a) is always (constant) eight times of the length of the radius $\left(\frac{a}{2}\right)$ of its inscribed circle.

Radius of the inscribed circle = $\frac{\text{Perimeter of the superscribed square}}{8}$

IV. The above constancy is extended to the relation between the end segment of the perimeter (esp) and the end segment of the circumference (esc).

"As radius is to the perimeter of the square $\left(\frac{1}{8}\text{th of it}\right)$*, so also, is the end segment of the circumference, to the end segment of the perimeter.*

$$esc = \frac{esp}{8} = x - 3a = \frac{4a - 2\sqrt{2}a}{8}$$

$$\therefore x = \frac{14a - \sqrt{2}a}{4} = \text{length of the circumference of the circle.}$$

$$esc = \frac{2 - \sqrt{2}}{4} = 0.1464466\ldots \text{ (when the diameter-1)}$$

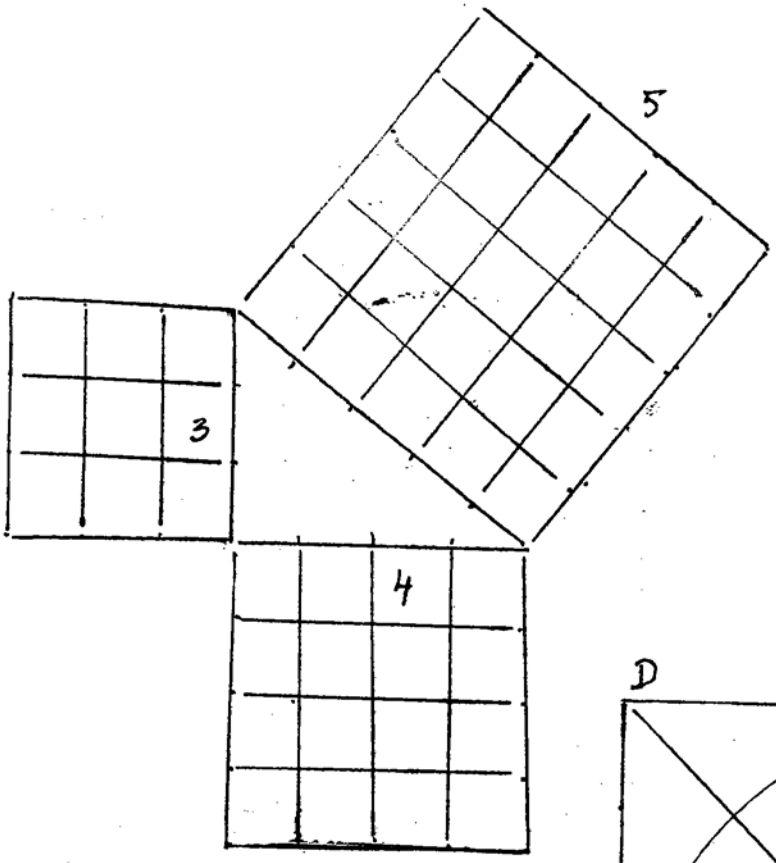
$$circumference = \pi d \text{ or } \pi a = \frac{14a - \sqrt{2}a}{4} \text{ (where a = d)}$$

$$\therefore \pi = \frac{14 - \sqrt{2}}{4}$$

3.1415926..... is only a number which does not exist in the circle but it exists as representing a variable length of the perimeter of the inscribed polygon of increasing number of sides of it, in a circle of Classical method of Archimedes, as an irrational and transcendental number. Whereas new one: 3.1464466.... exists in circle and thus is exact, finite, irrational and algebraic representing π constant.

Geometrical confirmation to the method of the area of a circle

Proof for the Pythagorean theorem



$3^2 + 4^2 = 5^2$

Thus constituent entities of two sides = constituent entities of hypotenuse

Area method of the circle

Square = ABCD

Side = AB = 1 = EF = diameter

Areas of C_1 and C_2

$$C_1 = \frac{6-\sqrt{2}}{128} \quad C_2 = \frac{2+\sqrt{2}}{128}$$

$16C_1 + 16C_2$ = Area of Square

$$16\left(\frac{6-\sqrt{2}}{128}\right) + 16\left(\frac{2+\sqrt{2}}{128}\right) = 1$$

$16C_1 + 8C_2$ = Area of circle

$$16\left(\frac{6-\sqrt{2}}{128}\right) + 8\left(\frac{2+\sqrt{2}}{128}\right) = \frac{14-\sqrt{2}}{16}$$

Square has 32 constituent parts

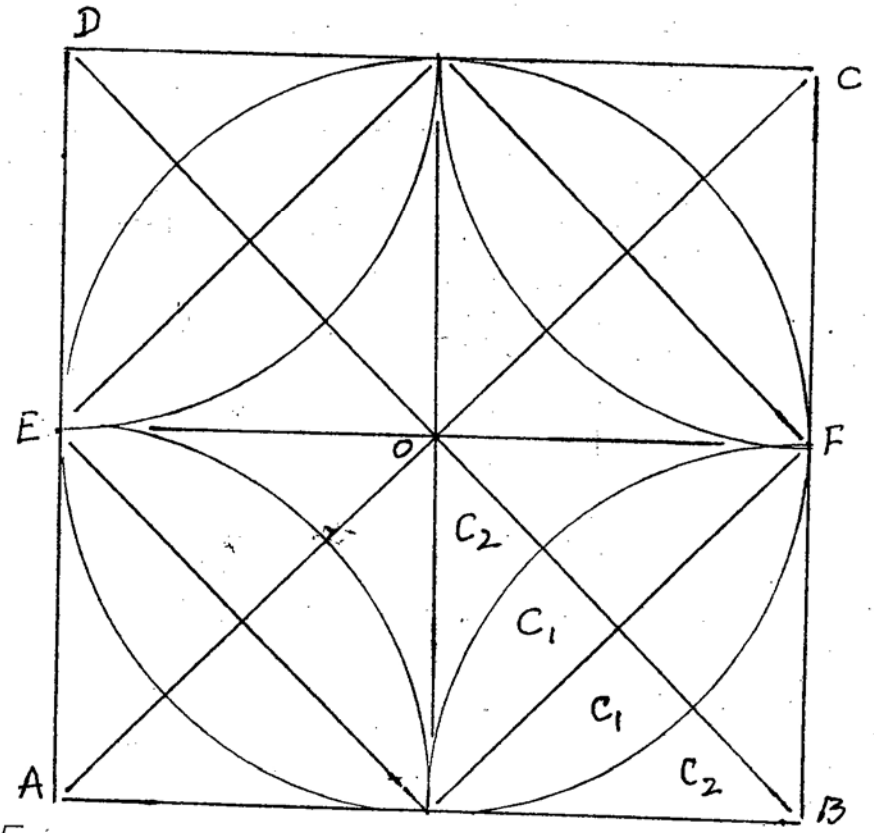


Fig:1

Geometrical Confirmation

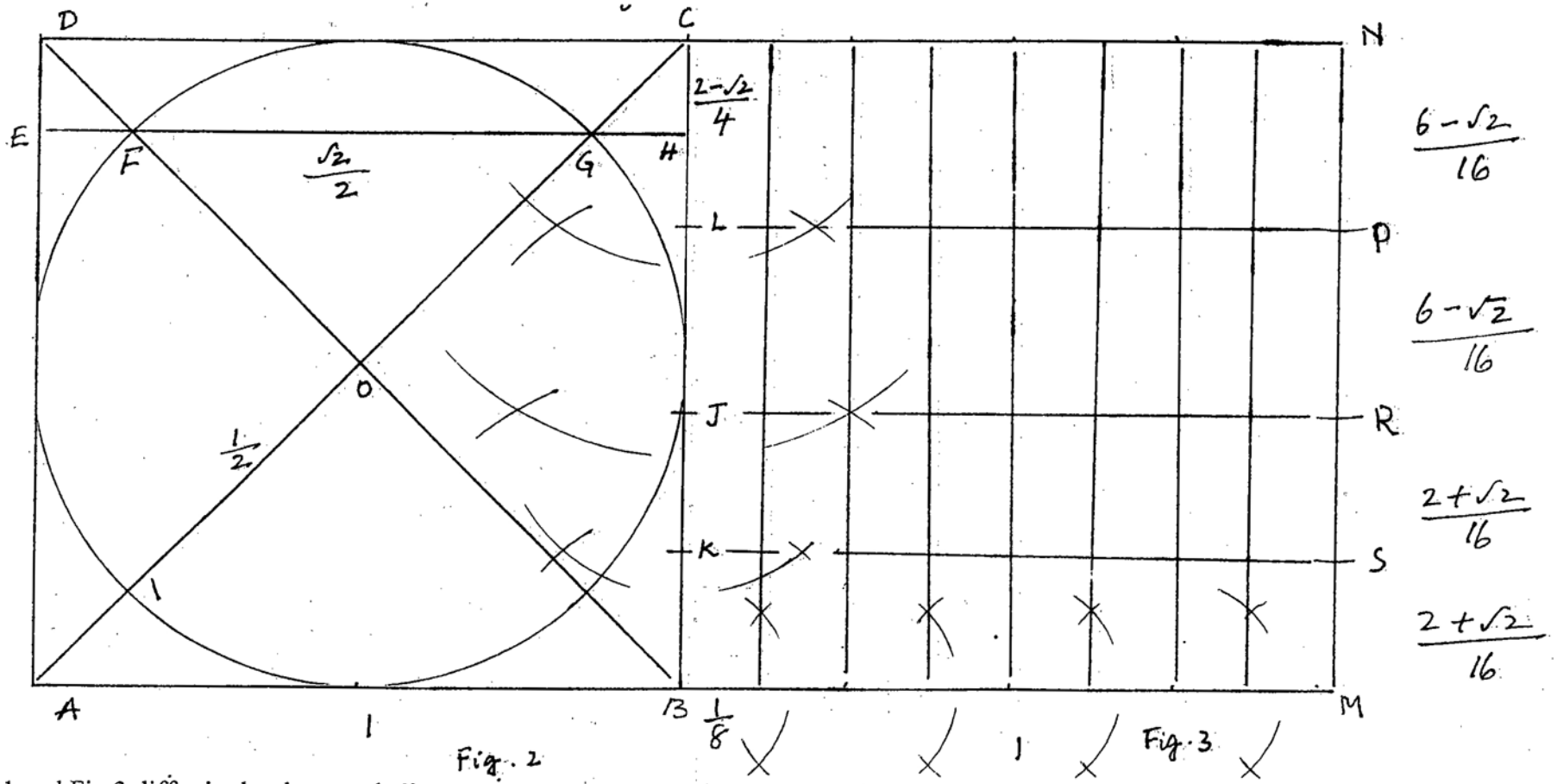


Fig 1 and Fig 3 differ in the shape and alignment of constituent parts. It is because of curvature lines of Fig 1. have totally been straightened in the Fig. 3.

Fig: 2

Square ABCD, side AB = 1 = diameter = 1 G

AC = BD = $\sqrt{2}$ OG = OF = radius = 0.5

$FG = \frac{\sqrt{2}}{2}$ $DE = EF = GH = HC = \frac{EH - FG}{2}$

$\frac{1 - \frac{\sqrt{2}}{2}}{2} = \frac{2 - \sqrt{2}}{4}$ $BC - HC = BH = 1 - \frac{2 - \sqrt{2}}{4} = \frac{2 + \sqrt{2}}{4}$

Bifurcate BH $HJ = JB = \frac{2 + \sqrt{2}}{8}$ Bifurcate JB $JK = KB = \frac{2 + \sqrt{2}}{16}$

$CK = \frac{1}{4}^{th}$ circumference $= \frac{\pi d}{4} = BC - BK = 1 - \frac{2 + \sqrt{2}}{16} = \frac{14 - \sqrt{2}}{16}$

Also this is the area of the circle $\frac{\pi d^2}{4}$

$CJ = BC - JB = 1 - \frac{2 + \sqrt{2}}{8} = \frac{6 - \sqrt{2}}{8}$

Bifurcate CJ CL = LJ = $\frac{6 - \sqrt{2}}{16}$

Fig 3: Square has 32 constituent parts,

Square BMNC Side = BM = 1 Divide BM into 8 equal segments

each segment is $= \frac{1}{8}$

Rectangle BMRJ has 16 equal smaller rectangles.

Each equal to in area $\left(\frac{2 + \sqrt{2}}{16}\right)\left(\frac{1}{8}\right) = \frac{2 + \sqrt{2}}{128}$

This is also the area of C2 entities of Fig. 1.

Rectangle JRNC has similarly 16 equal smaller rectangles and each equal to in area

$\left(\frac{6-\sqrt{2}}{16}\right)\left(\frac{1}{8}\right)=\frac{6-\sqrt{2}}{128}$ This is also the area of C1 entities of Fig. 1.

Area of KSNC rectangle $=1\times\left(\frac{6-\sqrt{2}}{16}+\frac{6-\sqrt{2}}{16}+\frac{2+\sqrt{2}}{16}\right)=\frac{14-\sqrt{2}}{16}$

which is also the area of the circle.

Perimeter of BMNC square is = 4.

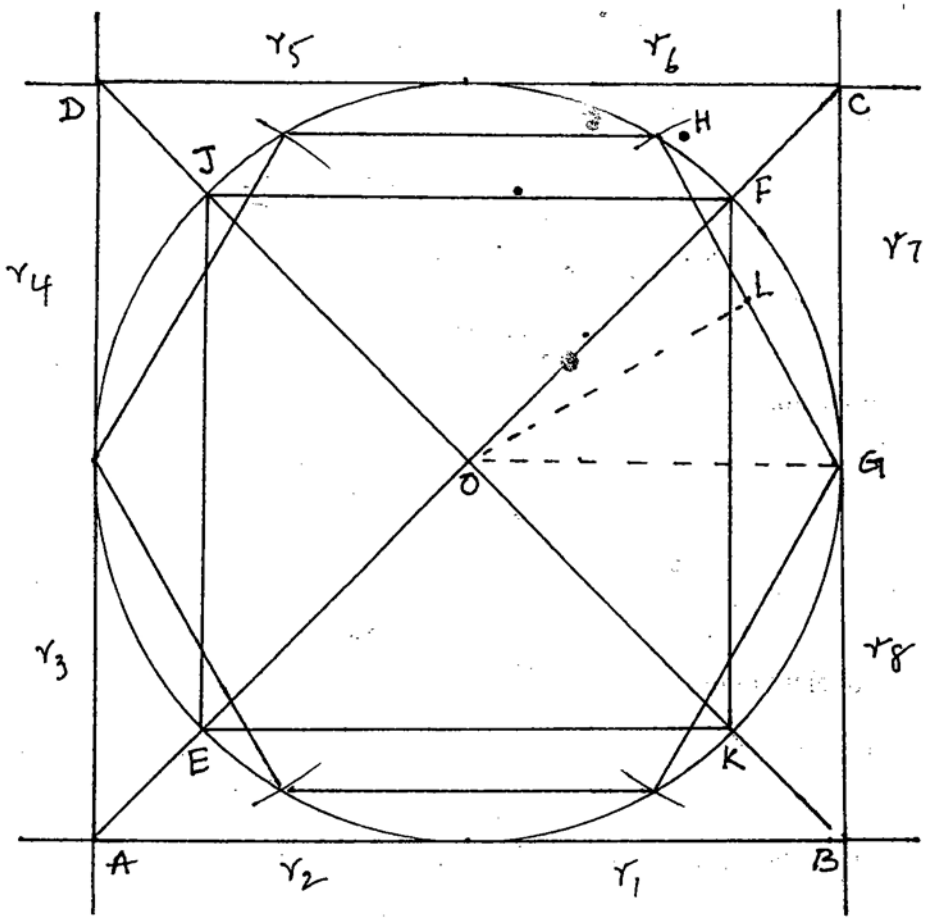
Perimeter of JRNC rectangle = JR + RP + PN +NC+CL+LJ

$=1+\frac{6-\sqrt{2}}{16}+\frac{6-\sqrt{2}}{16}+1+\frac{6-\sqrt{2}}{16}+\frac{6-\sqrt{2}}{16}=\frac{14-\sqrt{2}}{4}$

which is also the circumference of the circle.

Thus, constituent entities of Fig. 1 = constituent entities of Fig.3.

7th Radius method in the computation of π Value



O = centre of the circle

OF = radius EF = diameter

Circle has four parts: centre, radius, diameter and circumference

Four equidistant tangents on the circumference result in the creation of square ABCD

AC = BD = diagonals ABCD= Perimeter

Square has five parts:

O, intersecting point of two diagonals

A,B,C and D, intersecting points of four tangents

Two.diagonals

Four sides and perimeter

Two diagonals intersect circumference at E, K, F and J and result in the creation of another square EKFJ.

ABCD=Superscribed square of the circle

EKFJ = Inscribed square of the circle

Six equidistant chords result in the creation of a hexagon.

Finally this composite geometrical system has four geometrical constructions; circle, superscribed square, inscribed square and inscribed hexagon.

Dimensions

Radius = r = $\frac{a}{2}$ Diameter = d = 1 = a

Circumference = x

Side of the superscribed square = a = d = 1

Diagonal $\sqrt{2}\,a$ perimeter $4 \times a = 4a$

Side of the inscribed square $\frac{\sqrt{2}a}{2}$

Perimeter = $= 4\left(\frac{\sqrt{2}a}{2}\right)$

Hexagon: Chord = radius = $\frac{a}{2}$ perimeter $6 \times \frac{a}{2}$.

Relations

1. $\frac{\text{Perimeter of the superscribed square}}{\text{Perimeter of the inscribed square}} = \frac{4a}{2\sqrt{2}a} = \sqrt{2}$

2. $\frac{\text{Diagonal of the superscribed square}}{\text{Diameter of the inscribed circle}} = \frac{\sqrt{2}a}{a} = \sqrt{2}$

Thus circle and square are related by $\sqrt{2}$

Hexagon and circle

$$GH = \text{Chord} = \text{radius} = \frac{a}{2} \qquad GL = \frac{a}{4}$$

$$OG = \text{radius} = \frac{a}{2}$$

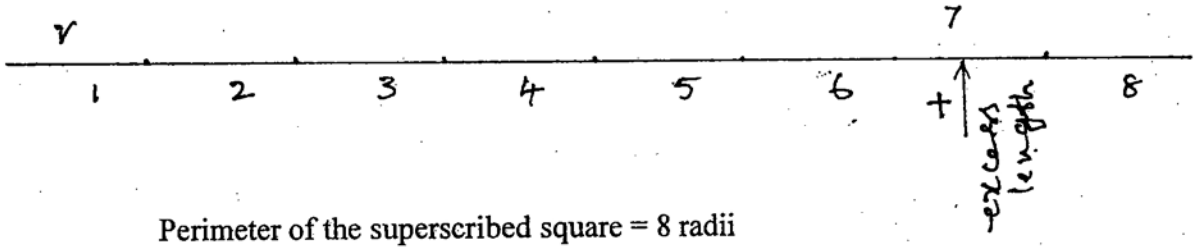
In Triangle OLG, $$OL = \frac{\sqrt{3}a}{4}$$

So, the circle by being in relation with the square and hexagon has proved itself that surds like $\sqrt{2}$ and $\sqrt{3}$ are integral part of the circle invisibly.

The length of the circumferenc

The perimeter of the inscribed hexagon is equal to the sum of the lengths of six radii. The perimeter of the superscribed square is equal to the sum of the lengths of eight radii.

As the circumference is in between the inscribed hexagon and superscribed square, the length of it is in between six radii and eight radii. Therefore, the end point (beginning with 1st radius) of the length of the circumference is some where in the seventh radius of the perimeter of the superscribed square.



Perimeter of the superscribed square = 8 radii

Perimeter of the hexagon = 6 radii

Circumference of the circle = x = 6 +

Excess length = 7 radii – 6+

The length of the circumference is the sum of the lengths of 7 radii and minus excess length of 7th radius.

Reasoning to obtain the excess length:

Any part of the square to be converted into part of its inscribed circle or square, we have to divide that part of the superscribed square by the surd $\sqrt{2}$.

For example

1. Perimeter of the inscribed square =

$$\frac{\text{Perimeter of the superscribed square}}{\sqrt{2}} = \frac{4a}{\sqrt{2}} = 2\sqrt{2}a$$

2. Diameter of the inscribed circle $= \frac{\text{Diagonal of the superscribed square}}{\sqrt{2}} = \frac{\sqrt{2}a}{\sqrt{2}} = a$

With the above two examples and applying the same reasoning to obtain the excess length of the 7^{th} radius, we have to divide the total length of the radius by $\sqrt{2}$.

Radius $= \frac{a}{2}$ diameter = side = a = d

$$\frac{\text{Radius}}{\sqrt{2}} = \frac{\frac{a}{2}}{\sqrt{2}} = \frac{\sqrt{2}a}{4} = \text{excess length in the } 7^{th} \text{ radius}$$

The length of the circumference = sum of the lengths of 7 radii – excess length in the 7^{th} radius.

$$= 7r - \frac{\sqrt{2}a}{4} = \frac{7a}{2} - \frac{\sqrt{2}a}{4} \qquad \text{where } r = \frac{a}{2}$$

Well known formula for computing the length of the circumference of a circle is π d or $2\pi r$ where d= diameter

r = radius

π = constant -= 3.1415926.....

The accepted π value is 3. 1415926.....

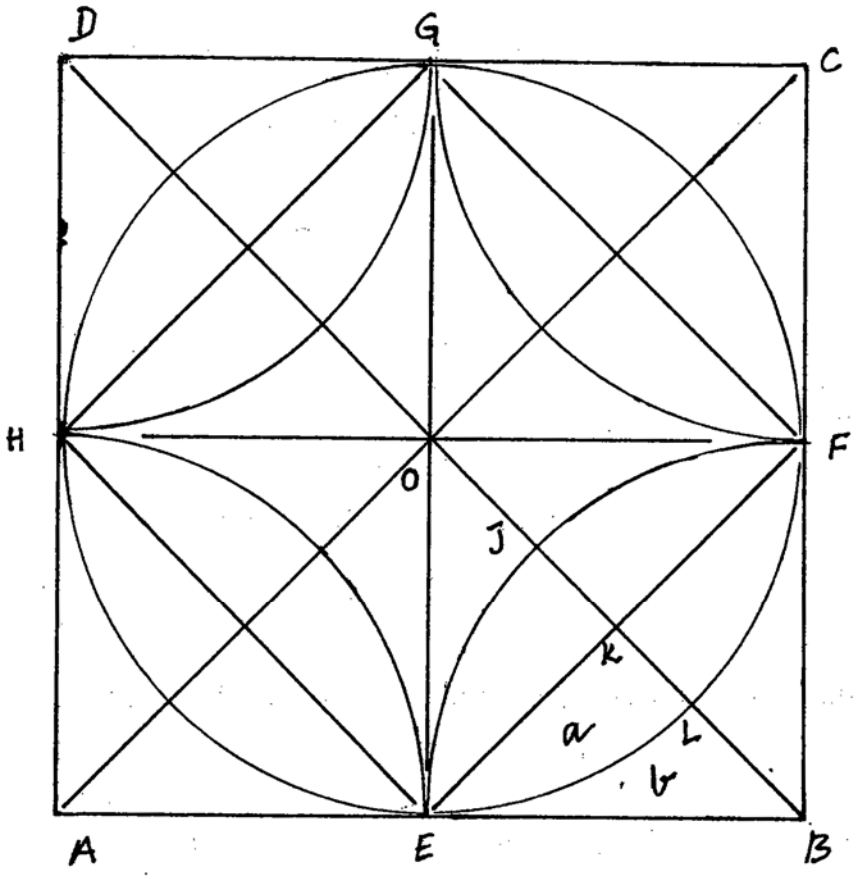
However, this method gives a different value to the constant π as shown below

$$\pi d = \pi a = \frac{7a}{2} - \frac{\sqrt{2}a}{4} = \text{Circumference}$$

$$\therefore \pi = \frac{14 - \sqrt{2}}{4} = 3.1464466....$$

Surd-based divided-triangle method

in the computation of π value



Square = = ABCD

AB = 1 = side

Circle = EFGH

HF = diameter = 1

Diagonal AC = BD= $\sqrt{2}$

Radius = OL = 0.5 = $\frac{1}{2}$

OB = $\frac{\sqrt{2}}{2}$

OK = KB = = $\frac{\sqrt{2}}{4}$

EK = KF = $\frac{\sqrt{2}}{4}$

KL = OL – OK =

$\frac{1}{2} - \frac{\sqrt{2}}{4} = \frac{2-\sqrt{2}}{4}$

LB = OB – OL

$\frac{\sqrt{2}}{2} - \frac{1}{2} = \frac{\sqrt{2}-1}{2}$

EB = $\frac{1}{2}$ Half of side

Draw a square and two diagonals. Inscribe a circle and draw four arcs with radius equal to half the side of the square. E, F, G and H are mid points of four sides. Join these mid points cross wise and in parallel.

This way the circle – square composite system is divided symmetrically into 32 parts. 16 a+ 16 b.= 32. Square has all the 32, and circle has 16a+8b parts, exactly.

Square is also divided into 16 equal triangles, each of EKB.

Area of the square = side x side = 1 x 1= 1 square unit

Area of EKB triangle = $\frac{1}{16}$; Half of EKB $= \frac{1}{32} = y$

Δ EKB is divided into two segments: a and b. a and b have a common side – a curvature. It's length is unknown. Sides of Δ EKB are EK, EB and KB. KB is further divided into KL and LB.

a = EKL b = ELB

From the visual observation, the area of EKL is greater than that of ELB. (It can be confirmed by subtracting the area of sector OEL from the Δ OEB. The area of the sector is $\frac{1}{8}$th of that of the circle). To calculate the area of the sector we can use the accepted π value 3.1415926……).

Before going further, the reader is requested to look into the following two quotations:

"According to intuitionistic philosophy, mathematics should be defined as a mental activity and not as a set of theorems (as was done in the section on logicism)".

"It is through intuition that the mathematical world remains in touch with the real world".

The author thanks the reader and requests to continue the study of the method.

We have seen above that EKL is greater than that of ELB and also ELB and also EKL, in area, is more than half of the area of Δ EKB.

i.e. area of EKL > half the area of EKB. $a = EKL > \frac{1}{32} = S$

Naturally, the other segment, b = ELB is lesser than half of the area of ΔEKB

i.e. area of ELB < half the area of EKB

$$b = ELB < \frac{1}{32} = t$$

in the ΔEKB, if EL were to be a straight line, the area of EKB would have been divided into two parts and their areas would have been easily calculated by the formula $\frac{1}{2} \times side \times side$

Here, EL is not a straight line. It is a curve. No doubt it has divided EKB into two segments with common curvature EL for both the segments.

Thus, the area of each segment has to be calculated applying **new relations** between the concerned parameters or line segments, keeping aside well established formula: $\frac{1}{2}ab$.

The Δ EKB has, no doubt, three sides. However, one side is further divided into two. i.e. KB is one side of EKB triangle, but KB is divided into KL and LB. Thereby, this Δ EKB has EK, EB, KL and LB. Their lengths are

$$EK = \frac{\sqrt{2}}{4} \qquad KL = \frac{2-\sqrt{2}}{4}$$

$$EB = \frac{1}{2} \qquad LB = \frac{\sqrt{2}-1}{2}$$

Instead of using the formula $\frac{1}{2}ab$, here we use new relations to arrive at the excess area $(= s)$ and lesser area $(= t)$.

One more thing. The sides of the EKB are related by a surd $\sqrt{2}$ and the divided triangle EKB is because of the arc in between 'a' and 'b'.

$$\frac{EB}{EK}=\frac{EB}{KB}=\frac{\frac{1}{4}}{\frac{2}{\sqrt{2}}}=\sqrt{2}\ , \qquad \frac{LB}{KL}=\frac{\frac{\sqrt{2}-1}{4}}{2-\sqrt{2}}=\sqrt{2}$$

The new relations do help in arriving at the values for the areas of S and t. These new relations detailed below look hypothetical and illogical at first sight but have a meaningful co-existence.

Four lengths of ΔEKB are EK, EB, KL and LB.

$$S=\frac{KL\times EB}{16}=\frac{\left(\frac{2-\sqrt{2}}{4}\right)\left(\frac{1}{2}\right)}{16}=\frac{2-\sqrt{2}}{128}$$

$$t=\frac{LB\times EK}{16}=\frac{\left(\frac{\sqrt{2}-1}{2}\right)\left(\frac{\sqrt{2}}{4}\right)}{16}=\frac{2-\sqrt{2}}{128}$$

From the above calculations it is clear that $S=t=\frac{2-\sqrt{2}}{128}$.

y = half of the area of Δ EKB = $\frac{1}{32}$

s = excess area needed to add to y

t = lesser area to be deducted from y

$a-y=s$ so $y+s=a$

$$\frac{1}{32}+\frac{2-\sqrt{2}}{128}=\frac{6-\sqrt{2}}{128}$$

Area of segment EKL = $a=\frac{6-\sqrt{2}}{128}$

Next

$y-b=t$ so $b=y-t$

$$\frac{1}{32}-\frac{2-\sqrt{2}}{128}=\frac{2+\sqrt{2}}{128}$$

Square has 16 a + 16 b

$$= 16\left(\frac{6-\sqrt{2}}{128}\right)+16\left(\frac{2+\sqrt{2}}{128}\right)=1$$

area of the square = 1 sq. unit

Circle has 16 a + 8 b

$$\text{Area of the circle } =\frac{\pi d^2}{4}$$

Where = d= diameter = side of the square = 1

$$\frac{\pi d^2}{4}=\left\{16\left(\frac{6-\sqrt{2}}{128}\right)+8\left(\frac{2+\sqrt{2}}{128}\right)\right\}$$

$$\therefore \pi=\frac{14-\sqrt{2}}{4}$$

A GEOMETRICAL LENGTH EQUAL TO THE SIDE OF THE SQUARE

Draw a square ABCD and two diagonals AC and BD

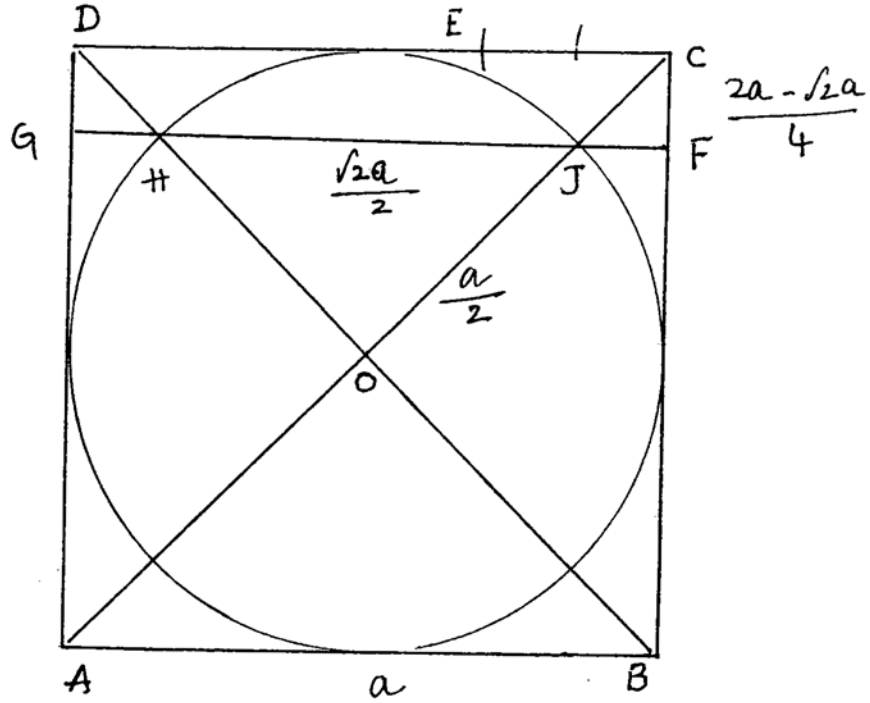


Fig. 1

$AB = a; \quad AC = \sqrt{2}\,a$

$DO = OC = \frac{\sqrt{2}a}{2}$

$DO = DE$

$EC = CD - DE =$

$$a - \left(\frac{\sqrt{2}a}{2}\right) = \frac{2a - \sqrt{2}a}{2}$$

$2CF = EC$

Obtain 4 times the length of

$$EC = 4\left(\frac{2a - \sqrt{2}a}{2}\right) = 4a - 2\sqrt{2}a$$

Second square

$FG = 4a - 2\sqrt{2}a$

$OM = OL =$ radius

$= 2a - \sqrt{2}a$

$LM = OM \times \sqrt{2}$

$= 2\sqrt{2}a - 2a$

$$KL = MN = GN = \frac{KN - LM}{2}$$

$GH - GN = NH$

$NH = BC$

This geometrical length NH could be obtained because of the inscribed circle only.

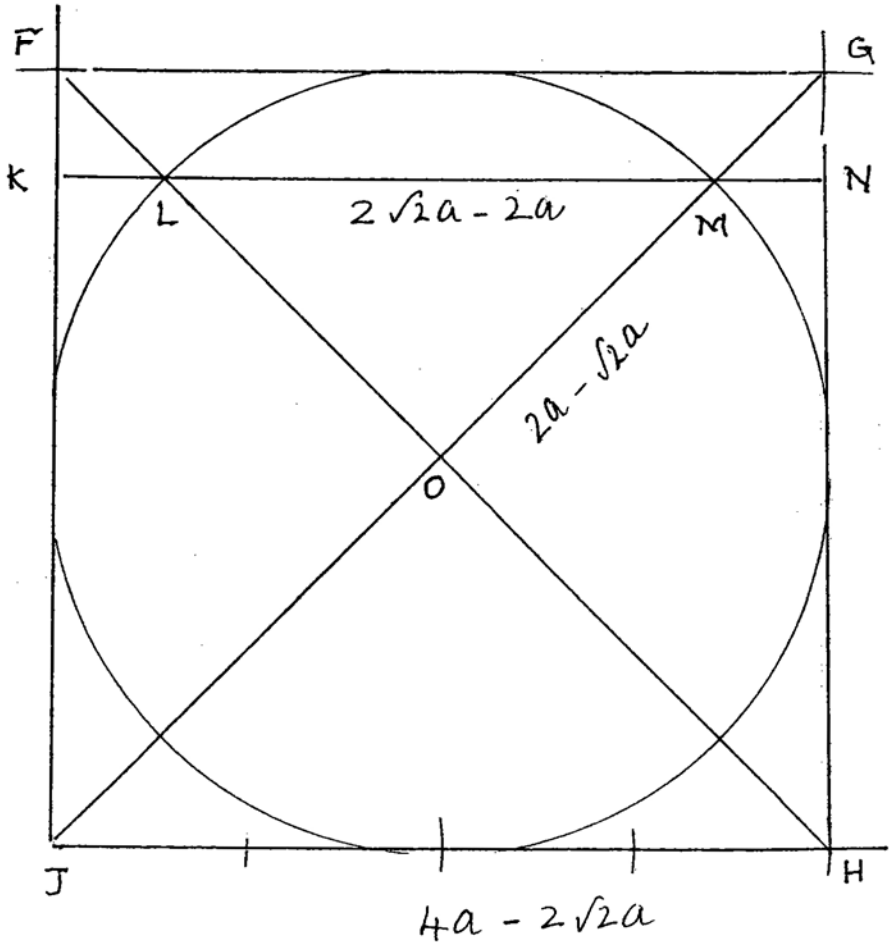


Fig. 2

THE TRUE LENGTH OF THE CIRCUMFERENCE OF THE CIRCLE

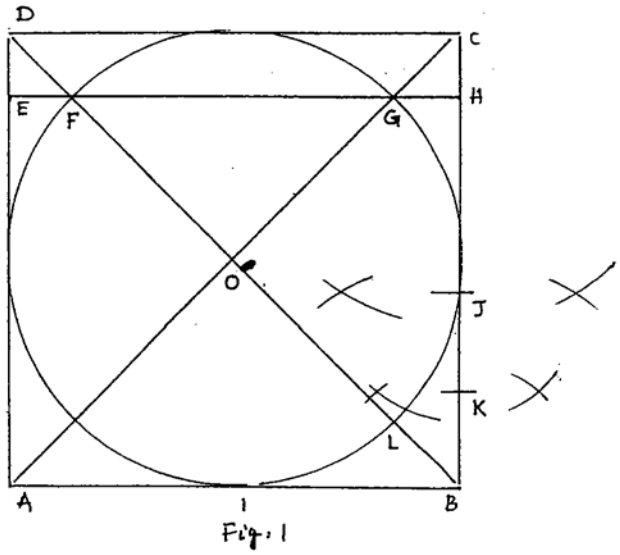


Fig. 1

ABCD = Square $\quad$ AB = BC = CD = DA = 1 = EH

Inscribe a circle

Diameter = Side = 1

Raidus = $OG = OF = \frac{1}{2}$

$$FG = \sqrt{2} \times OG = \sqrt{2} \times \frac{1}{2} = \frac{\sqrt{2}}{2} = OC$$

$$GH = HC = EF = DE$$

$$GH = \frac{EH - FG}{2} = \frac{1 - \frac{\sqrt{2}}{2}}{2} = \frac{2 - \sqrt{2}}{4}$$

$$HC = \frac{2 - \sqrt{2}}{4}$$

$$BC = 1$$

$$BC - CH = 1 - \frac{2 - \sqrt{2}}{4} = \frac{2 + \sqrt{2}}{4} = HB$$

Bifurcate HB twice = $= \frac{2 + \sqrt{2}}{4} \rightarrow \frac{2 + \sqrt{2}}{8} \rightarrow \frac{2 + \sqrt{2}}{16}$

$$\frac{1}{2}\left(\frac{2 + \sqrt{2}}{4}\right)$$

$$\frac{HB}{2} = \frac{BJ}{2} = KB = KJ = \frac{2 + \sqrt{2}}{16}$$

$$BC - KB = \frac{\pi}{4} = 1 - \left(\frac{2 + \sqrt{2}}{16}\right) = \frac{14 - \sqrt{2}}{16} = CK = \frac{1}{4} \text{ Circumference} = GJL$$

$$CK = GJL$$

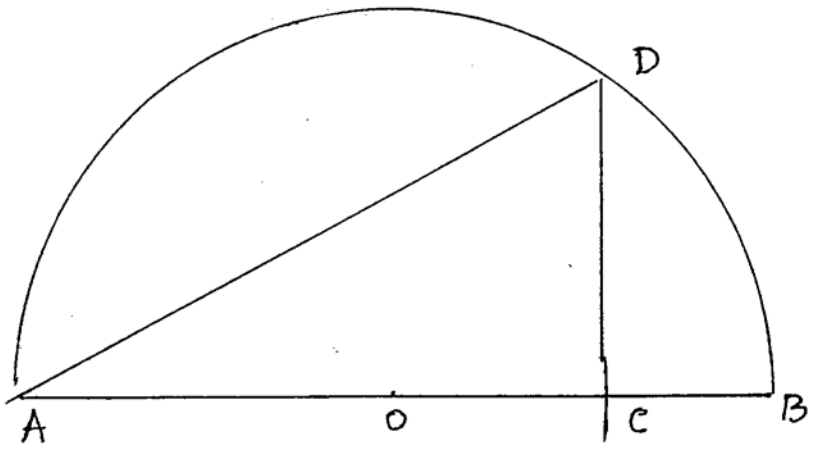


Fig: 2

Fig 2.

AB = 1 Draw a semicircle with radius $\frac{1}{2}$ and with centre O

$$AO = OB = \frac{1}{2}$$

$$AC = CK \text{ of Fig.1} = \frac{14-\sqrt{2}}{16}$$

$$CB = AB - AC = 1 - \frac{14-\sqrt{2}}{16} = \frac{2+\sqrt{2}}{16}$$

Draw a perpendicular line on AB at C, which intersects semicircle at D. Apply altitude theorem to obtain CD length.

$$CD = \sqrt{AC \times CB} = \sqrt{\frac{14-\sqrt{2}}{16} \times \frac{2+\sqrt{2}}{16}} = \frac{\sqrt{26+12\sqrt{2}}}{16}$$

Connect AD which is the side of the square whose area is equal to that of the inscribed circle of Fig.1. It is called squaring of the circle. Apply Pythagorean theorem to get AD from the triangle ACD.

$$\sqrt{AC^2 + CD^2} = \sqrt{\left(\frac{14-\sqrt{2}}{16}\right)^2 + \left(\frac{\sqrt{26+12\sqrt{2}}}{16}\right)^2} = \frac{\sqrt{14-\sqrt{2}}}{4}$$

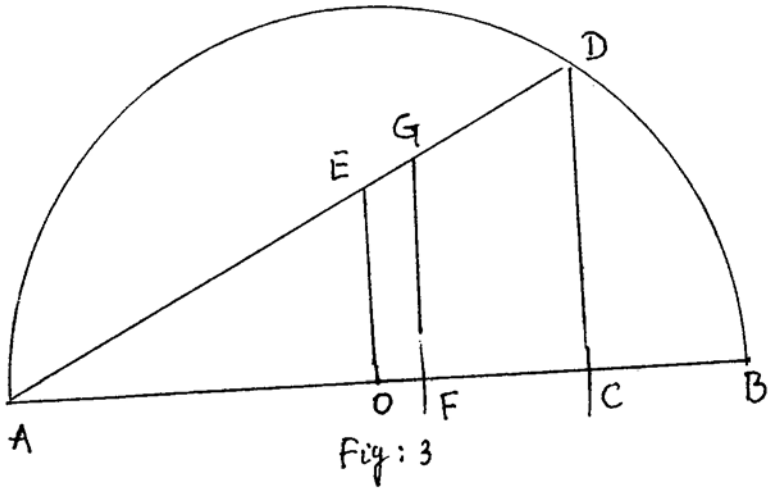


Fig : 3

Fig. 3

Reproduce Fig. 2 again here

$$AO = OB = \frac{1}{2}$$

Draw a perpendicular line on AB at O. Apply the concept of similar triangles to get AE

$$AE = \frac{AD \times AO}{AC} = \frac{\frac{\sqrt{14-\sqrt{2}}}{4} \times \frac{1}{2}}{\frac{14-\sqrt{2}}{16}} = \frac{2}{\sqrt{14-\sqrt{2}}}$$

$AE = AF$

Draw a perpendicular line on AB at F which intersects the semicircle at G. Apply the concept of similar triangles to get AG

$$AG = \frac{AD \times AF}{AC} = \frac{\frac{\sqrt{14-\sqrt{2}}}{4} \times \frac{2}{\sqrt{14-\sqrt{2}}}}{\frac{14-\sqrt{2}}{16}} = \frac{8}{14-\sqrt{2}}$$

II Arithmetic ratio

$$\frac{\pi}{2} = \frac{\frac{14-\sqrt{2}}{4}}{2} = \frac{14-\sqrt{2}}{8}$$

$$\frac{\sqrt{2}}{\frac{\pi}{2}} = \frac{2\sqrt{2}}{\pi} = \frac{2\sqrt{2}}{\frac{14-\sqrt{2}}{4}} = \frac{8\sqrt{2}}{14-\sqrt{2}}$$

In Fig.1 we have square and circle inscribed in the square and in them there are diagonals, diameter, circumference, side of the square and perimeter of the superscribed square.

Using one line segment of the square we can obtain another line segment of its inscribed circle. For it we have to get a length equal to the ratio of the line segment of the square and the line segment of the inscribed circle. For example, using **the circumference of the inscribed circle we can obtain the diagonal length of the superscribed square.**

III Geometric ratio

Let us obtain first the ratio of the diagonal ($\sqrt{2}$) of the square of unit side, and the half the length of the circumference $\left(\frac{\pi}{2} = \frac{14-\sqrt{2}}{2\times 4}\right)$ of the inscribed circle.

To get a length equal to $\frac{8\sqrt{2}}{14-\sqrt{2}}$ the following geometrical construction is adopted.

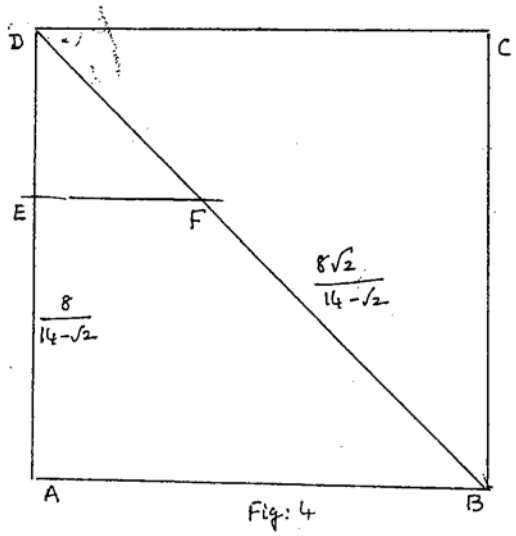


Fig: 4

Fig 4.

Draw a square AB = 1

AE is equal to AG of Fig. 3

$$AG = \frac{8}{14-\sqrt{2}} = AE$$

Draw a perpendicular line on AD at E which intersects BD diagonal at F.

$$AE \times \sqrt{2} = BF$$

$$\left(\frac{8}{14-\sqrt{2}}\right) \times \sqrt{2} = \frac{8\sqrt{2}}{14-\sqrt{2}} = BF$$

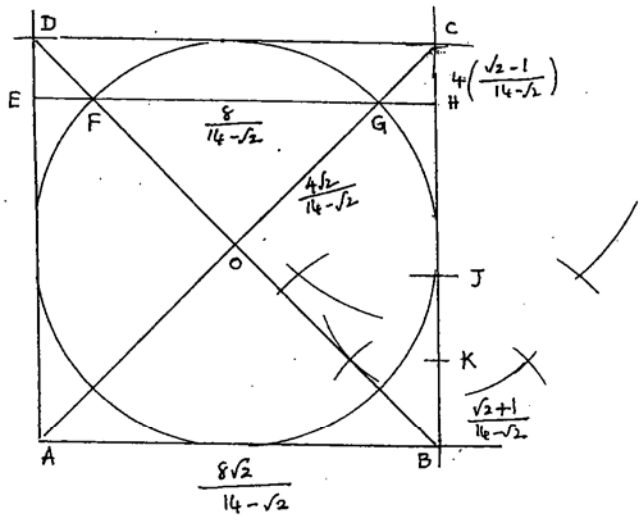


Fig. 5

Construct a square ABCD whose side is equal to the length of BF of Fig. 4 and repeat the geometrical construction process and arithmetic calculation that have been done in Fig.1 to get CK.

$$\frac{1}{2}\left(\frac{8\sqrt{2}}{14-\sqrt{2}}\right) = \frac{8}{14-\sqrt{2}}$$

$$FG = \text{radius} \times \sqrt{2} = \left(\frac{4\sqrt{2}}{14-\sqrt{2}}\right) \times \sqrt{2} = \frac{8}{14-\sqrt{2}}$$

(again this length is equal to AE of Fig.4)

$$\frac{1}{2}(EH - FG) = GH = CH$$

$$\frac{1}{2}\left\{\left(\frac{8\sqrt{2}}{14-\sqrt{2}}\right)-\left(\frac{8}{14-\sqrt{2}}\right)\right\}=4\left(\frac{\sqrt{2}-1}{14-\sqrt{2}}\right)$$

$$BC-CH=BH=\frac{8\sqrt{2}}{14-\sqrt{2}}-4\left(\frac{\sqrt{2}-1}{14-\sqrt{2}}\right)=\frac{4\sqrt{2}+4}{14-\sqrt{2}}$$

Bifurcate BH twice to get $BK=\frac{4\sqrt{2}+4}{14-\sqrt{2}}\rightarrow\frac{2\sqrt{2}+2}{14-\sqrt{2}}\rightarrow\frac{\sqrt{2}+1}{14-\sqrt{2}}$

$$CK=BC-BK=\frac{8\sqrt{2}}{14-\sqrt{2}}-\frac{\sqrt{2}+1}{14-\sqrt{2}}=\frac{\sqrt{2}}{2}$$

In Fig.1 ¼th of the circumference is obtained. In the Second Step the ratio of diagonal and half the circumference is obtained arithmetically and (in the third step) geometrically $\left(\frac{8\sqrt{2}}{14-\sqrt{2}}\right)$ in Fig 4.

IV Finally in Fig.5, CK is obtained geometrically whose length is equal to the half the diagonal of the square of Fig.1.

$$1/4^{th}\text{ of the circumference}=\frac{\pi}{4}=\frac{\frac{14-\sqrt{2}}{4}}{4}=\frac{14-\sqrt{2}}{16}$$

= (CK of Fig.1) = GJL of same Fig.1.

½ of the diagonal of the superscribed square = $\frac{\sqrt{2}}{2}$ (OC of Fig.1.)
or CK of Fig.5

$$\text{Ratio of }\frac{\sqrt{2}}{2}\text{ and }\frac{14-\sqrt{2}}{16}=\frac{\frac{\sqrt{2}}{2}}{\frac{14-\sqrt{2}}{16}}=\frac{8\sqrt{2}}{14-\sqrt{2}}$$

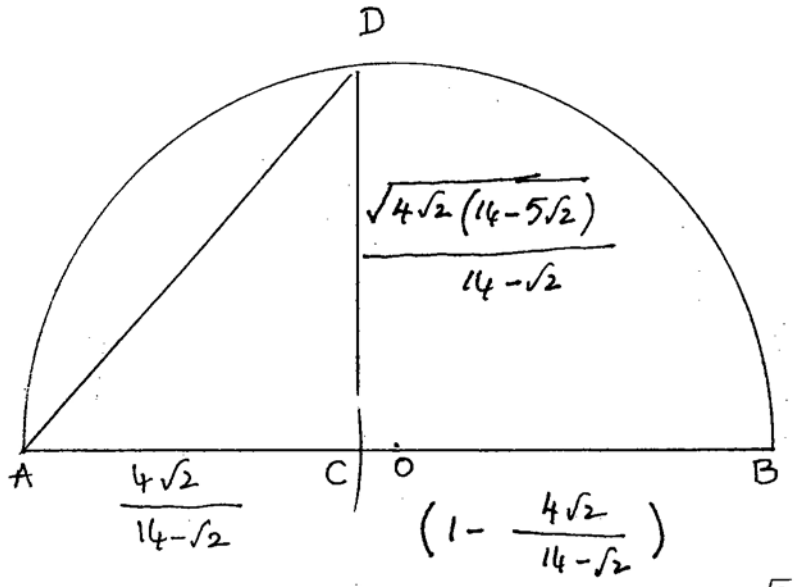
To conclude, the square ABCD of Fig.1. whose side is 1 has two diagonals, $\sqrt{2}$ length each $2\times\sqrt{2}=2\sqrt{2}$. $1/4^{th}$ of $2\sqrt{2}=\frac{\sqrt{2}}{2}$ (OC of Fig.1).

Secondly, the inscribed circle of above ABCD square of Fig.1. has one full circle i.e., circumference. It is presumed the length of the circumference is equal to $\frac{14-\sqrt{2}}{4}$

$$1/4^{th}\text{ of the circumference}=\frac{14-\sqrt{2}}{16}$$

As we have obtained exact diagonal length in the series of geometrical constructions from Fig.1, to Fig.5, our presumption of $\frac{14-\sqrt{2}}{4}$ as the length of the circumference of the inscribed circle of Fig.1 is true and represents the correct length of the circumference of the circle.

A CIRCLE OF $\sqrt{2}$ AREA



At the outset we have to get a line segment equal to $\frac{4\sqrt{2}}{14-\sqrt{2}}$. It has been explained in the earlier articles how to get it.

Draw a line segment AB of 1 unit. O, is the centre of AB, $\frac{1}{2}$. Draw a semicircle with O on AB. Then mark the length of $\frac{4\sqrt{2}}{14-\sqrt{2}}$ on AB. AC is the length.

Draw a perpendicular line at C on AB which intersects the semicircle at D.

$AB = 1$ $\qquad AC = \frac{4\sqrt{2}}{14-\sqrt{2}}$

$CB = AB - AC = 1-\left(\frac{4\sqrt{2}}{14-\sqrt{2}}\right) = \frac{14-5\sqrt{2}}{14-\sqrt{2}}$

Apply altitude theorem to get the value of CD

$$CD = \sqrt{AC \times CB} = \sqrt{\frac{4\sqrt{2}}{14-\sqrt{2}} \times \frac{14-5\sqrt{2}}{14-\sqrt{2}}} = \frac{\sqrt{4\sqrt{2}\left(14-5\sqrt{2}\right)}}{14-\sqrt{2}}$$

Apply Pythagorean theorem to get AD

$$AD = \sqrt{(AC)^2+(CD)^2} = \sqrt{\left(\frac{4\sqrt{2}}{14-\sqrt{2}}\right)^2 + \left\{\frac{\sqrt{4\sqrt{2}\left(14-5\sqrt{2}\right)}}{14-\sqrt{2}}\right\}^2} = \sqrt{\frac{4\sqrt{2}}{14-\sqrt{2}}}$$

So radius $= \sqrt{\frac{4\sqrt{2}}{14-\sqrt{2}}}$

Area of a circle $= \pi r^2 = \frac{14-\sqrt{2}}{4} \times \sqrt{\frac{4\sqrt{2}}{14-\sqrt{2}}} \times \sqrt{\frac{4\sqrt{2}}{14-\sqrt{2}}} = \sqrt{2}$

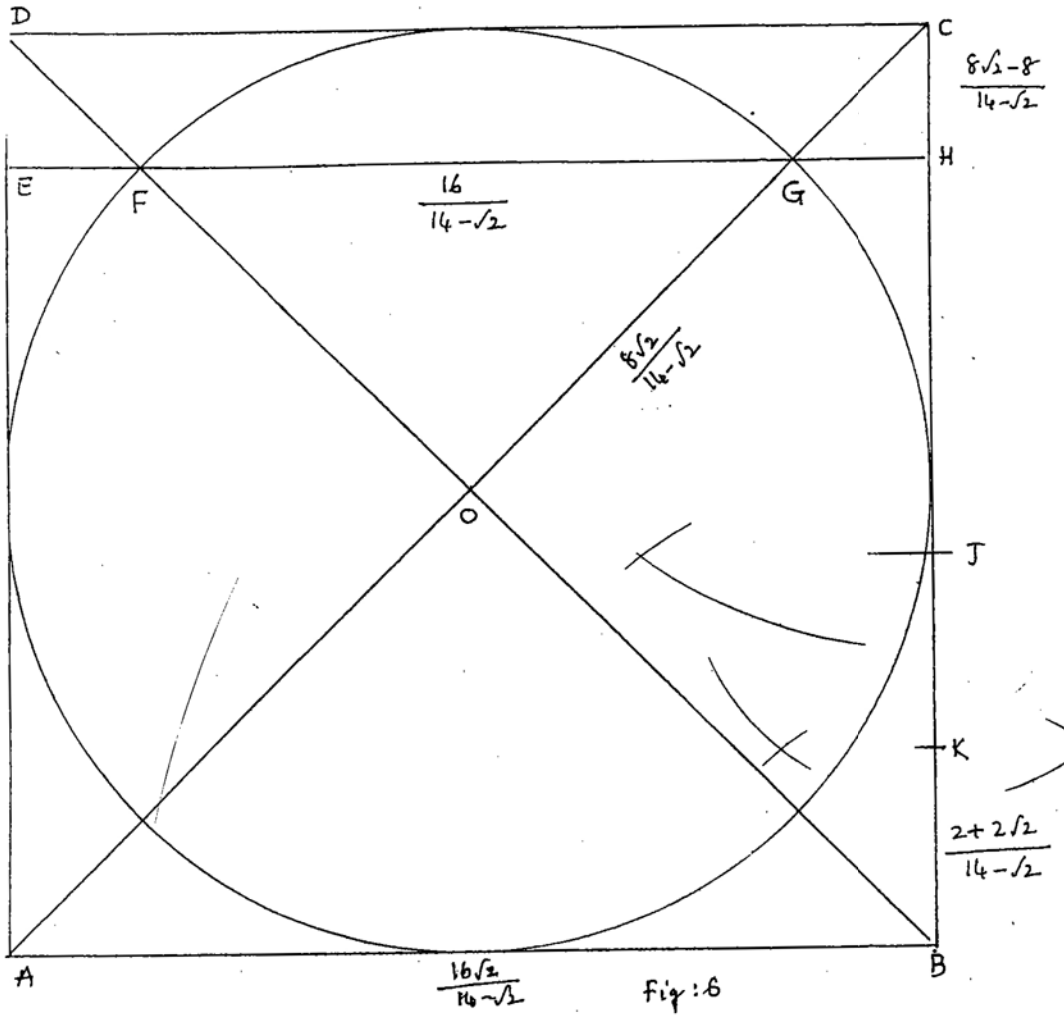


fig : 6

$\sqrt{2}$ Length from circle square ratio:

Square ABCD. AB = Side = AC of Fig.2 in the Page 194

$$AC = \left(\frac{16}{14-\sqrt{2}}\right)\sqrt{2} = \frac{16\sqrt{2}}{14-\sqrt{2}}$$

$$CH = GH = EF = \frac{EH-FG}{2} =$$

$$\left\{\left(\frac{16\sqrt{2}}{14-\sqrt{2}}\right)-\left(\frac{16}{14-\sqrt{2}}\right)\right\}\frac{1}{2} = \frac{8\sqrt{2}-8}{14-\sqrt{2}}$$

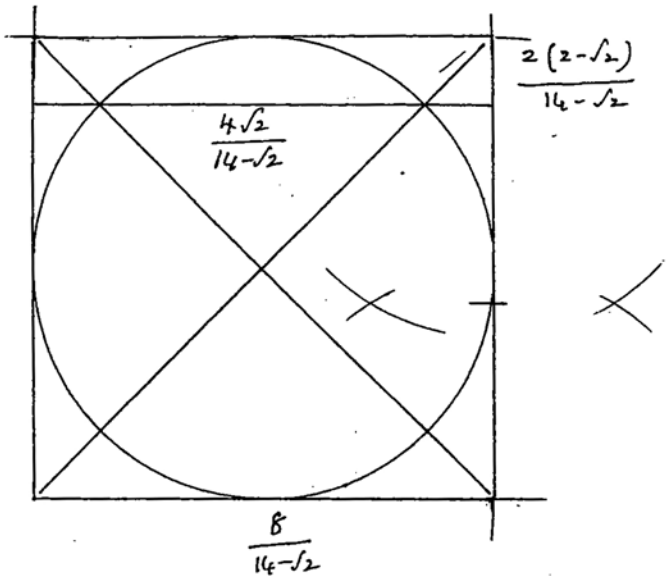
$$BC - CH = BH = \frac{8+8\sqrt{2}}{14-\sqrt{2}}$$

Bifurcate BH twice to get BK = JK = $\frac{2+2\sqrt{2}}{14-\sqrt{2}}$

$$BC - BK = CK = \sqrt{2}$$

$$\left(\frac{16\sqrt{2}}{14-\sqrt{2}}\right)-\left(\frac{2+2\sqrt{2}}{14-\sqrt{2}}\right) = \sqrt{2}$$

A CIRCLE OF $\sqrt{2}$ LENGTH OF ITS CIRCUMFERENCE



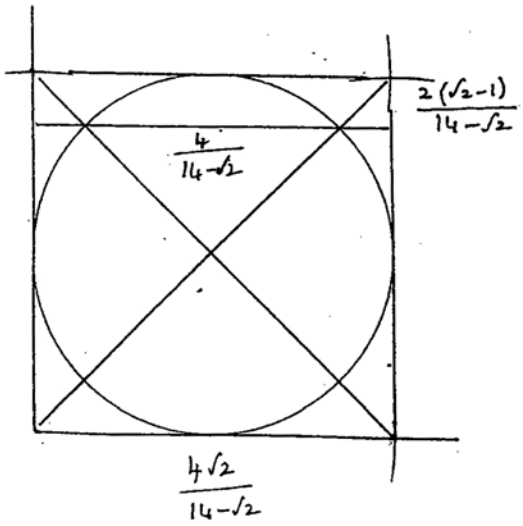
To obtain the circle whose circumference is of $\sqrt{2}$ we have to get the length equal to $\frac{8}{14-\sqrt{2}}$.

If the radius of the circle = $\frac{8}{14-\sqrt{2}}$ the circumference of that circle would be 4.

To obtain a length of $\frac{8}{14-\sqrt{2}}$ construction procedures were given in the earlier articles.

Radius = $\frac{4}{14-\sqrt{2}}$

Circumference = 2



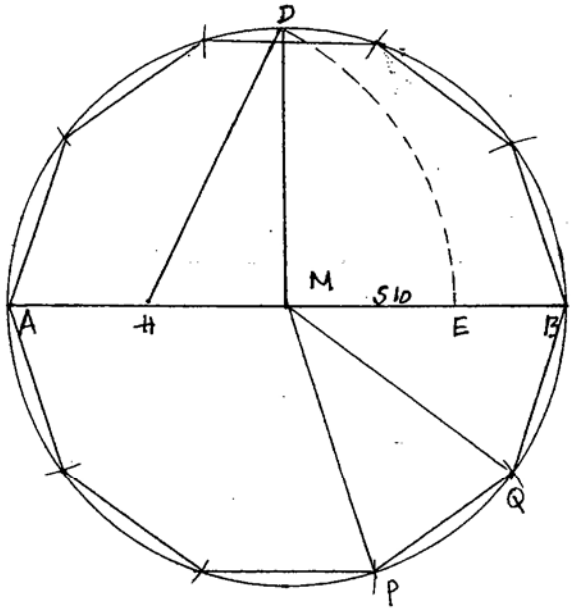
Radius = $\frac{2\sqrt{2}}{14-\sqrt{2}}$

$$3\left(\frac{4\sqrt{2}}{14-\sqrt{2}}\right)+\frac{2\left(\sqrt{2}-1\right)}{14-\sqrt{2}}=\sqrt{2}$$

$\sqrt{5}$ AND THE AREA OF A CIRCLE

In the earlier methods $\sqrt{2}$ and $\sqrt{3}$ were used for the area/circumference of a circle. Now decagon divides the area of a circle into two and two formulae-one new-are shown below using $\sqrt{5}$. This method is just to satisfy the desire. "But it is not the importance of mathematics that motivates this study; rather it is the desire to determine, if possible, how and why mathematical concepts such as "number" and geometry were created and developed".

Method: "The regular decagon can be constructed by the following method. One draws the diameter AB of the circle of radius r and constructs perpendicular to it the radius MD. Let H be the mid point of AM.



By Pythagorean theorem

$(HD)^2 = (DM)^2 + (HM)^2 = \left(\frac{r}{2}\right)^2 + r^2 = \left(\frac{5}{4}\right)r^2$ Thus $(HD) = r\sqrt{5}/2$.

The arc with radius HD about H intersects MB in E. Then

(ME) = (HE) – (HM) = $\frac{\left(r\sqrt{5}-2\right)}{2}$ = S10."

Area of the inscribed decagon + Area in between the decagon and circle = area of the circle = $\frac{\pi d^2}{4}$

$$10\left(\frac{d^2\left(\sqrt{5}-1\right)\left(\sqrt{5+\sqrt{5}}\right)}{32\sqrt{2}}\right)+d^2\left\{\frac{\left(28-2\sqrt{2}\right)-5\left(\sqrt{10}-\sqrt{2}\right)\left(\sqrt{5+\sqrt{5}}\right)}{32}\right\}$$

Where d = diameter = 2r, r = radius. The above one is a general formula based on the values of diameter, area of decagon and proposed π value = $\frac{\left(14-\sqrt{2}\right)}{4}$

Gayathri method

ABCD = Square

AB = Side = a = 1

JG = diameter = a = 1

$OF = OG = \text{radius} = \frac{a}{2} = 0.5$

$FG = \text{hypotenuse} = \frac{\sqrt{2}a}{2}$

DE = EF = GH = CH =

$$\frac{EH - FG}{2} = \frac{a - (\frac{\sqrt{2}a}{2})}{2}$$

$$= \frac{2a - \sqrt{2}a}{4}$$

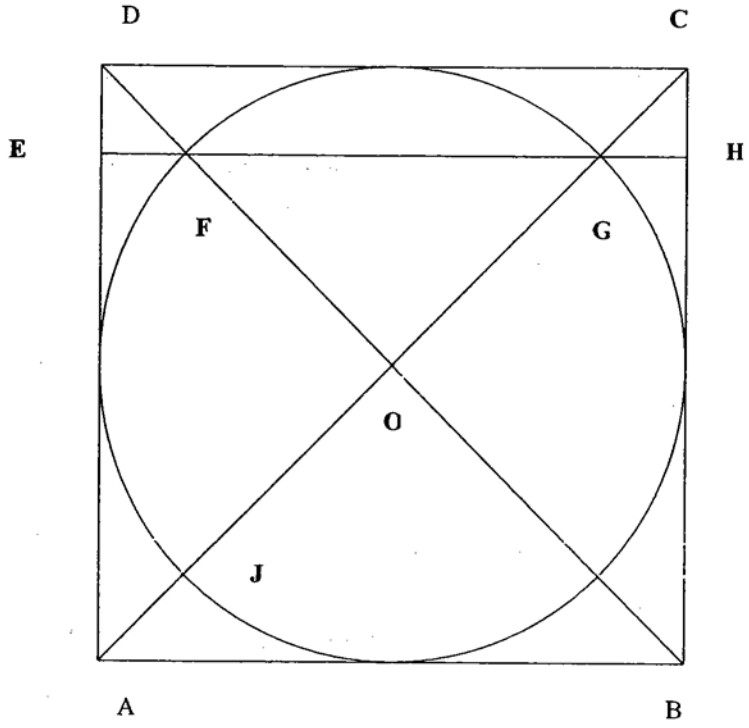


Fig. 1

The length of the circumference of the inscribed circle can bc earmarked in the perimeter of the superscribed square.

Circumference of the circle =

$$BA+AD+DC+CH = a + a + a + \frac{2a - \sqrt{2}a}{4} = \frac{14a - \sqrt{2}a}{4}$$

$$\pi d = \pi a = \frac{14a - \sqrt{2}a}{4}$$

$$\therefore \pi = \frac{14 - \sqrt{2}}{4}$$

Explanation

From the above simple geometrical construction the world came to know for the first time (in 1998) that the length of the circumference was $\frac{14-\sqrt{2}}{4}$ of unit diameter. We should know that there are more similarities than differences between diagonal and circumference.

Diagonal	Circumference
1. A straight line inside the square	1. A curvature inside the square
2. Having a finite length.	2. Having a finite length
3. Possesses two points at its two ends	3. Cut and straighten it, it will possess two points
4. finite length having a finite magnitude	4. finite length having a finite magnitude
5. Represented by $\sqrt{2}$	5. Should also be represented by $\sqrt{2}$. It is not a taboo (as we have been believing.)

The perimeter of the square contains four sides. The circumference of the circle contains sum of the lengths of three diameters plus, but less than the sum of the four diameters. If it were to be equal to four diameters it would be equal to the perimeter of the superscribed square. That is wrong.

We also know well, diameter of the inscribed circle is equal to the side of the superscribed square. So, our effort should be how to get the 'plus' segment of the circumference from the parameters such as diagonal, side (perimeter) of the superscribed square.

Thus, we are definite that the length of the circumference is in between three diameters and four diameters. The end point (beginning with the 1st diameter (=1st side of the square say B in BA of Fig.1) is somewhere in the 4th diameter (= somewhere in the 4th side of the square CB of Fig.1). How much part of the 4th side (here after we say side and not diameter) is equal to the 'plus' part of the circumference can be earmarked in the perimeter of the square, has to be reasoned meaningfully.

The inscribed circle in the square itself is giving a clue for how much part of the 4th side is equal to the 'plus' part of the circumference. The clue is the segment CH of the BC side of the perimeter of the square. How to confirm it? The reader is requested to follow the following reasoning:

The answer is, from understanding the new interrelationships in their right perspective between the parameters of the circle and that of the square.

Circle is mainly a circumference maintaining uniform distance from its centre. Whereas a square is an angular, four-sided, straightened entity, resulted due to the two diagonals inside, crossing or intersecting each other and connecting their four ends with straight lines i.e. sides. Thus the perimeter (just like circumference to its diameter) does not maintain uniform distance. At E, F, G, and H the perimeter is nearer to its center O equal to

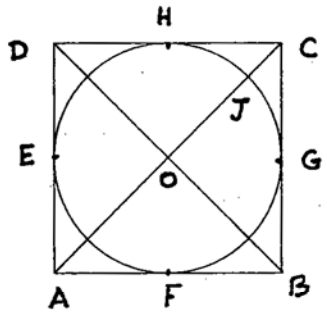


Fig.2

half of its side's length = $\left(\frac{a}{2}\right)$. And at A, B, C and D the perimeter is away from the centre O by half of its diagonal's length = $\frac{\sqrt{2}a}{2}$ or $\sqrt{2}$ times of its half of the side.

Half of the diagonal = OC = $\sqrt{2} \times \frac{a}{2} = \frac{\sqrt{2}a}{2}$. It is also greater than the radius OJ = $\frac{a}{2}$ of the inscribed circle. How much is greater than the radius?

$\frac{\sqrt{2}a}{2} - \frac{a}{2} = \frac{\sqrt{2}a - a}{2}$. It is the corner length beyond the circumference upto the point C of the perimeter of the square.

Corner length = JC (Fig. 2)

As there are two diagonals with four ends and are (diagonals) intersecting one another, we have four corner lengths of these two diagonals. These four corner lengths are outside the circumference and are responsible for the creation of the perimeter of the square (and greater than that of the circumference of the circle) and are again responsible for the creation of each side of the square in between two points A, B, C, and D. i.e. between A and B, B and C, C and D and D and A.

Reasoning 1

With the above brief analysis let us come to the 'plus' segment. The plus part of the circumference can be obtained by two ways of reasoning. One kind of reasoning is that (we are repeating here) the two diagonals project beyond the circumference by a length equal to

$$\frac{\sqrt{2}a - a}{2} = JC.$$

To get the length of the diameter (= side of the perimeter) we can divide the **whole** diagonal length by $\sqrt{2}$ (author may be permitted to call this process as **desquaring** of diagonal). $\frac{\sqrt{2}a}{\sqrt{2}} = a$ = diameter of inscribed circle.

On the same lines let us **desquare** that part of the diagonal (JC = Corner Length) which is beyond the circumference.

$$\text{i.e.} \frac{\frac{\sqrt{2}a - a}{2}}{\sqrt{2}} = \frac{\sqrt{2}a - a}{2\sqrt{2}} = \frac{2a - \sqrt{2}a}{4} \text{ and is CH of figure .1.}$$

This part is similarly "plus" part of the circumference beyond the sum of the lengths of three diameters.

$$3a + \frac{2a - \sqrt{2}a}{4} = \frac{14a - \sqrt{2}a}{4}$$

Reasoning 2

Earlier in this article we have seen some part of the 4th diameter = 4th side of the square = BC of Fig. 1 (3 diameters = 3 sides) has in it the plus part of the circumference.

To obtain **minus** part of the 4th diameter (= minus of 4th side = BC) let us again **desquare** a part of the diagonal. Which part of the diagonal?

$$AC=\text{Diagonal} = \sqrt{2}a$$

$$JC \text{ of Fig. 2} = \text{corner length} = \frac{\sqrt{2}a - a}{2}$$

Let us deduct JC length from the total length ($\sqrt{2}a$) of the diagonal (AC) and **desquare** the remaining part which gives the minus part of the 4th side of the perimeter of the square.

$$AC\text{-}JC = \sqrt{2}a - \left(\frac{\sqrt{2}a - a}{2}\right)$$

Desquare of $\dfrac{\sqrt{2}a - \left(\dfrac{\sqrt{2}a - a}{2}\right)}{\sqrt{2}}$ gives $\dfrac{2a + \sqrt{2}a}{4}$ This length is HB in the Fig. 1

This length when deducted from the **total** length of the perimeter (4 x a = 4a) of the superscribed square gives the **total** length of the circumference of the inscribed circle.

$$= 4a - \frac{2a + \sqrt{2}a}{4} = \frac{14a - \sqrt{2}a}{4}.$$

This length is equal to the circumference of the inscribed circle = BADCH segment of the perimeter of the square ABCD of Fig. 1.

As there are two diagonals with four ends and are (diagonals) intersecting one another, we have four corner lengths of these two diagonals. These four corner lengths are outside the circumference and are responsible for the creation of the perimeter of the square (and greater than that of the circumference of the circle) and are again responsible for the creation of each side of the square in between two points A, B, C, and D. i.e. between A and B, B and C, C and D and D and A.

Reasoning 1

With the above brief analysis let us come to the 'plus' segment. The plus part of the circumference can be obtained by two ways of reasoning. One kind of reasoning is that (we are repeating here) the two diagonals project beyond the circumference by a length equal to

$$\frac{\sqrt{2}a - a}{2} = JC.$$

To get the length of the diameter (= side of the perimeter) we can divide the **whole** diagonal length by $\sqrt{2}$ (author may be permitted to call this process as **desquaring** of diagonal). $\frac{\sqrt{2}a}{\sqrt{2}} = a$ = diameter of inscribed circle.

On the same lines let us **desquare** that part of the diagonal (JC = Corner Length) which is beyond the circumference.

$$\text{i.e.} \frac{\frac{\sqrt{2}a - a}{2}}{\sqrt{2}} = \frac{\sqrt{2}a - a}{2\sqrt{2}} = \frac{2a - \sqrt{2}a}{4} \text{ and is CH of figure .1.}$$

This part is similarly "plus" part of the circumference beyond the sum of the lengths of three diameters.

$$3a + \frac{2a - \sqrt{2}a}{4} = \frac{14a - \sqrt{2}a}{4}$$

SIVA METHOD

(A simple analysis)

Gayathri method was the first method. Siva method is the confirmation method. In this method the area of the square is divided – horizontally, vertically by the side length, crosswise by the two diagonals, and sub cross-wise again by half the diagonals, and circularly by circumference and four arcs – into orderly arranged 32 segments of S_1 and S_2 dimensions. Two formulae $\frac{a^2}{32}(\pi-2)$ and $\frac{a^2}{32}(4-\pi)$ for the areas of S_1 and S_2 segments, respectively arise spontaneously.

In this method geometrical lengths for $(\pi-2)$ and $(4-\pi)$ are obtained and areas of S_1 and S_2 , thus calculated in the ultimate desire for the true value of π, revealed by the circle itself. The circle divides the area of the square along with the other lengths and get itself its own share of the area fully –cent per cent - from that of the square and finally settles as an inscribed entity in the square.

We are blessed now, in that, the circle (= Nature) speaks to the world of mathematics through Siva method as it (circle) was not understood totally so far since the advent of human civilization, when it was alone, and hence get itself inscribed in the square, just for our sake and for our right understanding of it, and murmurs in our thought further, not to thrust 3.1415926..... on its personality and distort any more. So far so good.

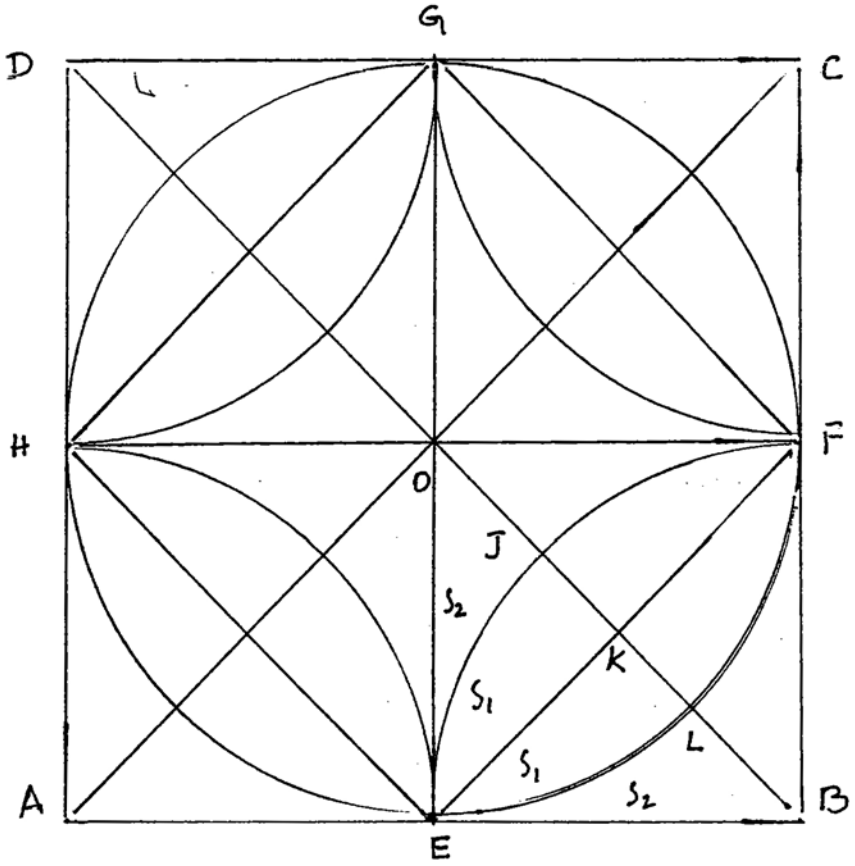


Fig. 1

ABCD = square AB = side = diameter = a Radius = OL = AE = EB = $\frac{a}{2}$

AC = diagonal = $\sqrt{2}a$ AO = OC =EF = OB $\frac{\sqrt{2}a}{2}$ OK = KB = EK

= KF = $\frac{\sqrt{2}a}{4}$ JK = KL = OL – OK = $\frac{a}{2} - \frac{\sqrt{2}a}{4} = \frac{2a-\sqrt{2}a}{4}$

OJ = OK – JK = $\frac{\sqrt{2}a}{4} - \frac{2a-\sqrt{2}a}{4} = \frac{\sqrt{2}a-a}{2}$

The square has 16 S_1 and 16 S_2 segments

The circle has 16 S_1 and 8 S_2 segments

Area of the square = a^2

Area of the circle = $\pi \times \frac{d^2}{4}$ $(d = a)$

16 S_1 + 16 S_2 = a^2

16 S_1 + 8S_2 = $\frac{\pi a^2}{4}$

From the above two equations we can obtain the areas of S_1 and S_2

Area of $S_1 = \frac{a^2}{32} \times \pi - 2$

Area of $S_2 = \frac{a^2}{32} \times 4 - \pi$

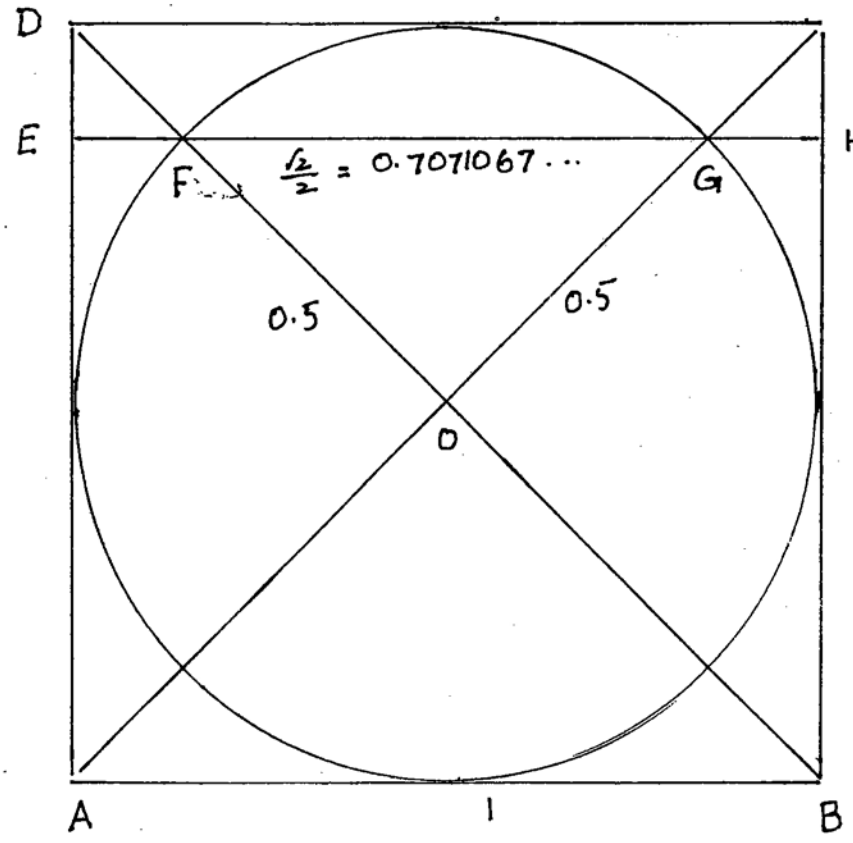
We have two π values

Official value = 3.1415926.....

Proposed value $= \frac{14-\sqrt{2}}{4} = 3.1464466.....$

3.1415926.....	3.1464466.....
1. $\pi - 2 = 1.1415926.....$	1. $\pi - 2 = 1.1464466.....$
2. $4 - \pi = 0.8584074.....$	2. $4 - \pi = 0.8535534.....$

We can not get geometrical lengths either for 1.1415926..... or for 0.8584074 ..... However, the following geometrical construction gives lengths equal to 1.1464466.... and 0.8535534 ..... of proposed π value.



$\frac{2-\sqrt{2}}{4} = 0.1464466.$

$\frac{2+\sqrt{2}}{4} = 0.8535534$

Fig. 2

ABCD = Square AB = 1 = Side = Diameter = 1 OG = Radius = 0.5, AC = BD = Diagonal = $\sqrt{2}$, Half the diagonal = OA = OC = FG = $\frac{\sqrt{2}}{2}$

side = EH = 1

$$DE = EF = GH = CH = \frac{EH - FG}{2} = \frac{1 - \frac{\sqrt{2}}{2}}{2} = \frac{2-\sqrt{2}}{4} = 0.1464466.....$$

$$BC - CH = HB = 1 - \frac{2-\sqrt{2}}{4} = \frac{2+\sqrt{2}}{4} = 0.8535534.....$$

DC + CH = 1 + 0.1464466 = 1.1464466.....

BH = 0.8535534.....

If the above numbers truly represent $\pi - 2$ and $4 - \pi$ we should also earmark areas of each segment in the square. The square has 16 S_1 and 16 S_2 segments. Area of each segment

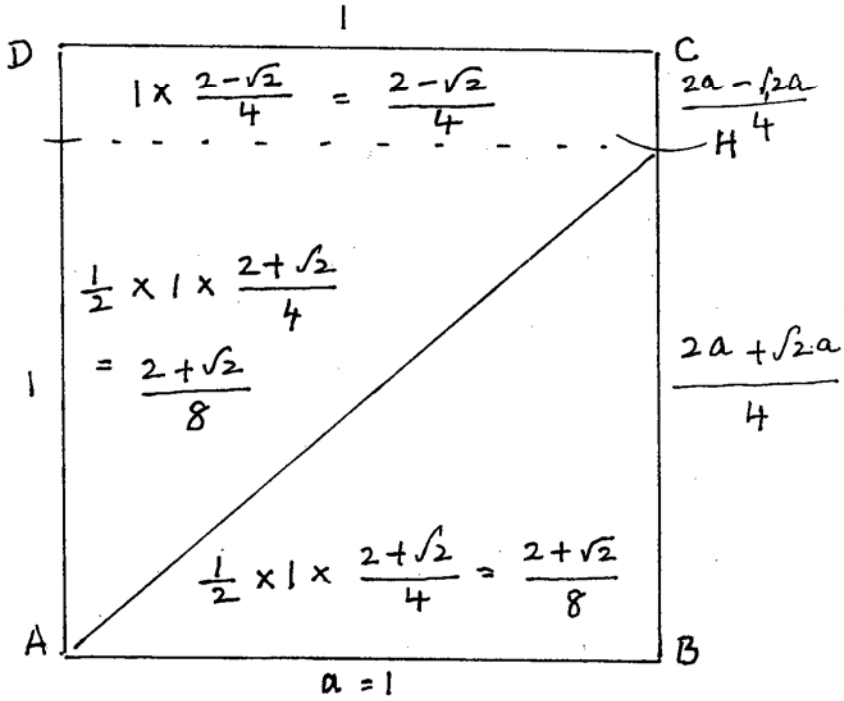
$$S_1 = \frac{a^2}{32} \times \pi - 2$$

$$16\,S_1 = \left(\frac{a^2}{32} \times \pi - 2\right) 16 = \frac{a^2}{2} \times \pi - 2$$

$$S_2 = \frac{a^2}{32} \times 4 - \pi$$

$$16\,S_2 = \left(\frac{a^2}{32} \times 4 - \pi\right) 16 = \frac{a^2}{2} \times 4 - \pi$$

$$(\pi - 2) + (4 - \pi) = 2$$



CH of Fig 3 is equal to CH of Fig.2 = $\frac{2-\sqrt{2}}{4}$

Fig. 3

Fig.3: Perimeter of the square = BA + AD + DC + CH + HB = 1 + 1 + 0.1464466+0.8535534 = 4

Area of the triangle ABH = $\frac{1}{2}$ab

$\frac{1}{2} \times AB \times BH = \frac{1}{2} \times 1 \times 0.8535534 = 0.4267767$

It is the sum of the areas of 16 S_2 segments.

8 segments inside the circle and

8 segments outside the circle but within the square.

Area of ADCH quadrilateral = 1 – 0.4267767 = **0.5732233**

(This value can also be obtained by drawing a parallel line to DC at H. This becomes a rectangle + a triangle).

It is the sum of the areas of 16 S_1 segments in the circle totally (in the square anyway)

---

1. The **formula** for the area of 16 S_1 segments

$\frac{a^2}{2} \times 1.1464466 = 0.5372233$ (where a = 1)

2. The **formula** for the area of 16 S_2 segments

$\frac{a^2}{2} \times 0.8535534 = 0.4267767$

Thus the areas obtained from the two formulae of Fig. 1 above are **in total agreement** with the areas earmarked in the square of Fig.3.

Now let us derive π value

Area of the circle has 16 S_1 + 8 S_2 Segments

$$\left(\frac{a^2}{32}\times\pi-2\right)16 + \left(\frac{a^2}{32}\times 4-\pi\right)8 = \left(\frac{1}{32}\times 1.1464466\right)16 + \left(\frac{1}{32}\times 0.8535534\right)8 = \frac{\pi d^2}{4}$$

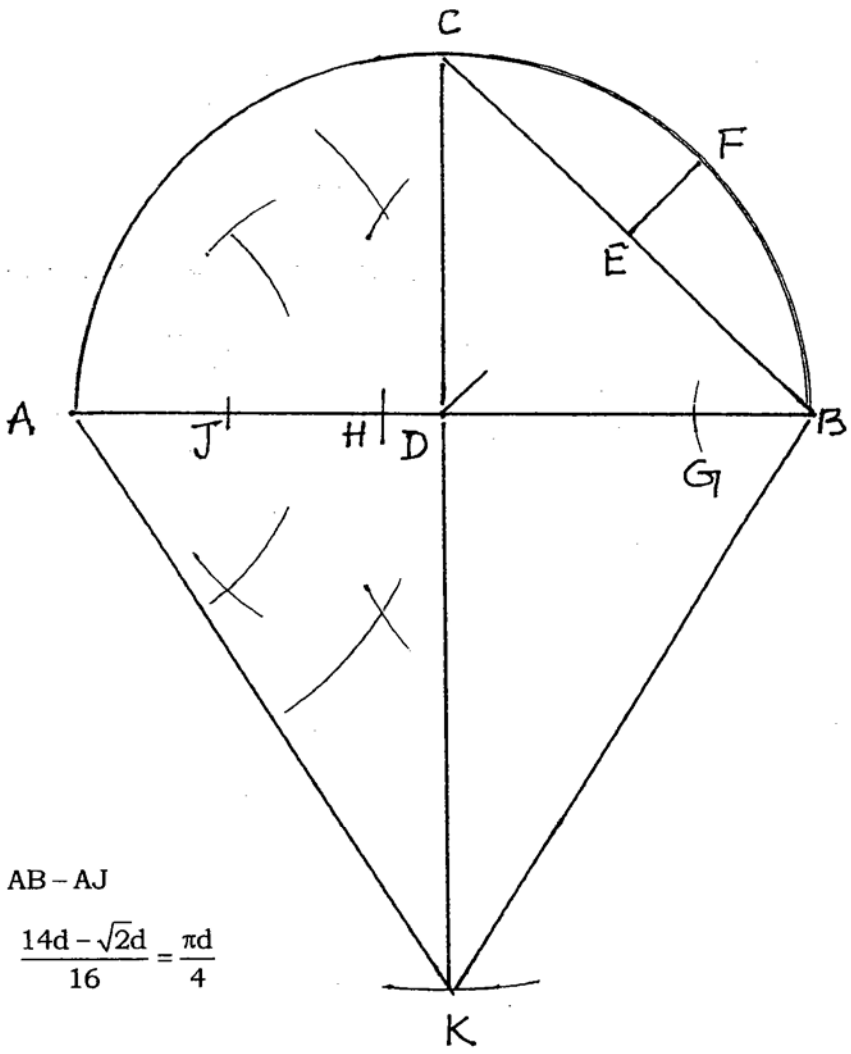
$$= 0.5732232 + 0.2133883 = \frac{\pi}{4} = 0.7866115 = \frac{\pi}{4}$$

$$\therefore \pi = 3.1464466..... \qquad (\text{where } d = 1)$$

Finally In Fig. 3 we have geometrical lengths for the following

1. $\pi-2 = 1.1464466..... =$ DCH
2. $4-\pi = 0.8535534..... =$ HB
3. $\pi-2+4-\pi = 2 =$ DCHB

CONVERSION OF SEMICIRCLE INTO A TRIANGLE IN AREA



Semicircle = ABC
Diameter = AB = d

Radius = DF = $\frac{d}{2}$

BC = $\frac{\sqrt{2}d}{2}$

DE = $\frac{\sqrt{2}d}{4}$

EF = DF – DE = $\frac{2d-\sqrt{2}d}{4}$

EF = BG

AG = AB – BG

$= d - \frac{2d-\sqrt{2}d}{4}$

$= \frac{2d+\sqrt{2}d}{4}$

Bisect AG twice

$\frac{AG}{2} \rightarrow \frac{AG}{4} = AJ = \frac{2d+\sqrt{2}d}{16}$

JB = arc length CFB = $\frac{\pi d}{4}$ = AB – AJ

JB = $d - \frac{2d+\sqrt{2}d}{16}$ = $\frac{14d-\sqrt{2}d}{16} = \frac{\pi d}{4}$

Triangle: AKB

Base = b = AB = diameter = d

Altitude = a = DK = JB = $\frac{\pi d}{4}$

Area = $\frac{1}{2}ab$ = $\frac{1}{2} \times DK \times AB$ = $\frac{1}{2} \times \frac{\pi d}{4} \times d$

Semicircle: ABC

Area of the semicircle ABC = Area of the triangle AKB

$$\frac{\pi d^2}{8} = \frac{1}{2} \times \frac{\pi d}{4} \times d$$

Area of the circle is equal to the sum of the areas of semicircle and triangle (It looks as though wheel turns and top spins)

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From: "Simon Plouffe" <simon.plouffe@gmail.com>
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To: rsjreddy1341946@yahoo.co.in

Subject: received a letter

Hello Mr. Reddy

I received your letter from India this week, someone forwarded the letter from Canada to France where I live now by email.

I read carefully your constructions and formulas about pi which is an approximation of pi to about 3 decimals.

These kind of constructions are known to give approximations and were done a couple of centuries ago by Viete and some followers but this was in the 16'th century. Despite the fact that the construction ingeniously approximates pi it is NOT pi. Pi cannot be expressed by radicals like sqrt(2) and this was proved in 1882 once and for all by Lindemann.

Thank you for your trouble of sending that letter. I would suggest that you read wikipedia on the internet if you have access ? There are a lot of pages and sites about pi explaining what I just mentioned.

Good luck with your work.

To

Prof. Simon Plouffe
Canada

Tirupati,
Date: 25-03-2008.

Dear Professor

Namasthe !

Thank you very much. I have received e- mail and I wish to submit the following:

I am particularly attracted by your statement.

I. "..... that the construction ingeniously approximates Pi it is not Pi ....."

Answer :

(1) In Siva method we have

$16\,S_1 + 16\,S_2 = a^2 =$ area of the square (exact)

$16\,S_1 + 8\,S_2 = \frac{\pi d^2}{4}$ = area of the in scribed circle (also exact)

where $S_1 = \frac{6-\sqrt{2}}{128}$ and $S_2 = \frac{2+\sqrt{2}}{128}$ and $a = d = 1$

(2) In the squaring of a circle we have obtained a geometrical length equal to the **required side** of square $\frac{\sqrt{14-\sqrt{2}}}{4}$

Area of the square $= a^2 = \left(\frac{\sqrt{14-\sqrt{2}}}{4}\right)^2 = \frac{\pi d^2}{4}$ (exact)

where $\pi = \frac{14-\sqrt{2}}{4}$, $d = a = 1$

(3) In the circling of a square we have obtained a length equal to the **required radius** as $\frac{8}{14-\sqrt{2}}$

Circumference $= 2\pi r = 2 \times \frac{14-\sqrt{2}}{4} \times \frac{8}{14-\sqrt{2}} = 4$ (exact)

In biology we have a saying " Life begets life".

Similarly, approximation begets approximation.

3.1415926..... is an approximation and hence it has succeeded **partially** in "Squaring of circle" and "Circling of a square". Exactness has become **impossible** for Pi number and consequently to the resulting geometrical constructions also.

II. Second statement

"…….Pi cannot be expressed by radicals like sqrt (2) ……"

Answer:

Let us inscribe a square in the circle - the magnitude of the constituent parts have to be represented by $\sqrt{2}$

If triangle inscribed - by $\sqrt{3}$

If decagon inscribed - by $\sqrt{5}$ and $\sqrt{10}$

Thus different surds are coming into the picture depending upon the regular polygon inscribed in the circle. **Out of all the surds $\sqrt{2}$ has become the soul of the circle.** Thus area, circumference etc. are expressed by $\sqrt{2}$ only.

In view of the above situation with the circle is it possible to disassociate $\sqrt{2}$ from the circle and consequent Pi?

I am very happy that you have sent your good wishes to the efforts made by this author for the universal constant Pi.

Prof. Constantine Karapappopulos of Greece squared the circle; Prof. Gernot Hoffmann of Germany circled the square. Whether you believe it or not as Pi on academic interest please be the **first** and top most in giving infinite series to the proposed Pi number and you will be remembered with honour when this Pi is proved true.

My hearty thanks to you Professor once again

With regards

Yours sincerely,

(R. Sarva Jagannadha Reddy)

SQUARING OF CONSTITUENT SEGMENTS INCLUDING CIRCLE OF THE SQUARE

INTRODUCTION

π equal to 3.1415926 ..... is a transcendental number. Consequently, squaring of a circle has become an unsolved geometrical problem. For centuries, people have been trying. It implies, that the idea of squaring of a circle is not wrong. If this thought is not wrong, π number 3.1415926 ..... is not exact π then. Further, every one agrees 3.1415926 .... is an approximation.

Therefore, we have to find out a number to be called as π number which can satisfy the following conditions/ requirements:

1. the number must be very close to 3.1415926 .....
2. it must be exact,
3. its derivation should be based on the dimensions of the circle or circle – square or circle – triangle or circle – polygon etc. composite geometrical constructions,
4. it should become an integral part in squaring of circle,
5. it must have an arithmetic/ geometric proof,
6. it should be backed by diversified geometrical constructions,
7. methods that are adopted to derive formulas to find out area/ circumference should be as simple as that are present for triangle, square, rectangle, other polygons etc.
8. the visible Universe contains 10^{79} protons – when it is possible to represent a cosmic magnitude by a simple number (10^{79}), a simple circle can also be understood similarly by a simple number ..... thus the π is not as big as (having billions of decimals) we think.

9. the age of the Universe range from $\sim 13\times10^9$ to $\sim 18\times10^9$ years and the age of man (Homo sapiens) is about 160, 000 years old. The Universe consists of spherical celestial bodies, visible light, invisible electromagnetic radiation, an unknown cosmic entity of unlimited speed of propagation i.e. appears instantantly at every place and at every time etc. Thus celestial bodies which are spherical have surface area/ volume and are measured by $4\pi r^2 / \frac{4}{3}\pi r^3$. Therefore π is in existence without the tag of past/ present/ future. **It is cosmic.**

Human effort therefore involves a simple imagination to dis .....cover it and not the application of higher mathematics or super computers. Their contribution does not serve the purpose. These are not only superfluous but ultimately will result in distortion of π number. (This has actually what happened with 3.1415926....).

METHOD

The individual areas of S_1 and S_2 segments of square in Fig. 3. have been calculated exactly. Here we have an arithmetic proof.

Square has

$$16\,S_1 + 16S_2 = 16\left(\frac{6-\sqrt{2}}{128}\right) + 16\left(\frac{2+\sqrt{2}}{128}\right) = 1$$

where side = diameter = 1

Circle has

$$16\,S_1 + 8S_2 = 16\left(\frac{6-\sqrt{2}}{128}\right) + 8\left(\frac{2+\sqrt{2}}{128}\right) = \frac{14-\sqrt{2}}{16}$$

Square has been divided exactly into 16 S_1 and 16 S_2 curvilinear segments. Circle, as inscribed entity has in it 16 S_1 and 8 S_2 segments.

When the area of the square is 1, the area of the inscribed circle is $\frac{14-\sqrt{2}}{16}$.

If the values given to the areas of S_1 and S_2 were not correct their multiplication in the calculation with 32 will not result in one square unit as area of the square. Further, the value given to the area of the inscribed circle will not be close to 3.1415926…. an approximate accepted number.

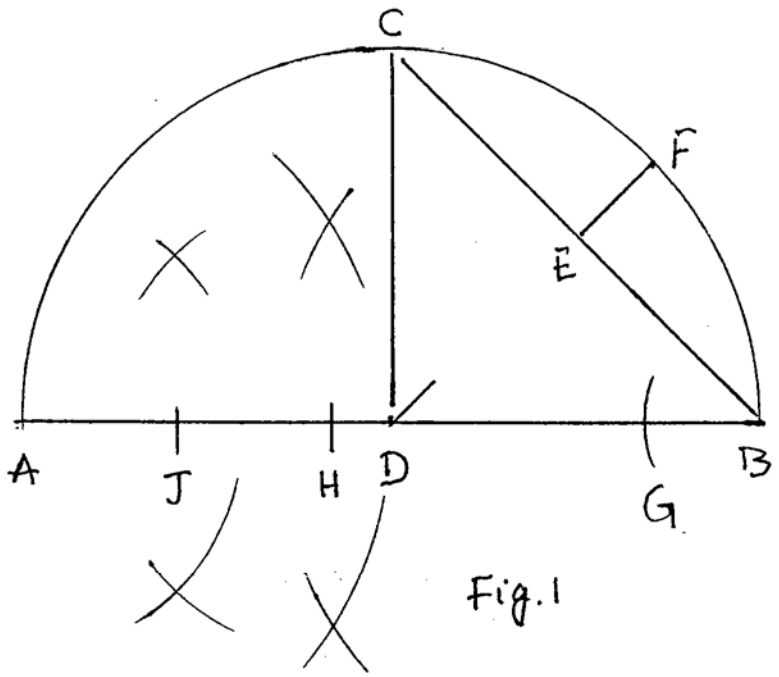


Fig. 1

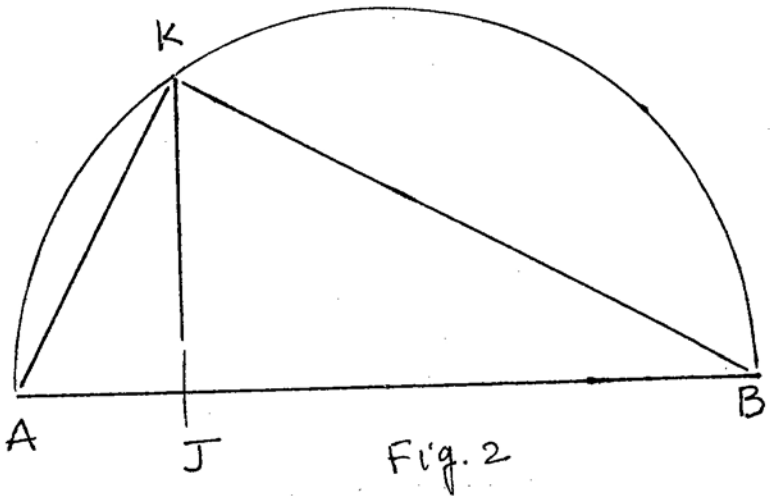
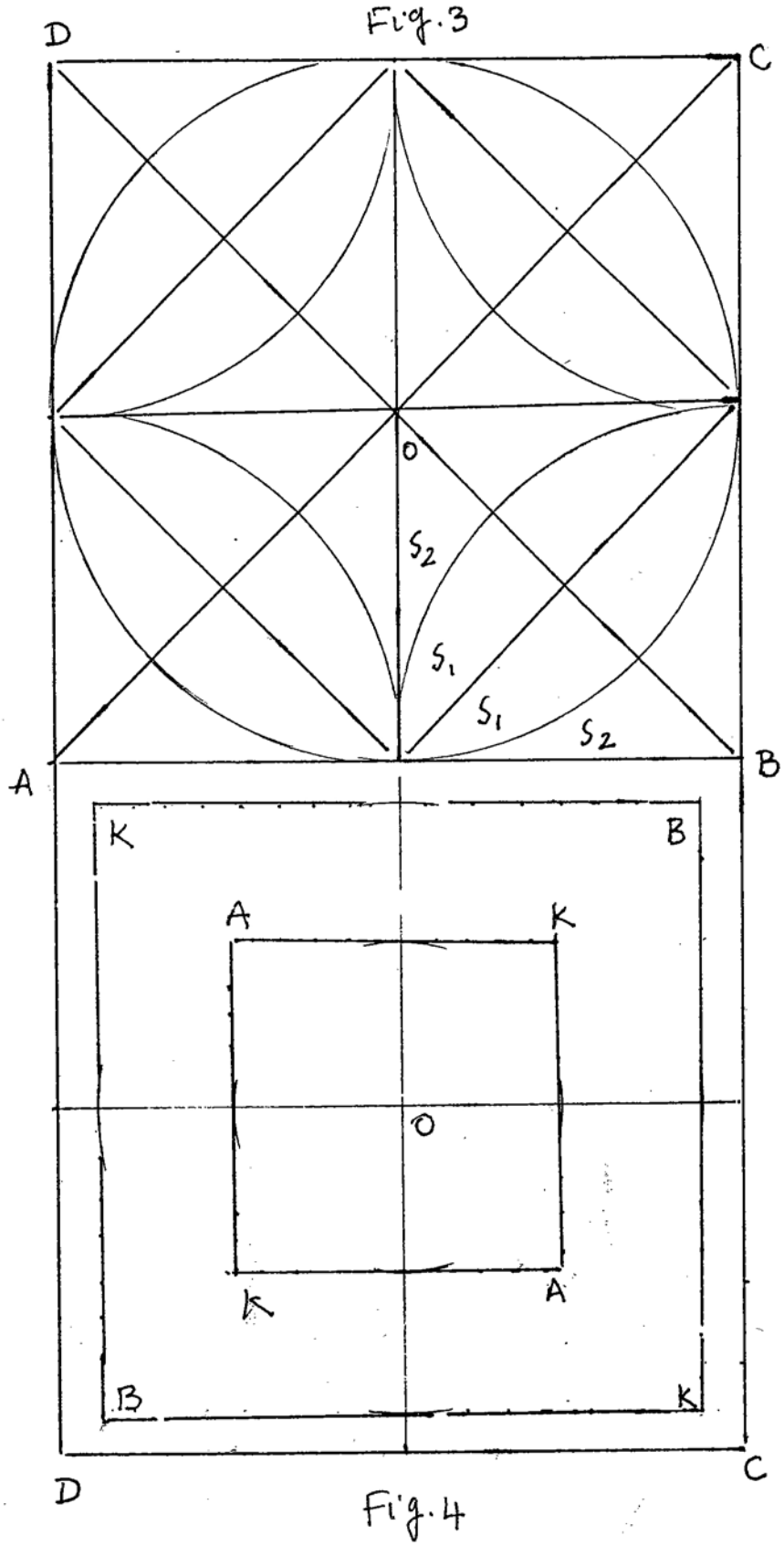


Fig. 2



To be exact the square has three broad divisions (Fig .3)

a) Core part $= 8\ S_2 = 8\left(\frac{2+\sqrt{2}}{128}\right) = \frac{2+\sqrt{2}}{16}$

b) Middle part $= 16S_1 = 16\left(\frac{6-\sqrt{2}}{128}\right) = \frac{6-\sqrt{2}}{8}$

c) Outer part $= 8\ S_2 = 8\left(\frac{2+\sqrt{2}}{128}\right) = \frac{2+\sqrt{2}}{16}$

Fig .1. $AB = \text{diameter} = 1$

$DF = \text{radius} = \frac{1}{2}$

$BC = \frac{\sqrt{2}}{2}$

$DE = \frac{\sqrt{2}}{4}$

$EF = \frac{2-\sqrt{2}}{4}$

$EF = BG$

Bisect AG twice $\frac{AG}{2} \rightarrow \frac{AG}{4} = \frac{2+\sqrt{2}}{16}$

$AJ = \frac{2+\sqrt{2}}{16}$

$JB = 1 - \frac{2+\sqrt{2}}{16} = \frac{14-\sqrt{2}}{16}$

Fig.2

$AB = 1$

$AJ = \frac{2+\sqrt{2}}{16}$

$JB = \frac{14-\sqrt{2}}{16}$

$BK = \sqrt{JB} = \frac{\sqrt{14-\sqrt{2}}}{4}$

Fig. 3

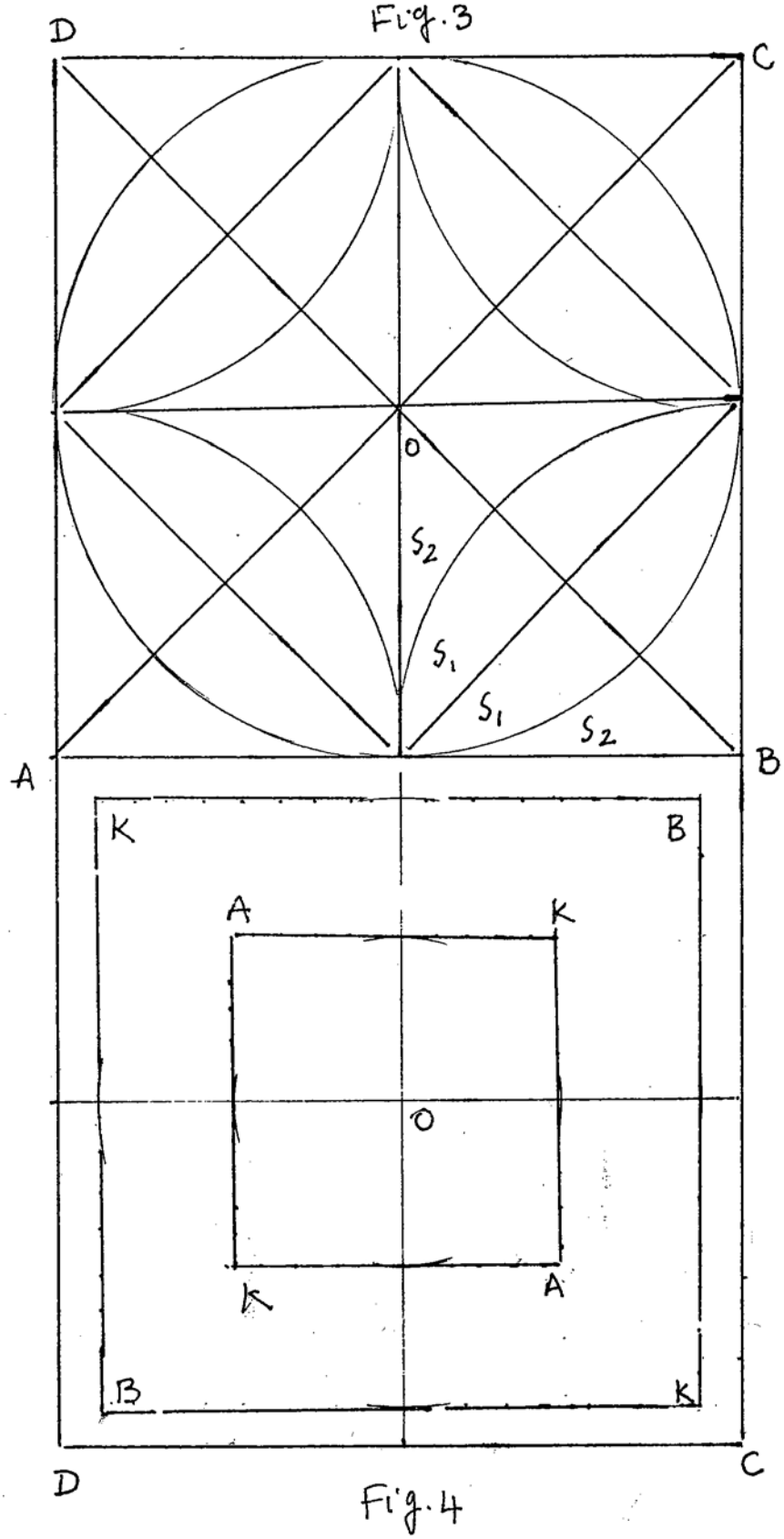


Fig. 4

To be exact the square has three broad divisions (Fig .3)

a) Core part $= 8\ S_2 = 8\left(\frac{2+\sqrt{2}}{128}\right) = \frac{2+\sqrt{2}}{16}$

b) Middle part $= 16S_1 = 16\left(\frac{6-\sqrt{2}}{128}\right) = \frac{6-\sqrt{2}}{8}$

c) Outer part $= 8\ S_2 = 8\left(\frac{2+\sqrt{2}}{128}\right) = \frac{2+\sqrt{2}}{16}$

Fig .1. AB = diameter = 1

$DF = \text{radius} = \frac{1}{2}$

$BC = \frac{\sqrt{2}}{2}$

$DE = \frac{\sqrt{2}}{4}$

$EF = \frac{2-\sqrt{2}}{4}$

EF = BG

Bisect AG twice $\frac{AG}{2} \rightarrow \frac{AG}{4} = \frac{2+\sqrt{2}}{16}$

$AJ = \frac{2+\sqrt{2}}{16}$

$JB = 1 - \frac{2+\sqrt{2}}{16} = \frac{14-\sqrt{2}}{16}$

Fig.2

AB =1

$AJ = \frac{2+\sqrt{2}}{16}$

$JB = \frac{14-\sqrt{2}}{16}$

$BK = \sqrt{JB} = \frac{\sqrt{14-\sqrt{2}}}{4}$

$$AK = \sqrt{AJ} = \frac{\sqrt{2+\sqrt{2}}}{4}$$

Fig. 4

1. Square AKAK

$$\text{Side} = AK \text{ (fig 2)} = \frac{\sqrt{2+\sqrt{2}}}{4}$$

$$\text{Area} = \frac{2+\sqrt{2}}{16}$$

2. Square BKBK

$$\text{Side} = BK \text{ (fig 2)} = \frac{\sqrt{14-\sqrt{2}}}{4}$$

$$\text{Area} = \frac{14-\sqrt{2}}{16}$$

3. Square ABCD

Side = 1
Area = 1

4. Area in between ABCD and BKBK

$$= ABCD - BKBK$$
$$= 1 - \frac{14-\sqrt{2}}{16} = \frac{2+\sqrt{2}}{16}$$

5. Area in between BKBK and AKAK

$$= \frac{14-\sqrt{2}}{16} - \frac{2+\sqrt{2}}{16} = \frac{6-\sqrt{2}}{8}$$

Finally the constituent areas in the Fig .3

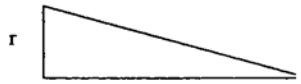
Core part	**= 8 S_2**	**= AKAK of Fig 4**
Middle part	**= 16 S_1**	**= BKBK – AKAK of Fig 4.**
Outer part	**= 8 S_2**	**= ABCD – BKBK of Fig 4.**

Deficiency of the Method

Formulas for the areas of S_1 and S_2 have been derived from the concerned dimensions without going into the details of justification.

CONTRIBUTIONS / CAPABILITIES OF 3.1415926 AND 3.1464466

S. No.	3.1415926....		3.1464466....
1	a.	This number was derived from Archimedes' Exhaustion method of B.C. period.	First time it was known from Gayatri method in 1998.
	b.	The following simplified formula of 1a gives $\pi = 3.2$ $b = \left\{2r^2 - r\left(4r^2 - a^2\right)\right\}^{\frac{1}{2}}$ Thus exhaustion method is undependable	Siva method gives an arithmetic proof $16S_1 + 16S_2 = a^2$ $16S_1 + 8S_2 = \frac{\pi a^2}{4}$
2	It survives now on infinite series only, the latest one is $\pi = \sum_{n=0}^{\alpha}\left(\frac{4}{8.n+1} - \frac{2}{8.n+4} - \frac{1}{8.n+5} - \frac{1}{8.n+6}\right)\left(\frac{1}{16}\right)^n$		$\frac{14-\sqrt{2}}{4}$ is a finite number. Infinite series can be had but it will become superfluous for $\frac{14-\sqrt{2}}{4}$
3	World of mathematics has been believing it for more than 20 centuries		It's age is just 10 years.
4	It is a number. It exists in the human imagination Geometrical length for it is yet to be drawn. William		It is also a number. It is known from geometrical constructions. It does not exist without its corresponding physical form: area / length.
5	Lot of human effort is needed to understand it. Shanks spent 20 long years to know even its partial personality.		School kids can derive it in no time.
6	Super computer is a must for its calculation		Straightedge, compass etc are enough.
7	Certain areas in the different geometrical constructions are represented in terms of π only. Formulae are given with their creators.		

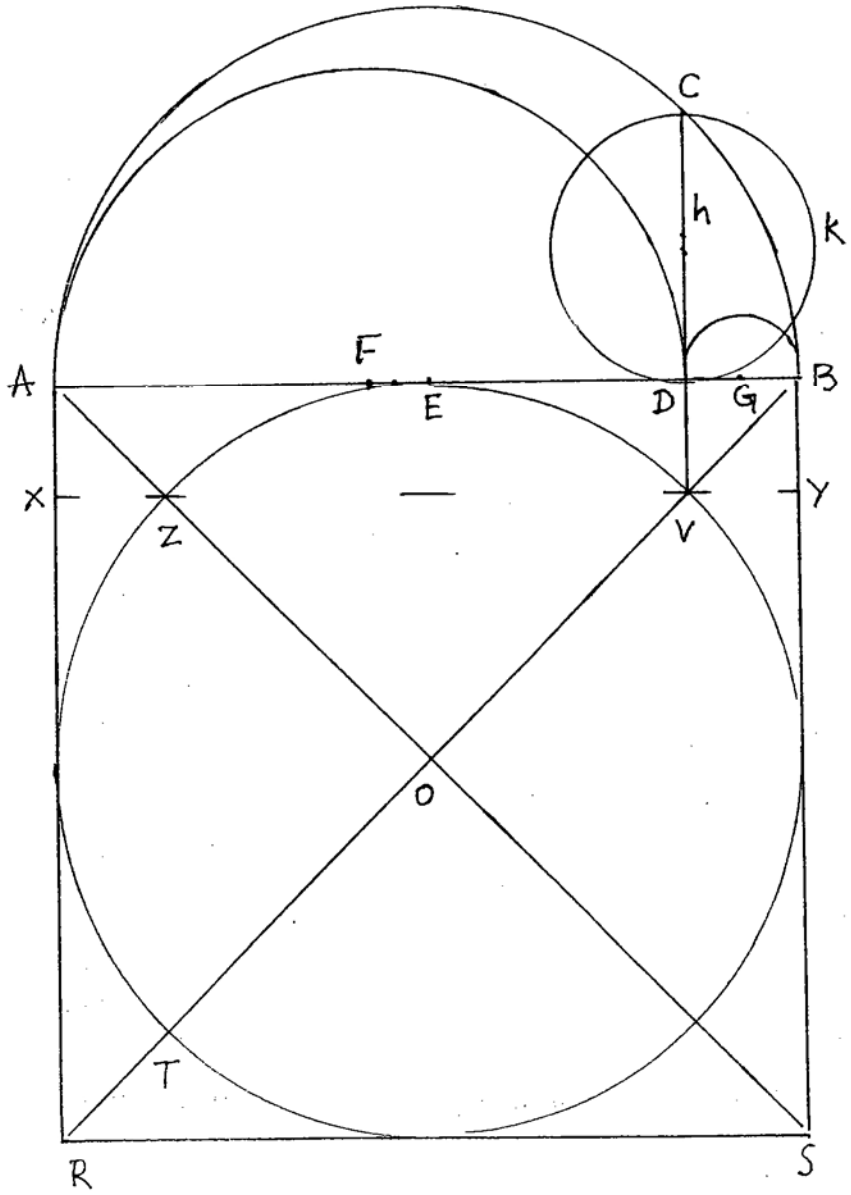
	a.	Archimedes:  = area of the circle	Construction done
	b.	Hippocrates of Chios	
		Lune = triangle **in area**	Arithmetic values are given
		Lune + circle = triangle + hexagon	,,
		2 Lunes = triangle	,,
		3 Lunes + semi circle = trapezium	,,
		2 Lunes = triangle (right angled)	,,
		4 lunes = square	,,
	c.	Trevor Easingwood $\frac{1}{2} - \frac{\pi}{8}$ $\frac{\pi}{3} - \frac{\sqrt{3}}{4}$ $\left(\frac{\pi}{3} - \frac{\sqrt{3}}{4}\right) - \frac{\pi}{8}$	Different lengths of **their constructions** give this π value and not 3.1415926
	d.	C.R. Fletcher $1 - \frac{1}{4}\pi$	,,
	e.	J.V. Narlikar $\left\{\frac{\pi}{3} - \frac{\sqrt{3}}{2}\right\} a^2$	,,

	f.	Stephen Ainely $\left(\frac{\pi}{6}+\frac{\sqrt{3}}{4}\right)-\frac{\pi}{4}$	,,
8	It is a transcendental number		It is an algebraic number.
9	Squaring of circle is impossible		Possible - done
10	Circle a squre is also impossible		Also possible - done
11	Unit area of circle is unthinkable		Done
12	Louis Loynes could approximate quadrature of the circle		With Loynes' method exact quadrature of the circle is done
13	Crockett Johnson could show an approximate geometrical construction for $\sqrt{\pi}$		With Johnson's method exact length for $\sqrt{\pi}$ is obtained.
14	A geometrical ensemble to generate the squaring of the circle by Helmut Sander for the NN (theoretical diagram is devised, available in the internet.)		Exact geometrical construction is submitted for Sander's idea.
15	Impossible		Unit side of the square from the length equal to $\frac{\pi}{4}$ is obtained.
16	Impossible		Pythagorean way of calculating arc's length is submitted.
17	Impossible		$\sqrt{2}$ length is obtained from the ratio of circle and square
18	Impossible		The circumference equal to $\sqrt{2}$ length is obtained.

3.1415926….. is **not** wrong. It has satisfied millions of people for many centuries.

Dimensions - based derived π value from the area of Arbelos

The arbelos was investigated by Archimedes. "An arbelos consists of two small semicircles inside a large one, that such the sum of the diameters of the small semicircles is the diameter of the large one" (VNR Concise Encyclopedia of Mathematics, 1975 page – 175)



1. **Square ABSR**

Side = AB = XY = d

E = Mid point of AB

2. **Inscribe Circle** in the square ABSR

O = Centre

TV = Diameter = Side = AB = d

$$OV = \text{Radius} = \frac{d}{2}$$

XY = Side = d

$$ZV = \frac{\sqrt{2}d}{2}$$

$$XZ = VY = \frac{XY - ZV}{2} = \frac{d - \frac{\sqrt{2}d}{2}}{2} = \frac{2d - \sqrt{2}d}{4}$$

3. **Square DVYB**

$$DB = \text{Side} = \frac{2d - \sqrt{2}d}{4}$$

G = mid point of DB

$$AD = AB - DB = d - \frac{2d - \sqrt{2}d}{4} = \frac{2d + \sqrt{2}d}{4}$$

F= mid point of AD

4. **Semicircle ACB**

AB = Diameter = d

$$EB = \text{Radius} = \frac{d}{2}$$

5. **Semicircle on DB**

6. **Semicircle on AD**

7. **Altitude theorem** to obtain DC length = h

$$\sqrt{AD \times DB} = \sqrt{\left(\frac{2d+\sqrt{2}d}{4}\right)\left(\frac{2d-\sqrt{2}d}{4}\right)} = \frac{\sqrt{2}d}{4}$$

8. Circle k has diameter = DC = h

9. Arbelos is the area ~~in between~~ inside the larger semicircle on AB ~~and~~, but minus the two areas of smaller semicircles on AD and DB

10. **Area of Arbelos = area of Circle k**

11. Area of Arbelos = $\frac{\text{Area of semicircle on AB}}{4}$

When AD $= \frac{2d+\sqrt{2}d}{4}$ and DB $= \frac{2d-\sqrt{2}d}{4}$

some interesting relations exist between different lengths.

(i) $ZV \times \sqrt{2} = AB$

$$\frac{\sqrt{2}d}{2} \times \sqrt{2} = d$$

(ii) Circumference of k circle $\times \sqrt{2}$ = circumference of larger semicircle ABC on AB diameter

(iii) Semichord DC length × 4 = RB diagonal of ABSR square

(iv) Area of k circle x 4 = Area of ABC semicircle on AB.

(v) Semichord DC length x 2 = ZV length of ABSR square .

(vi) $DC + DB = \frac{\sqrt{2}d}{4} + \frac{2d-\sqrt{2}d}{4} = \frac{d}{2}$ = EB = radius

(vii) $3AB + DB = 3d + \frac{2d - \sqrt{2}d}{4} = \frac{14d - \sqrt{2}d}{4}$

is the length of the **full** circle of the inscribed circle in the square and is the **double** the length of the semicircle ABC on AB diameter .

12. Area of the larger semicircle on AB diameter

$EB = \text{radius} = \frac{d}{2}$

$DB = \frac{2d - \sqrt{2}d}{4}$

a) Length of the circumference of the semicircle on AB diameter $= 3EB + \frac{DB}{2}$

$= \frac{3d}{2} + \frac{2d - \sqrt{2}d}{4 \times 2} = \frac{14d - \sqrt{2}d}{8}$

b) The area of the circle is the product of half the circumference and the radius.

Full Circle area = half circumference x radius

$= \frac{14d - \sqrt{2}d}{8} \times \frac{d}{2} = \left(\frac{14 - \sqrt{2}}{16}\right)d^2$

c) Area of the **semicircle**

$= \left(\frac{14 - \sqrt{2}}{16}\right)\frac{d^2}{2} = \left(\frac{14 - \sqrt{2}}{32}\right)d^2$

13. **Area of Arbelos**

$= \frac{\text{area of larger semicircle on AB}}{4} = \left(\frac{14 - \sqrt{2}}{128}\right)d^2$

14 a) **Area of Circle k** = area of Arbelos

Formula for the area of Arbelos $= \dfrac{\pi q(d-q)}{4}$

where,

$d = AB$ = diameter

$q = DB = \dfrac{2d-\sqrt{2}d}{4}$

$d - q = AD = \dfrac{2d+\sqrt{2}d}{4}$

$\therefore \quad \dfrac{\pi q\,(d-q)}{4} = \left(\dfrac{14-\sqrt{2}}{128}\right) d^2$

$= \dfrac{\pi\left(\dfrac{2d-\sqrt{2}d}{4}\right)\left(\dfrac{2d+\sqrt{2}d}{4}\right)}{4} = \left(\dfrac{14-\sqrt{2}}{128}\right) d^2$

$\therefore \quad \pi = \dfrac{14-\sqrt{2}}{4}$

b) **Area of Circle K** $= \pi\left(\dfrac{h}{2}\right)^2$

where h = DC semichord $= \dfrac{\sqrt{2}d}{4}$

$\pi\left(\dfrac{h}{2}\right)^2 = \pi\left(\dfrac{\dfrac{\sqrt{2}d}{4}}{2}\right)^2 = \left(\dfrac{14-\sqrt{2}}{128}\right) d^2$

$\therefore \quad \pi = \dfrac{14-\sqrt{2}}{4}$

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* π **: A biography of the world's most mysterious number"**, by Alfred S. Posamentier and Ingmar Lehmann, Prometheus Books, 2004, Page – 212.

Acknowledgements

This author is grateful to Archimedes of Syracuse, Greece, for his ingenious geometrical construction which has helped to derive the proposed π value.

This author is also thankful to Prof. Alfred S. Posamentier and Prof. Ingmar Lehmann whose book has just arrived form New York, USA – fortunately – on the midnoon of 6-5-2008 and helped to have the formula for the area of circle k, at the fag end of the preparation of this article.

Further, this author is indebted to Prof. Lennart Berggren, Prof. Peter Borwein and Prof. Jonathan Borwein of Canada whose classic on π entitled **Pi – A Source Book** helped to learn many concepts on Pi number. This author has read no no studied the book four times – beginning to end. In this article "…. the area of the circle is the product of half the circumference and the radius ...." - (Page – 21) has helped in giving sensible shape to this though a simple article. Indeed , it (book) is an encyclopedia on Pi.

Born from Circle Square Union

OABC = Square

Side = OA=AB=BC=CO= r

OB = diagonal = $\sqrt{2}\,r$

O as centre, draw an arc CDA, with radius OA = r

D, an intersection point of arc and diagonal

OD = radius = r

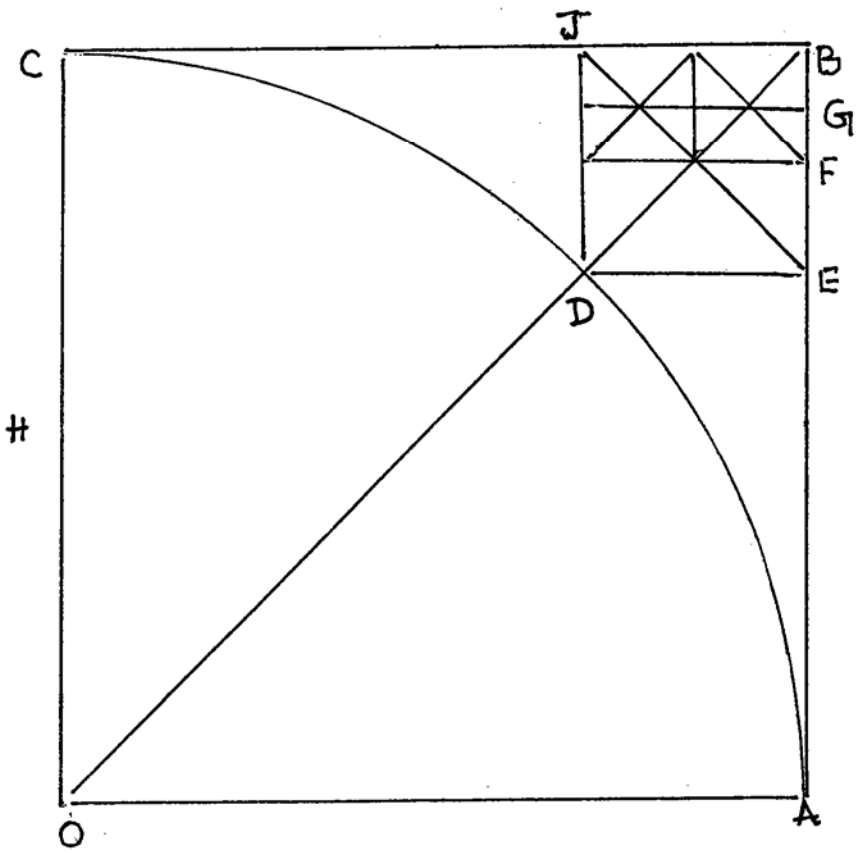
DB = OB – OD = $\sqrt{2}\,r - r$

Desquare DB (any length of the square when it is divided by $\sqrt{2}$ is called "desquaring" of that length) to get EB length.

$$\frac{DB}{\sqrt{2}} = \frac{\sqrt{2}\,r - r}{\sqrt{2}} = EB$$

DE BJ = small square

Side = EB = $\frac{\sqrt{2}\,r - r}{\sqrt{2}}$



Divide EB twice to get $BG = \frac{EB}{4} = \frac{\sqrt{2}\,r - r}{4\sqrt{2}}$

H is the mid point of OC, $OH = HC = \frac{r}{2}$

Length of the arc CDA

$$= \mathbf{HC + CB + BG} = \frac{r}{2} + r + \frac{\sqrt{2}\,r - r}{4\sqrt{2}}$$

Arc CDA is $\frac{1}{4}$ of the circumference of the full circle i.e., $\frac{2\pi r}{4}$

$$\text{Arc CDA} = \left(\frac{r}{2} + r + \frac{\sqrt{2}r - r}{4\sqrt{2}}\right) = \frac{\pi r}{2}$$

$$\therefore \quad \pi = \frac{14 - \sqrt{2}}{4}$$

POST SCRIPT

From the above method we have obtained $\pi = 3.1464466..... = \frac{14-\sqrt{2}}{4}$. People have been sending messages (to the author) from all over the world that $\frac{14-\sqrt{2}}{4}$ is not π. They cite, reasons similar to the following passage:

"..... *But even more fascinating is the fact that* π *cannot be calculated by a combination of the operation of addition, subtraction, multiplication, division, and* ***square root extraction.*** *This means* π *is a type of non- rational number called a transcendental number"* ..... ***ALFRED S. POSAMENTIER*** *(2004).*

The above method is against *in toto* with the current view of the world.

The challenging fight now is in between inborn $\frac{14-\sqrt{2}}{4}$ and non-geometrical, imposed, 1.24 trillion decimal strong 3.1415926.....

The former has one supporter: a zoology teacher, whereas 3.1415926..... is backed by the whole world.

DIMENSIONAL EVALUATION OF 3.1415926.....

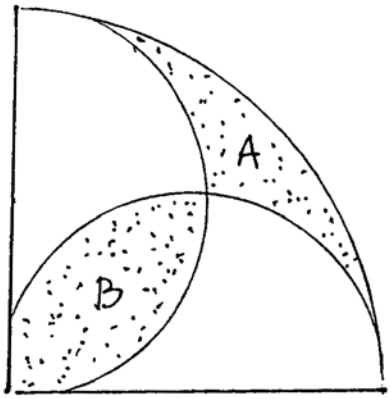


Fig.1

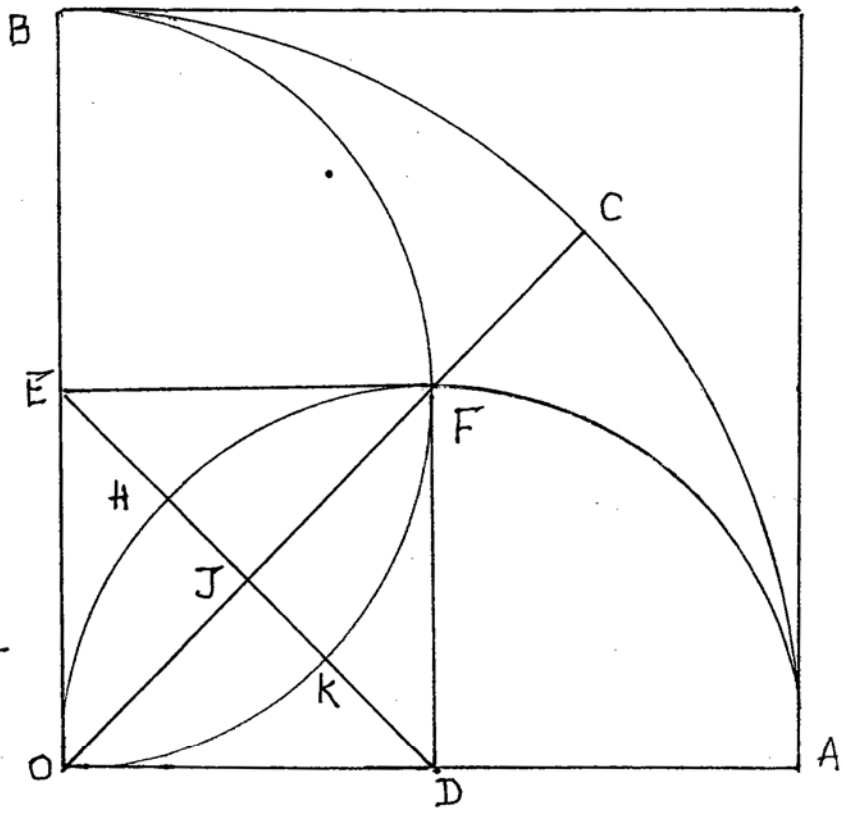


Fig.2

The above geometrical construction (fig.1) is obtained from the book **"Pi – A biography of the world's most mysterious number"** authored by Prof. Alfred S. Posamentier and Prof. Ingmar Lehmann and Published by Prometheus books, in 2004, Page 180.

In this method an attempt has been made to interpret geometrically the π value of 3.1415926 ..... in other words, its identity / agreement with any one of the dimensions / lengths of construction has been probed.

METHOD (Fig. 2)

1. Arc : BCA : $\frac{1}{4}^{th}$ of the circumference

O = Centre

OA = OB = OC = Radius = r

2. Semicircle : BFO

E = Centre

EO = EB = Radius = $\frac{r}{2}$

3. Semicircle : AFO

D = Centre

DO = DA = $\frac{r}{2}$

4. Semicircle BFO = Semicircle AFO

5. BFAC Region (A region of Fig. 1.)

OC = Radius = r

OF = $\frac{\sqrt{2}r}{2}$

FC = OC – OF = r - $\frac{\sqrt{2}r}{2}$ = $\frac{2r-\sqrt{2}r}{2}$

6. OKFH Petal (B region of Fig. 1.)

ODFE = Square

OD = Side = $\frac{r}{2}$

DE = OF = $\frac{\sqrt{2}r}{2}$

7. Semicircle BFO

Radius = EK = $\frac{r}{2}$

EJ = JD = OJ = JF = $\frac{\sqrt{2}r}{4}$

JK = EK - EJ = $\frac{r}{2} - \frac{\sqrt{2}r}{4} = \frac{2r-\sqrt{2}r}{4}$

JK = JH

HK = JH + JK = $\frac{2r-\sqrt{2}r}{2}$

8. HK = FC = $\frac{2r-\sqrt{2}r}{2}$

Thus HK and FC lengths of Fig. 2 are same.

9. Further, **BFAC = OKFH** i.e. areas of A and B regions of Fig.1 are equal. (established by Prof. A.S.Posamentier)

And the formula derived to find out the area for both the regions is

$$r^2\left(\frac{\pi}{2}-1\right)$$

From this step onwards this author is trying to **rewrite** the above forumula (with a sole motive of explaining 3.1415926.....in terms of **HK and FC lengths** of fig.2).

In 8th step we have seen HK = FC = $\frac{2r-\sqrt{2}r}{2}$

10. Let us rewrite $r^2\left(\frac{\pi}{2}-1\right)$ as $\frac{r^2}{2}(\pi-2)$

11. **π = 3.1415926.....**

Let us incorporate this number in the above formula

$$\frac{r^2}{2}(3.1415926-2) = \frac{r^2}{2}\times 1.1415926$$

12. **Proposed π = 3.1464466 ..... =** $\frac{14-\sqrt{2}}{4}$

Similarly, let us incorporate the above number

$$\frac{r^2}{2}(3.1464466-2) = \frac{r^2}{2}\times 1.1464466$$

13. **In step** **11 we have** $\frac{r^2}{2}\times 1.1415926$

12 we have $\frac{r^2}{2}\times 1.1464466$

9 we have HK = FC = $\frac{2r-\sqrt{2}r}{2}$

When radius = $\frac{1}{2}$

HK = FC = $\frac{2r-\sqrt{2}r}{2}$ becomes $\frac{\left(2\times\frac{1}{2}\right)-\left(\sqrt{2}\times\frac{1}{2}\right)}{2}$

$$= \frac{2-\sqrt{2}}{4} = 0.1464466 .....$$

14. i. a^2 is for the area of square where a is its side

ii 4a is for the perimeter of square

iii. $\frac{1}{2}$ ab is for the area of triangle where a is the altitude, b is the base.

Thus, side, base, altitude are different dimensions / lengths of triangle / square.

Similarly HK and FC are the dimensions / lengths of A region and B region of fig.1.

15. Is it inappropriate to evaluate π number in terms of dimensions / lengths of the construction?

Let us not forget π number is not only represents π ratio but also it represents as a measure of the length of the circumference of a circle of its unit diameter.

16. 0.1464466 ..... of HK / FC of step 13 is seen exactly in the 12th step of formula i.e. $\frac{r^2}{2}\times 1.1464466$ and **not** seen in the 11th step

$$\text{i.e.}\frac{r^2}{2}\times 1.1415926$$

17. Can we deny the following:

i. 3.1415926.....is the perimeter of the inscribed and circumscribed polygons of the circle.

ii. Equating perimeter to the circumference is purely logical conclusion

iii. Perimeter is **always** angular in nature

iv. From John Wallis (1650) on words π 3.1415926.....has gradually become **non -** geometrical. It has transformed itself more a number than a symbol of circumference – diameter ratio.

v. With the proofs of Johann Heinrich Lambert and Carl Louis Ferdinand Lindemann, area / circumference of a circle have to be represented in the **decimal form** only and never in the square root two expression.

vi. **Sides** of the different inscribed regular polygons are really the **Chords** of the superscribed circle.

In trigon side = $r\sqrt{3}$ where r = radius

tetragon side = $r\sqrt{2}$

pentagon side = $\frac{r}{2}\sqrt{\left(10-2\sqrt{5}\right)}$

Thus geometrical lengths for example side which is **inside** the circle have necessarily to be represented by surds $\sqrt{2},\sqrt{3},\sqrt{5}$ etc.

Interestingly, the same surds are **forbidden** for the circumference / area of a circle.

vii. (a). Hippocrates of Chios squared the lunes. He **did** square the circle: **Area of lune + area of circle = area of triangle + area of hexagon.**

Ref: **Mathematics and Mathematicians by P.Dedron and J. Itard.**

(b) Archimedes of Syracuse has said " the area of any circle is equal to a right angled triangle in which one of the sides of about the right angle is equal to the radius and the other to the circumference.

(c) S. Ramanujan of India squared the circle partially. These three great people did believe squaring of circle not **un**geometrical.

(d) 3.1415926.....however says every thing – squaring or circling etc is impossible.

From the above facts is it unmathematical to call 3.1415926.....as an **abstract number** and has no base in geometry?

Squaring and conversion of Arbelos of Archimedes into a triangle in area

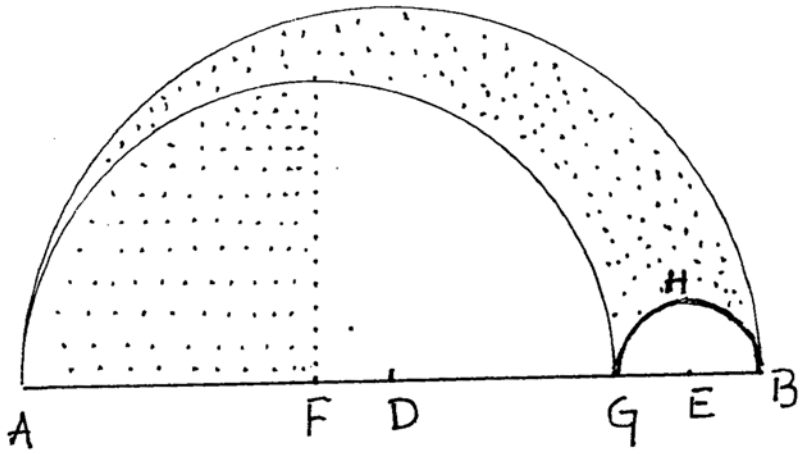


Fig. 1

1. Semicircle on AB (Fig . 1)

   AB = diameter = 1

   D = midpoint of AB

   DA = DB = radius = $\frac{1}{2}$

2. Semicircle on GB

   GB = diameter = $\frac{1}{5} = 0.2$

   E = mid point of GB

   EG = EB = radius = $\frac{1}{10} = 0.1$

3. Semicircle on AG

   AG = diameter = $\frac{4}{5} = 0.8$

   F = mid point of AG

   FA = FG = radius = $\frac{2}{5} = 0.4$

4. Arbelos = ABHG (shaded region)

5. **Areas of three semicircles and Arbelos.**

Here $\pi = \frac{14-\sqrt{2}}{4}$ is used, as 3.1415926 ..... does not fit in, in any one of the following geometrical constructions.

(a) AB semicircle, d = 1

$$\frac{1}{2}\left(\frac{\pi d^2}{4}\right)=\frac{1}{2}\left(\frac{14-\sqrt{2}}{4}\times 1\times 1\right)\frac{1}{4} = \frac{14-\sqrt{2}}{32}$$

(b) GB semicircle, d = $\frac{1}{5}$

$$\frac{1}{2}\left(\frac{\pi d^2}{4}\right)=\frac{1}{2}\left(\frac{14-\sqrt{2}}{4}\times\frac{1}{5}\times\frac{1}{5}\right)\frac{1}{4} = \frac{14-\sqrt{2}}{800}$$

(c) AG semicircle, d = $\frac{4}{5}$

$$\frac{1}{2}\left(\frac{\pi d^2}{4}\right)=\frac{1}{2}\left(\frac{14-\sqrt{2}}{4}\times\frac{4}{5}\times\frac{4}{5}\right)\frac{1}{4} = \frac{14-\sqrt{2}}{50}$$

(d) Area of Arbelos

= AB semicircle - (GB semicircle + AG semicircle)

$$= \left(\frac{14-\sqrt{2}}{32}\right)-\left(\frac{14-\sqrt{2}}{800}+\frac{14-\sqrt{2}}{50}\right) = \frac{14-\sqrt{2}}{100}$$

6. Area of Arbelos
   is half of AG semicircle (shaded region AF of Fig. 1) and
   eight times of GB semicircle.

7. Area of GB semicircle is $\frac{1}{16}$ th of AG semicircle.

8. Area of larger semicircle on AB is 25 times of GB semicircle.

9. Thus all the three semicircles along with the arbelos are related among themselves **in clear proportions.**

II. Conversion of Arbelos into a Triangle in Area

In this direction, at the outset, let us obtain a length equal to $\frac{2\sqrt{2}}{5}$, for which, the following geometrical construction, is required.

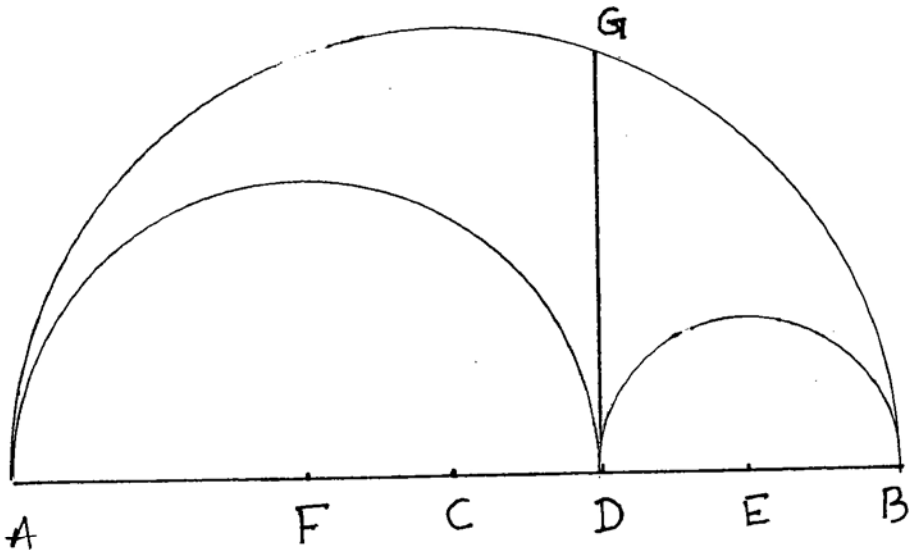


Fig. 2

10. Semicircle on AB

AB = diameter = $\frac{6}{5}$ = 1.2

C = mid point of AB

11. Semicircle on DB

DB = diameter = $\frac{2}{5}$ = 0.4

E = mid point of DB

12. Semicircle on AD

AD = diameter = $\frac{4}{5}$ = 0.8

F = mid point of AD

13. Draw a perpendicular line at D on AB which meets circumference at G.

Apply altitude theorem to get DG length

$$\sqrt{AD \times DB} = \sqrt{\frac{4}{5} \times \frac{2}{5}} = \frac{2\sqrt{2}}{5} = DG$$

14. Fig 3

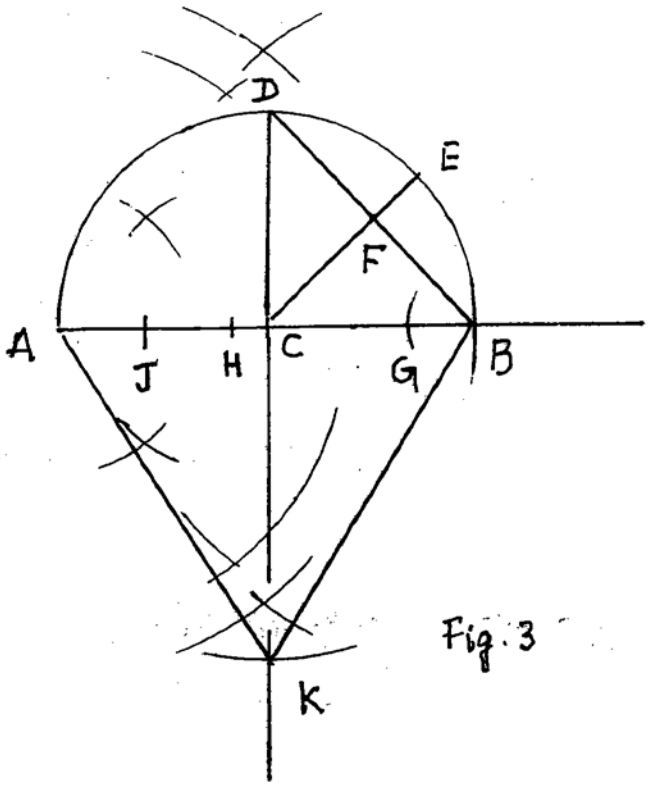


Fig. 3

AB of Fig 3 = DG of Fig .2 = $\frac{2\sqrt{2}}{5}$

AB = diameter = $\frac{2\sqrt{2}}{5}$

C = mid point of AB

Draw a semicircle with centre C on AB

CA = CB = CD = CE = radius = $\frac{\sqrt{2}}{5}$

$$BD = CB \times \sqrt{2} = \left(\frac{\sqrt{2}}{5}\right)\sqrt{2} = \frac{2}{5} = 0.4$$

$$BF = CF = DF = \frac{1}{5} = 0.2$$

$$EF = CE - CF = \frac{\sqrt{2}}{5} - \frac{1}{5} = \frac{\sqrt{2}-1}{5}$$

$$\mathbf{EF = GB = \frac{\sqrt{2}-1}{5}}$$

$$AG = AB - GB = \frac{2\sqrt{2}}{5} - \frac{\sqrt{2}-1}{5} = \frac{\sqrt{2}+1}{5}$$

Bisect AG twice

$$\frac{AG}{2} \rightarrow \frac{AG}{4} = \frac{\sqrt{2}+1}{5} \rightarrow \frac{\sqrt{2}+1}{10} \rightarrow \frac{\sqrt{2}+1}{20}$$

$$AJ = \frac{\sqrt{2}+1}{20}$$

$$JB = AB - AJ = \frac{2\sqrt{2}}{5} - \frac{\sqrt{2}+1}{20} = \frac{7\sqrt{2}-1}{20}$$

$$JB = \frac{7\sqrt{2}-1}{20}$$

$$JB = CK = \frac{7\sqrt{2}-1}{20}$$

Triangle = ABK

where a = altitude = $CK = \frac{7\sqrt{2}-1}{20}$

b = base = $AB = \frac{2\sqrt{2}}{5}$

$$\text{area} = \frac{1}{2}ab = \frac{1}{2}(CK \times AB) =$$

$$= \frac{1}{2}\left(\frac{7\sqrt{2}-1}{20}\right)\left(\frac{2\sqrt{2}}{5}\right) = \frac{14-\sqrt{2}}{100}$$

Thus the area of ABK triangle is same as the area of Arbelos of Fig. 1 in step 5 (d).

III. Squaring of Arbelos

15. Square ABCD (Fig .4)

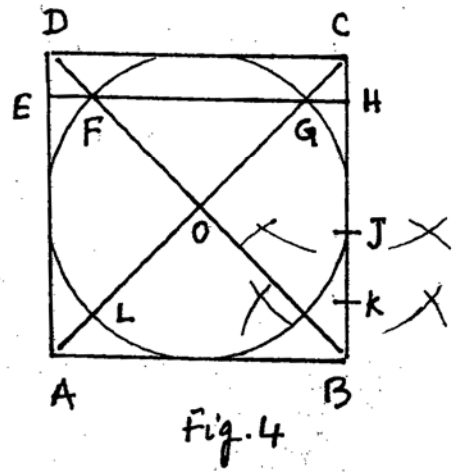


Fig. 4

Side $AB = EH = \frac{2}{5} = 0.4$

Inscribe a circle in the square

O = Centre

$LG = \text{Diameter} = \text{Side} = \frac{2}{5} = 0.4$

$OG = OF = \text{radius} = \frac{1}{5} = 0.2$

$$FG = \left(\frac{1}{5}\right)\sqrt{2} = \frac{\sqrt{2}}{5}$$

$$DE = EF = GH = CH = \frac{EH-FG}{2} = \left(\frac{2}{5}-\frac{\sqrt{2}}{5}\right)\frac{1}{2} = \frac{2-\sqrt{2}}{10}$$

$$HB = BC - CH = \frac{2}{5} - \frac{2-\sqrt{2}}{10} = \frac{2+\sqrt{2}}{10}$$

Bisect HB twice

$$HB \rightarrow \frac{HB}{2} \rightarrow \frac{HB}{4} = \frac{2+\sqrt{2}}{40} = KB$$

$$CK = CB - KB = \frac{2}{5} - \frac{2+\sqrt{2}}{40} = \frac{14-\sqrt{2}}{40}$$

16. Semicircle ABC (Fig .5)

D = Centre

$$AB = \text{diameter} = \frac{2}{5} = 0.4$$

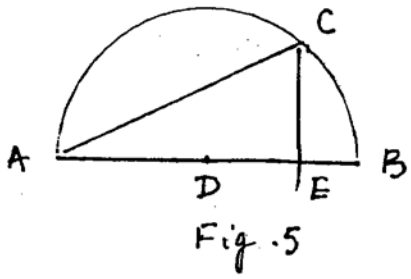


Fig .5

$$AE \text{ of Fig 5} = CK \text{ of Fig.4} = \frac{14-\sqrt{2}}{40}$$

$$EB = AB - AE = \frac{2}{5} - \frac{14-\sqrt{2}}{40} = \frac{2+\sqrt{2}}{40}$$

Draw a perpendicular line at E on AB which meets circumference at C

Apply altitude theorem to get CE length

$$\sqrt{AE \times EB} = \sqrt{\left(\frac{14-\sqrt{2}}{40}\right)\left(\frac{2+\sqrt{2}}{40}\right)} = \frac{\sqrt{26+12\sqrt{2}}}{40} = CE$$

Join AC

Triangle AEC

$$AC = \sqrt{AE^2 + CE^2} = \sqrt{\left(\frac{14-\sqrt{2}}{40}\right)^2 + \left(\frac{\sqrt{26+12\sqrt{2}}}{40}\right)^2}$$

$$= \frac{\sqrt{14-\sqrt{2}}}{10}$$

$AC = \frac{\sqrt{14-\sqrt{2}}}{10}$ is the required side of the square whose area is equal to the area of the Arbelos of Fig. 1. in step 5(d).

SIVA METHOD

$\frac{22}{7}$ is being used in our class rooms as equal to π. For more accuracy 3.1415926..... is used. Both the values of π are approximate numbers. Even Super Computers have failed to give an exact value to π.

Again you will notice how the more sides the polygon has, the closer its perimeter gets to the circumference of the circle—which we now know is π.

Archimedes, as we said before, saw the inscribed and the circumscribed polygons "sandwiching in" the circle, as seen below by the inscribed and circumscribed dodecagons ($n = 12$).

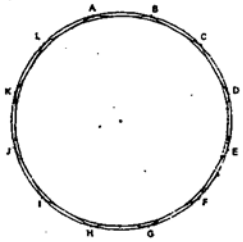


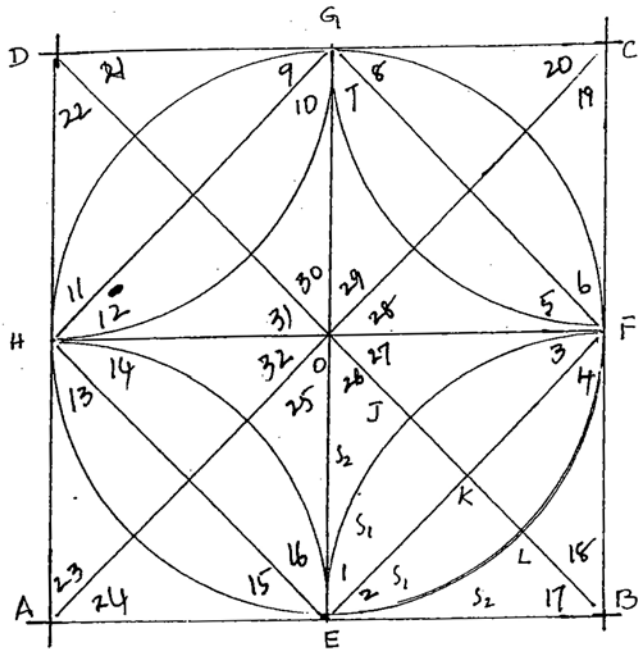
Fig. 3-8

He essentially suggested taking the average of the two perimeters for each type of polygon to get a better approximation of π.

n	Perimeter of inscribed polygon of *n* sides	Perimeter of circumscribed polygon of *n* sides	Average of the perimeters of the inscribed and circumscribed polygons of *n* sides
3	2.59807621135331594029116951225 88...	5.19615242270663188058233902451 76...	3.89711431702997391043675426838 75...
4	2.82842712474619009760337744841 94...	4.00000000000000000000000000000 00...	3.41421356237309504880168872420 95...
5	2.93892626146236564584352977319 54...	3.63271264002680442947733787403 1...	3.28581945074458503766043178029 9...
6	3.00000000000000000000000000000 00...	3.46410161513775458705489268301 17...	3.23205080756887729352744634150 55...
7	3.03718617382290684333037832993 85...	3.37102233165270051032513647139 88...	3.20410425273780367682775740066 8...
8	3.06146745892071817382767987224 32...	3.31370849898476039041350979367 76...	3.18758797895273928212059483296...
9	3.07818128993101859739689653214 03...	3.27573210839582125215943094499 15...	3.17695669916341992477816373856 55...
10	3.09016994374947424102293417182 82...	3.24919696232906326155871412215 13...	3.16968345303926875129082414698 95...
11	3.09905812525572667452559706881 28...	3.22989142332203385420668296859 44...	3.16447477378888026451614001870 3...
12	3.10582854123024914818678605144 86...	3.21539030917347247767064390192 95...	3.16060942520186081292871497670 85...
13	3.11110363573825097293379844138 28...	3.20421221941570764730031492162 91...	3.15765792757697931011705668150 55...
14	3.11529307538840166004463590295 51...	3.19540864146209913308655906885 42...	3.15535085842525039656559748590 45...
15	3.11867536226639005652613426606 69...	3.18834842505033187889387490855 12...	3.15351189365836096771000458731 35...
24	3.13262861328123819716174946949 17...	3.15965994209750048331166349778 332...	3.14614427768936934023919222366 2...
36	3.13760673891569424809031375014 9...	3.14959188693326418799267209965 86...	3.14359931292447921804149292490 35...
54	3.13982076116569474109239290974 19...	3.14514184337910393914934210860 04...	3.14248130227239934012086750917 05...
72	3.14059589030419198428622155911 6...	3.14358788941286845956260303991 74...	3.14209188985853022192441229951 65...
90	3.14095470322508744813956634628...	3.14286925425729574503623631963 53...	3.14191197874119159658790133295 75...
96	3.14103195089050963811135292645 97...	3.14271459964536429816885909377 21...	3.14187327526793896814010601011 55...
120	3.14123379694477831327340226649 35...	3.14231058830243146672365927534 28...	3.14177219262360488999853077091 75...
180	3.14143315871103230749541613293 69...	3.14191168707916543772320113955 1...	3.14167242289509887260930863624 35...
250	3.14150997083815197856364728719 87...	3.14175803084489443537076906133 84...	3.14163400084152320696970817426 8...
500	3.14157198277947562486765507897 99...	3.14163399594488606459529576947 32...	3.14160298936218084473147542422 6...
1,000	3.14158748587955633519332270354959...	3.14160289056156126041343290105 4...	3.14159523746785973898728516 28...
10,000	3.14159260191266569297934647928 9...	3.14159275694405291972467077191 18...	3.14159267942835930635200862 56...

The above extract is from: "π : **A biography of the world's most mysterious number**" by A.S. Posamentier (2004), speaks 3.1415926..... is actually the length of the perimeter of the polygon.

Siva method gives an exact value to π and is explained below.



Draw a circle with centre O and radius ½ . Diameter is 1. Draw four equidistant tangents on the circle. They intersect at four points A, B, C and D resulting a square ABCD. The side is also equal to 1 just like the length of the diameter of the circle. Draw two diagonals. E, F, G, and H are the mid points of four sides of the square. Join EG, FH, EF, FG, GH and HE. Draw four arcs with radius ½ and with centres A, B, C and D. Now, the circle square system is divided into 32 segments and number them 1 to 32. 1 to 16 are called S_1 segments and 17 to 32 are called S_2 segments. Let us calculate the areas of S_1 and S_2 and finally from S_1 and S_2 the area of circle

Square = ABCD AB = Side = a = 1 O = centre

Circle = EFGH HF = diameter = d = a =1

OL = Radius = ½ BD = diagonal = $\sqrt{2}$

$OB = \frac{\sqrt{2}}{2}$ $OK = KB = \frac{\sqrt{2}}{4}$

$JK = KL = OL - OK = \frac{1}{2} - \frac{\sqrt{2}}{4} = \frac{2-\sqrt{2}}{4}$ 1-16. are S_1 segments

$OJ = BL = OB - OL = \frac{\sqrt{2}}{2} - \frac{1}{2} = \frac{\sqrt{2}-1}{2}$ 17-32 are S_2 segments

$$\text{Area of } S_1 \text{ segments} = \frac{\Delta EKB \text{ area}}{2} \times (1 + KL) = \frac{\frac{1}{16}}{2} \times \left(1 + \frac{2-\sqrt{2}}{4}\right) = \frac{6-\sqrt{2}}{128}$$

$$\text{Area of } S_2 \text{ segments} = \frac{\Delta EKB \text{ area}}{8\sqrt{2} \times BL} = \frac{\frac{1}{16}}{8\sqrt{2} \times \frac{\sqrt{2}-1}{2}} = \frac{2+\sqrt{2}}{128}$$

$$\text{Area of square} = (16.S_1) + (16.S_2) = 16\left(\frac{6-\sqrt{2}}{128}\right) + 16\left(\frac{2+\sqrt{2}}{128}\right) = 1$$

$$\text{Area of circle} = (16.S_1) + (8.S_2) = 16\left(\frac{6-\sqrt{2}}{128}\right) + 8\left(\frac{2+\sqrt{2}}{128}\right) = \frac{14-\sqrt{2}}{16}$$

$$\pi r^2 = \frac{14-\sqrt{2}}{16}, \quad \pi = \frac{\text{Area of circle}}{\text{radius} \times \text{radius}} = \frac{\frac{14-\sqrt{2}}{16}}{\frac{1}{2} \times \frac{1}{2}}$$

$$\therefore \pi = \frac{14-\sqrt{2}}{4}$$

Explanation of formulas of S_1 and S_2 areas:

32 segments are present in the square and 16 S_1 +8S_2°= 24 segments are in the circle. The area of the circle is total. Whether the values for areas of S_1 and S_2 are correct ? We will get doubt in the formulas of S_1 and S_2 for [illegible] accuracy.

Let us clarify ourselves.

In the square, side is a, in the circle, diameter = d = a

$16\,S_1 + 16S_2 = a^2$ = area of square (1)

$16\,S_1 + 8\,S_2 = \frac{\pi a^2}{4}$ = area of circle (2) (where d = a)

$$(1) - (2) \Rightarrow \quad 8S_2 = a^2 - \frac{\pi a^2}{4}$$

$$8S_2 = \frac{4a^2 - \pi a^2}{4}$$

$$S_2 = \frac{(4-\pi)a^2}{32} = \frac{a^2}{32}(4-\pi)$$

$(2)\times 2 \Rightarrow \qquad 32S_1 + 16S_2 = \frac{2\pi a^2}{4}$

$$16S_1 + 16S_2 = a^2$$

$(3)-(1) \Rightarrow \qquad 16S_1 = \frac{\pi a^2}{2} - a^2$

$$= \frac{\pi a^2 - 2a^2}{2}$$

$$S_1 = \frac{a^2(\pi-2)}{32} = \frac{a^2}{32}(\pi-2)$$

Finally, we have for the areas of S_1 segments $= \frac{a^2}{32}(\pi-2)$,

S_2 segments $= \frac{a^2}{32}(4-\pi)$

Let us equate Area of $S_1 = \frac{\Delta EKB \text{ area}}{2} \times (1+KL) = \frac{\text{Square area}}{32}(\pi-2)$

$2\cdot\sqrt{2}/4$

$$= \frac{\frac{1}{16}}{2} \times \left(1 + \frac{2-\sqrt{2}}{4}\right) \qquad = \frac{1}{32} \times \left(\frac{14-\sqrt{2}}{4} - 2\right)$$

Area of $S_2 = \frac{\Delta EKB \text{ area}}{8\sqrt{2} \times BL}$ $\quad 1\cdot 1/2$ $\qquad = \frac{\text{Square area}}{32}(4-\pi)$

$$= \frac{\frac{1}{16}}{8\sqrt{2} \times \frac{\sqrt{2}-1}{2}} \qquad = \frac{1}{32} \times \left(4 - \frac{14-\sqrt{2}}{4}\right)$$

In the above equalization process present official π value 3.1415926..... does not find a place for it, because it is an approximate value and is the length of the perimeter of the polygon, where as the new value $\frac{14-\sqrt{2}}{4}$ agrees *in toto* with the every line segment (KL of S_1 and BL of S_2). Therefore $(14-\sqrt{2})/4$ is the true value of π.

SIVA METHOD

It is a simple method and a confirmation method. This author came to know of $\frac{14-\sqrt{2}}{4}$ from the Gayatri method for the first time in 1998.

ABCD = Square

Side = a = 1

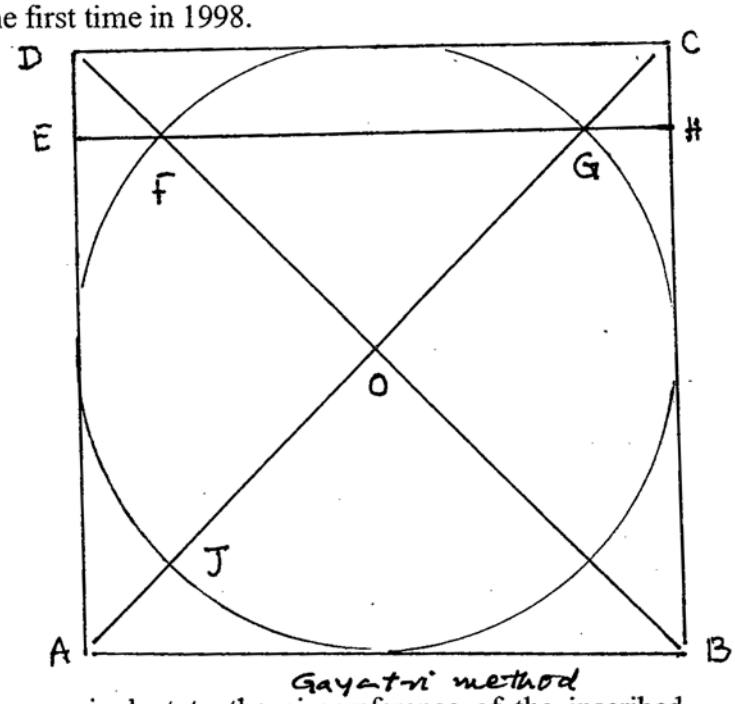
JG = diameter = side = 1

OG = radius = $\frac{1}{2}$

FG = $\frac{\sqrt{2}}{2}$

DE = EF = GH=HC = $\frac{2-\sqrt{2}}{4}$

BAEDCH = $1+1+1+\frac{2-\sqrt{2}}{4} = \frac{14-\sqrt{2}}{4}$



Gayatri method

It was thought by this author as equivalent to the circumference of the inscribed circle. After Gayatri method many more geometrical constructions were made later and all gave $\frac{14-\sqrt{2}}{4}$ and after Siva method, alone, this author believed this new number really represents π value. Siva method is as follows:

ABCD = Square

AB = Side = a = 1

EG = diameter = a = 1

OF = radius = 0.5

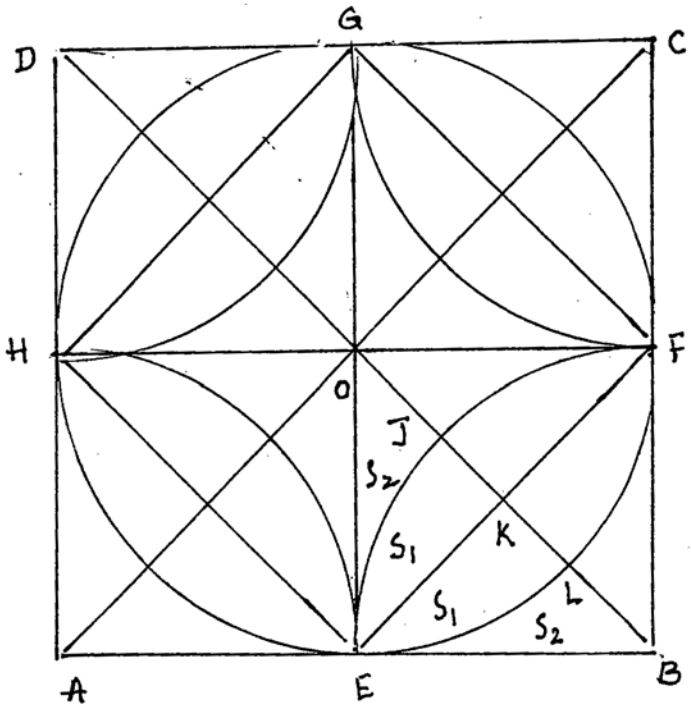
AC = diagonal = 1.4142135....

AO = OC = EF = OB = 0.7071067....

OK = KB = EK = KF = 0.3535533....

JK = KL = OL – OK = 0.1464466....

OJ = OK – JK = 0.2071067.....



Siva method

The square has 16 S_1 and 16 S_2 segments. Whereas the circle has 16 S_1 and 8 S_2 segments. To arrive at the areas of S_1 and S_2 a selection method is adopted. We have official π value 3.1415926….. and proposed π value of Gayatri method 3.1464466….

As a first step, areas of S_1 and S_2 are calculated with the above two π values and are shown in the Table

	π value	Official 3.1415926	Proposed 3.1464466
1.	Area of OEB triangle = 0.125		
2.	Area of OEL sector	0.0981747	0.0983264
3.	Area of OEK or EKB triangle = 0.0625		
4.	Area of S_1 = Sector – OEK	0.0356747	**0.0358264**
5.	Area of S_2 = OEB – Sector	0.0268253	**0.0266736**

The above values for S_1 and S_2 are taken as guidance in the making of formulae in conjunction with the respective line segments of S_1 and S_2. With KL line segment (=0.1464466) for S_1 and with BL line segment (=0.2071067) for S_2 the following formulae are devised.

Area of EKB triangle = $S_1 + S_2 = 0.0625$

(0.0356747+0.0268253=0.0625) (0.0358264+0.0266736=0.0625)

1. Area of S_1 = $\frac{\text{EKB triangle area}}{2} \times \left(1 + \frac{2-\sqrt{2}}{4}\right)$ $= \frac{0.0625}{2} \times 1.\mathbf{1464466} = \mathbf{0.0358264} = \frac{6-\sqrt{2}}{128}$

2. Area of S_2 = $\frac{\text{EKB triangle area}}{8\sqrt{2} \times BL}$ = $\frac{0.0625}{8\sqrt{2} \times \mathbf{0.2071067}} = \mathbf{0.0266735...} = \frac{2+\sqrt{2}}{128}$

Siva method calculates **total area** of the inscribed circle using line segments of the square circle nexus. Official π value does not agree with the line segments of S_1 and S_2 and hence 3.1415926….. has remained 'π' of the inscribed polygon in a square of Archimedean method and not as π of the circle.

Till yesterday formula with **radius** was impossible and an unthinkable idea.

Area of the circle = $r\left(\frac{7r}{2} - \frac{\sqrt{2}r}{4}\right) = \pi r^2$ Circumference = $6r + \frac{2r - \sqrt{2}r}{2} = 2\pi r$

Where 'r' is radius of a circle.

$\frac{\pi}{4}$ from $\sqrt{2}$ length

Step I: O as centre and $\frac{1}{2}$ as radius draw a circle. Draw four equidistant tangents. They intersect at A, B, C and D, creating a square ABCD. (without circle a square can also be drawn).

AB = Side = diameter = 1

AC = BD = diagonal = $\sqrt{2}$

Step II: Draw second square AEFC with side AC equal to $\sqrt{2}$. Draw two diagonals AF and CE. Inscribe a circle with centre B.

AEFC = Square

AE = AC = CF = EF = Side = diameter = $\sqrt{2}$

AF = CE = diagonal = AE x $\sqrt{2}$ = $\sqrt{2}\times\sqrt{2}=2$

Circle intersects two diagonals at G and H. Join GH and extend, which meets EF at J.

AE = Side = $\sqrt{2}$ = CF = EF = $\sqrt{2}$

$$\text{BH = radius} = \frac{AE}{2} = \frac{\sqrt{2}}{2}$$

$$\text{GBH = triangle,}\quad \text{GH = Hypotenuse} = \left(\frac{\sqrt{2}}{2}\right)\times\sqrt{2}=1$$

$$HJ = JF = \frac{CF-GH}{2} = \frac{\sqrt{2}-1}{2} = \frac{\sqrt{2}-1}{2}$$

$$JE = EF - JF = \sqrt{2}-\left(\frac{\sqrt{2}-1}{2}\right) = \frac{\sqrt{2}+1}{2}$$

Bisect JE twice. $\frac{\sqrt{2}+1}{2} \rightarrow \frac{\sqrt{2}+1}{4} \rightarrow \frac{\sqrt{2}+1}{8}$

$$\text{So LE = KL} = \frac{\sqrt{2}+1}{8}$$

$$FL = EF - EL = \sqrt{2}-\left(\frac{\sqrt{2}+1}{8}\right) = \frac{7\sqrt{2}-1}{8}$$

Step III: Draw third square LNPF with side FL

$$\text{FL} = \text{Side} = \frac{7\sqrt{2}-1}{8}$$

Draw two diagonals LP and FN

M is the centre

Inscribe a circle with centre M and radius equal to half the side LF

$$\text{i.e., } \frac{7\sqrt{2}-1}{16} = \text{MR} = \text{MQ}$$

Circle intersects two diagonals LP and FN at Q and R.

QMR = triangle

$$\text{QR} = \text{hypotenuse} = \text{MR} \times \sqrt{2}$$

$$= \left(\frac{7\sqrt{2}-1}{16}\right) \times \sqrt{2} = \frac{14-\sqrt{2}}{16}$$

QR is the length of the $\frac{1}{4}$ of the circumference of the 1st circle whose diameter (d), is 1.

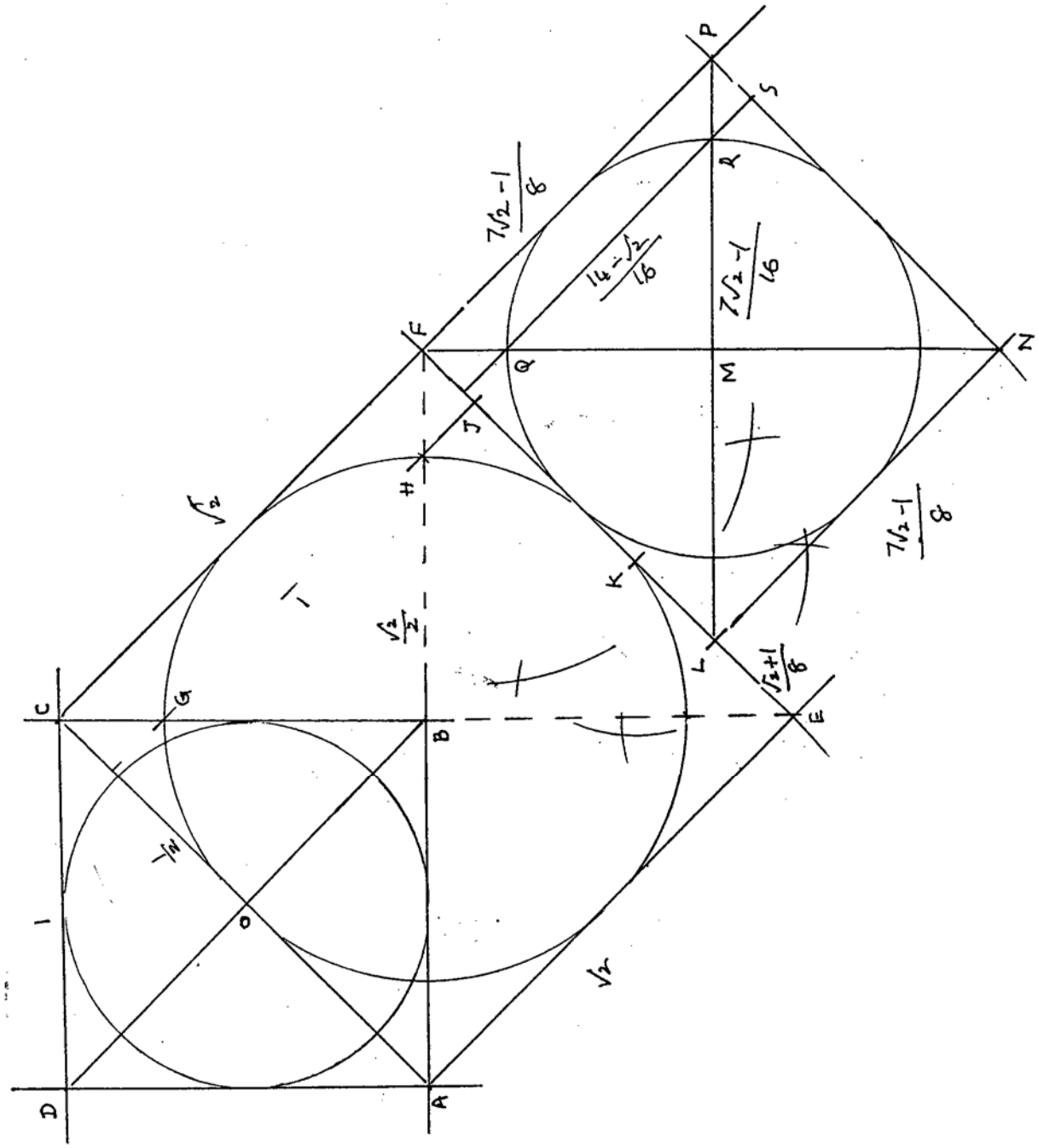
Circumference of the circle = πd

Where diameter = d = 1

$$1/4^{\text{th}} \text{ circumference} = \frac{\pi d}{4} = \frac{\pi \times 1}{4} = \frac{14-\sqrt{2}}{16}$$

$$\frac{\pi}{4} = \frac{14-\sqrt{2}}{16}$$

$$\therefore \pi = \frac{14-\sqrt{2}}{4}$$


P
S
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A
O
I
$\frac{7\sqrt{2}-1}{8}$
$\frac{14-\sqrt{2}}{16}$
$\frac{7\sqrt{2}-1}{16}$
$\frac{\sqrt{2}}{2}$
$\frac{\sqrt{2}+1}{8}$
$\sqrt{2}$

LATEST WORK

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Note: Some pages are deleted where methods computing new π value show that new Pi value is **not wrong.**

What is wrong with the Classical method of Archimedes

1. We have only **one geometrical method** to compute the value of π.
2. 3.1415926... of Classical method represents actually the length of the **perimeter** of the inscribed **polygon** in a circle.
3. One polygon gives **two** 'π' values.

   For example: a regular polygon of 24 sides

   Side $= r\sqrt{\left[2-\sqrt{(2+\sqrt{3})}\right]}$ where r = radius of the superscribed circle

   I. Perimeter $= 24\left\{r\sqrt{\left[2-\sqrt{(2+\sqrt{3})}\right]}\right\} = 2\pi r$

   $$\pi = \frac{24\left\{r\sqrt{2-\sqrt{(2+\sqrt{3})}}\right\}}{2r} = 3.13262862...$$

   II. Area $= 6r^2\sqrt{(2-\sqrt{3})} = \pi r^2$

   $$\pi = \frac{6r^2\sqrt{(2-\sqrt{3})}}{r^2} = 3.10582854...$$

   Let us see the following two formulae (discovered by this author in March, 1998). One for the circumference and the other for the area of a circle and both give one π value.

   Formula for circumference $= 6r + \frac{2r-\sqrt{2}r}{2} = 2\pi r$

   Formula for area $= r\left(\frac{7r}{2} - \frac{\sqrt{2}r}{4}\right) = \pi r^2$

   Circle gives one π value i.e. $\frac{14-\sqrt{2}}{4} = 3.1464466...$

4. Archimedes (240 B.C) has said that the π value does not exceed $3\frac{1}{7} = 3.142857...$ (In the Archimedean days of non-decimal numbers $3\frac{1}{6}$

would have been much bigger. Archimedes was right in choosing $3\frac{1}{7}$. **He was a great man).**

Hence official π value which has been in vogue is 3.1415926... agrees with the views of Archimedes. 3rd decimal is less than 2 i.e. 3.14**2** of Archimedes.

5. A modern method called **Trepezoidal approximation** has given another range to π. i.e. 3.133... to 3.14**6**5... exceeding Archimedes' upper limit 3.14**2** ... (Source: **Calculus and Analytic Geometry by, C.H. Edwards Jr and D.E. Penney, 1986 Prentice-Hall, Page 295.)** However, **Simpson's approximation** is in tune with the official π value 3.1415926....
6. Further, the number 3.1415926... (of perimeter of the inscribed polygon in a circle) does not agree with any **line segment** of a circle.

What is wrong with the Lindemann's proof

1. Lindemann's proof is based on the Euler's formula $e^{i\pi}+1=0$. On survey of the literature, we find this formula has no universal acceptance. In the formula $i=\sqrt{-1}$ has no meaning. Here π means, π radians = 180^0. The transcendental status has been given to the π constant. The Euler's formula can be rewritten as follows:

$e^{\sqrt{-1}\times 180}+1=0$ is right

$e^{\sqrt{-1}\times 3.14}+1=0$ is wrong

We incorporate π radians 180^0 in the Euler's formula and the result of transcendency is attributed to the π constant 3.14....

It is not acceptable.

π radians is not identical to π constant.

2. As 3.1415926... does not exist in the circle, squaring of circle has become an **unsolved geometrical problem.**

If f is either increasing ($f'(x) > 0$) on $[a, b]$ or decreasing ($f'(x) < 0$) on $[a, b]$, then $\int_a^b f(x)\,dx$ lies between the end-point approximations L_n and R_n. If f is either concave upward ($f''(x) > 0$) on $[a, b]$ or concave downward ($f''(x) < 0$) on $[a, b]$, then this integral lies between the midpoint and trapezoidal approximations M_n and T_n. So in these four cases the value of the integral can—in principle—be approximated with any desired degree of accuracy by choosing n sufficiently large.

In addition, there are *error estimates* that can be used to predict in advance the maximum possible error in a particular approximation. Here we will simply state and briefly illustrate two such error estimates and defer further discussion until Section 11-5.

Trapezoidal Error Estimate

Suppose that f'' is continuous on $[a, b]$ and that $|f''(x)| \leq M$ for all x in $[a, b]$. Then the difference between T_n and $\int_a^b f(x)\,dx$ is at most $M(b - a)^3/12n^2$.

Simpson's Error Estimate

Suppose that $f^{(4)}$ is continuous on $[a, b]$ and that $|f^{(4)}(x)| \leq M$ for all x in $[a, b]$. Then the difference between S_n [n even] and $\int_a^b f(x)\,dx$ is at most $M(b - a)^5/180n^4$.

For example, beginning with the integrand $f(x) = 4/(1 + x^2)$ of the integral in (7), repeated differentiation gives the second and fourth derivatives as

$$f''(x) = \frac{4(6x^2 - 2)}{(1 + x^2)^3} \quad \text{and} \quad f^{(4)}(x) = \frac{96(5x^4 - 10x^2 + 1)}{(1 + x^2)^5}.$$

By application of the closed-interval maximum-minimum method of Section 3-4, it can be verified (see Example 2 in Section 11-5) that

$$|f''(x)| \leq 8 \quad \text{and} \quad |f^{(4)}(x)| \leq 96$$

for x in $[0, 1]$. Hence with $n = 10$ as in Example 2, the maximum error in the trapezoidal approximation is

$$\frac{(8)(1)^3}{12(10)^2} \approx 0.006666667,$$

so the actual value of π, according to the integral (7), is

$$3.139925990 \pm 0.006666667;$$

that is, π lies between

$$3.133259323 \approx 3.133 \quad \text{and} \quad 3.146592657 \approx 3.147.$$

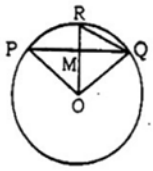
Why 3.1415926… is lower value at its 3rd decimal place onwards ?

3.1415926… is the value of inscribed/ circumscribed polygon where Pythagorean theorem is applied. Hence 3.1415926… is a **Pythagorean π value**.

Secondly, from the second doubling onwards, **all the decimals** of apothem, sagetta and second side (hypotenuse) are **not involved in the calculations.** We believe **last decimal** is approximation, but actually from the 3rd decimal it is a lower value.

Therefore Exhaustion method (below) is not a suitable method for computation of π of the circle.

Classical method of computing π

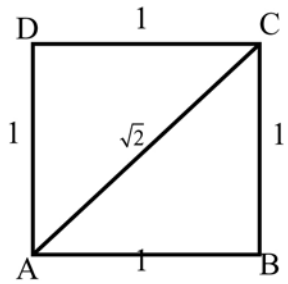


Classical method involves the inscription of polygon in the circle. Starting with the known perimeter of a regular polygon (here we start with a regular hexagon) of n sides inscribed in a circle, the perimeter of the inscribed regular polygon of 2 n sides can be calculated by the application of Pythagorean theorem. Let C be a circle with centre O and radius r, and let PQ = s be a side of a regular inscribed polygon of n sides having a known perimeter. Then the apothem, OM = u is given by $\sqrt{r^2-(s/2)^2}$ hence the sagetta, MR = v = r – u is known. Then the side RQ = w of the inscribed polygon of 2 n sides is found from $w=\sqrt{v^2+(s/2)^2}$ hence the perimetper of this polygon is known.

First Successive

S.No.	n	Side PQ=s	ns Perimeter = circumference = π	s/2 = t	Apothem $\sqrt{r^2-t^2}=u$	Sagetta r – u = v	Side $\sqrt{v^2-t^2}=w$
1.	6	0.5	3.0	0.25	0.433012701	0.066987299	0.258819045
2.	12	0.258819045	3.1058828544	0.129409522	0.482962913	0.17037087	0.13052619
3.	24	0.13052619	3.132628565	0.065263095	0.49572243	0.00427757	0.065403128
4.	48	0.065403125	3.139350157	0.032701564	0.498929461	0.001070539	0.032719082
5.	96	0.032419082	3.141031907	0.016359541	0.499732293	0.000267707	0.016361731
6.	192	0.016361731	3.141452431	0.00818065705	0.499933069	0.000066931	0.008181139495
7.	384	0.008181139495	3.141557566	0.004090569748	0.499983267	0.000016733	0.004090603972
8.	768	0.004090603972	3.14158385	0.002045301986	0.499995816	0.000004184	0.002045306262
9.	1536	0.002045306262	3.141590424	0.001022653133	0.499998654	0.000001046	0.001022653368
10.	3072	0.001022653668	3.141592067	-	-	-	-

π times of area of the circle is equal to area of the triangle
(Arthanaareeswara method)



Square ABCD
Side AB = 1
Diagonal = AC = $\sqrt{2}$

Take a paper and construct a square whose side is 1 (=10 cm) and diagonal $\sqrt{2}$. Fold the paper along the diagonal AC. Then bring the two points of A and C of the folded triangle touching each other in the form of a ring, such that AC becomes the length of the circumference of the circle whose value is $\sqrt{2}$. Now the folded paper finally looks like a paper crown.
Let us find out the area of the circle

Circumference = $\sqrt{2} = \pi d$; $d = \frac{\sqrt{2}}{\pi}$; Area = $\frac{\pi d^2}{4} = \pi \times \frac{\sqrt{2}}{\pi} \times \frac{\sqrt{2}}{\pi} \times \frac{1}{4} = \frac{2}{4\pi}$

Area of the triangle = $\frac{1}{2}$; π x Area of circle = $\pi \times \frac{2}{4\pi} = \frac{1}{2}$

Second method

This time let us bring A and D or D and C close together, touching just in such a way they form a ring (= circle)

Side = AD = 1 (=10 cm); Circumference = 1 = πd;

d = $\frac{1}{\pi}$; $\frac{\pi d^2}{4} = \pi \times \frac{1}{\pi} \times \frac{1}{\pi} \times \frac{1}{4} = \frac{1}{4\pi}$; 2π x Area of circle = $2\pi \times \frac{1}{4\pi} = \frac{1}{2}$

The interrelationship between two areas of circle and triangle by π, shows that π is not a special number called transcendental number. Everything about circle since the days of Archimedes till today is all confusing. Euler's formula $e^{i\pi} + 1 = 0$ is one solid example. e, i, 180^0, 1 and 0 which are unrelated but combined and synthesized into Euler's formula, and crowned "famous".

Attributing polygon's 3.1415926… to circle as its π value is second example. Forbidding vehemently the association of "Square root extraction" in the arrival of 3.1415926… on one side, though it was based on $\sqrt{3}$ application in Classical method of Archimede's inscribing/ circumscribing polygons in and around circle is third example.

The world of Mathematics has succeeded in rejecting the new π value **always** quoting Archimedes (240 BC) range of $3\frac{10}{71} to 3\frac{1}{7}$, Madhava's (1450) infinite series, Euler's formula (1740) and C.L.F. Lindemann's (1882), so called proof of transcendency to π (Lindemann is doubly right in calling 3.1415926… as transcendental but this number is not exact π no no 3.1415926… is not π at all).

The fight: One man vs. World, cannot go on forever, when the mathematical establishment is reluctant to accept the new π value and at the same time unable to disprove it without quoting 1. Archimedes, 2. Francois Viete (1579), 3. John Wallis (1655), 4. William Brouncker (1658), 5. James Gregory, Gottfried Wilhelm Leibnitz (1670), 6. Isaac Newton (1666), 7. Machin (1706-1776), 8. Leonard Euler (1748), 9. Srinivas Ramanujan (1914), 10. Louis Comtet (1974), 11.Eugene Salamin, Richard Brent (1976), 12. Jonathan Borwein, Peter Borwein (1991), 13. David Chudnovsky & Gregory Chudnovsky (1989), 14. Roy North (1989) and 15. David Bailey and Simon Plouffe (1996).

Either one or two or more of the above giants/ stalwarts is/ are quoted and dubbed new π value is wrong. The purpose for which I was born is partially fulfilled. I die like a warrier, defeated.

Half body of Lord Shivaa and the remaining half body of His consort goddess Parvathi is Arthanaareeswara

Gayathri method to demarcate the length of the circumference and the area of the inscribed circle in the square.

ABCD = Square

AB = Side = a

JG = Diameter = a

$OF = OG = \text{Radius} = \frac{a}{2}$

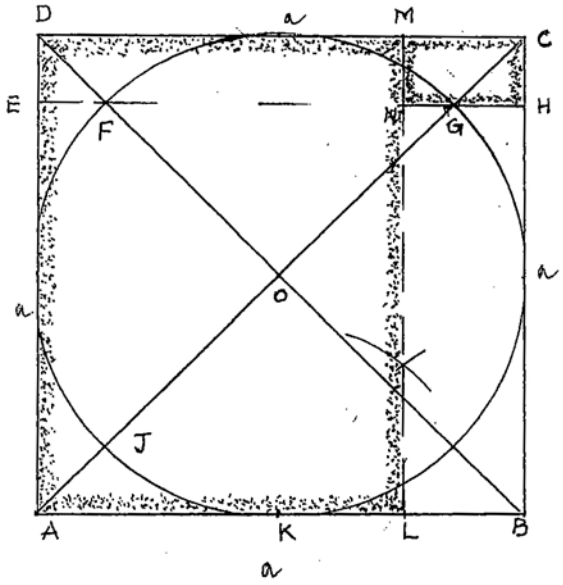
FOG = Triangle

$FG = \text{Hypotenuse} = \frac{\sqrt{2}a}{2}$

DE = EF = GH = CH

$= \frac{EH - FG}{2} = \left(a - \frac{\sqrt{2}a}{2}\right)\frac{1}{2}$

$= \left(\frac{2-\sqrt{2}}{4}\right)a$



1. The length of the circumference of the circle

= BA + AD + DC + CH

$= a + a + a + \left(\frac{2-\sqrt{2}}{4}\right)a = \left(\frac{14-\sqrt{2}}{4}\right)a = \pi a = \pi d$

where a = d = diameter $\quad \therefore \pi = \frac{14-\sqrt{2}}{4}$

k = Midpoint of AB $= \frac{\text{Side}}{2} = \frac{a}{2} = AK = KB$

Bisect KB into KL and LB = $KL = \frac{a}{4}$

$AL = AK + KL = \frac{a}{2} + \frac{a}{4} = \frac{3a}{4} = DM$

Join ML. $\quad LB = MC = \frac{a}{4} = NH = MC = \frac{a}{4}$

There are two rectangles ALMD and MNHC

Area of ALMD = AD x AL $= a \times \frac{3a}{4} = \left(\frac{3}{4}\right)a^2$

Area of MNHC = MN x NH $= \left(\frac{2-\sqrt{2}}{4}\right)a \times \frac{a}{4} = \left(\frac{2-\sqrt{2}}{16}\right)a^2$

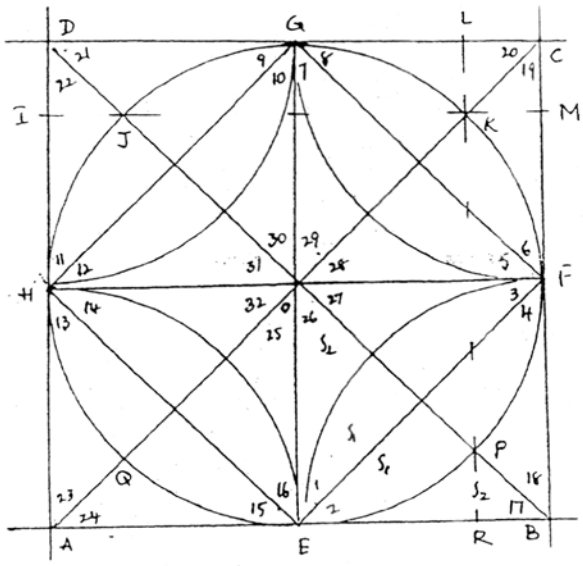
2. Area of the circle = Areas of ALMD + MNHC

$= \left(\frac{3}{4}\right)a^2 + \left(\frac{2-\sqrt{2}}{16}\right)a^2 = \left(\frac{14-\sqrt{2}}{16}\right)a^2 = \frac{\pi a^2}{4} = \frac{\pi d^2}{4}$

where a = d = diameter $\quad \therefore \pi = \frac{14-\sqrt{2}}{4}$

SIVA METHOD TO COMPUTE π VALUE FROM THE AREA OF THE CIRCLE

Construction procedure:



Draw a circle with center '0' and radius *a*/2. Diameter is 'a'. Draw 4 equidistant tangents on the circle. They intersect at A, B, C and D resulting in ABCD square. The side of the square is also equal to diameter 'a'. Draw two diagonals. E, F, G and H are the mid points of four sides. Join EG, FH, EF, FG, GH and HE. Draw four arcs with radius *a*/2 and with centres A, B, C and D. Now the circle square composite system is divided into 32 segments and number them 1 to 32. 1 to 16 are of one dimension called S_1 segments and 17 to 32 are of different dimension called S_2 segments.

Area of the square = $16S_1 + 16S_2 = a^2$ … eq. (1),

Area of the circle = $16S_1 + 8S_2 = \pi a^2/4$ … eq. (2),

By solving above two equations, we get two formulae for the areas of segments S_1 and S_2.

They are $S_1 = \frac{a^2}{32} \times \pi - 2$ and $S_2 = \frac{a^2}{32} \times 4 - \pi$

In the subject of Geometry, the role of line segments is supreme. As there are two π values 1. Official π value 3.1415926… and 2. New π value 3.1464466… = $\left(14-\sqrt{2}\right)/4$, a selective method is adopted here.

a =1 and π = 3.1415926… and 3.1464466…

Let us search for line segments representing π - 2 and 4 - π.

π - 2 = 3.1415926 – 2 = 1.1415926… and 3.1464466 – 2 = 1.1464466…

4 - π = 4 – 3.141592 = 0.8584074 … and 4 – 3.1464466 = 0.8535534…

Square ABCD; side = a = 1 = diameter

IM = LR = also sides parallel to CD and CB; Corner square = LKMC

Side = LK = KM = MC = CL = (IM – JK)/2

OJ = OK = radius = 1/2; JK = $\sqrt{2}/2$

$$\frac{IM - JK}{2} = \left(1 - \frac{\sqrt{2}}{2}\right)\frac{1}{2} = \left(2 - \sqrt{2}\right)/4$$

$$DC + CM = 1 + \frac{2-\sqrt{2}}{4} = \frac{6-\sqrt{2}}{4} = 1.1464466... = \pi - 2$$

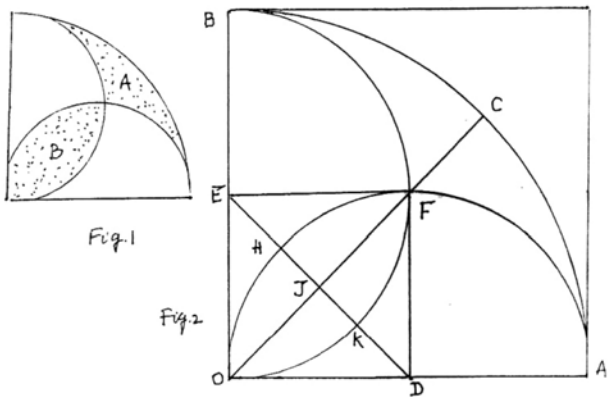
$$MB = CB - CM = 1 - \frac{2-\sqrt{2}}{4} = \frac{2+\sqrt{2}}{4} = 0.8535534... = 4 - \pi$$

Thus, there are no line segments representing for official π value, 1.1415926… and 0.8584074. Hence, it is argued that $\frac{14-\sqrt{2}}{4}$ is the real π.

NEW YORK – TIRUPATI MATH – ZOO METHOD

(Decimal similarity between an area and its line segments)

(Reference book: **Pi – A biography of the world's most mysterious number** – by **Alfred S. Posamentier** and **Ingmar Lehmann,** 2004, Prometheus Books, Page 180)



Prof. A.S. Posamentier has established that areas of A and B regions are equal. His formula is $r^2\left(\frac{\pi}{2}-1\right)$ for the above regions. This author is grateful to the professor of New York for the reason through his idea this author tries to show that his new π value equal to $\frac{1}{4}\left(14-\sqrt{2}\right)$ is **exactly** right.

1. Arc = BCA; O = Centre; OB = OA = OC = Radius = r
2. Semicircles : BFO = AFO; E and D = Centres; OD=DA = BE = OE= radius= $\frac{r}{2}$

   OF = $\frac{\sqrt{2}r}{2}$; FC = OC – OF = $r-\frac{\sqrt{2}r}{2}=\frac{2r-\sqrt{2}r}{2}$
3. Petal = OKFH; EK = $\frac{r}{2}$; ED = $\frac{\sqrt{2}r}{2}$; EJ = $\frac{\sqrt{2}r}{4}$; JK = EK – EJ = $\frac{r}{2}-\frac{\sqrt{2}r}{4}=\frac{2r-\sqrt{2}r}{4}$;

   JK = JH, HK = JH + JK = $\frac{2r-\sqrt{2}r}{2}$
4. So, FC of region A = HK of region B = $\frac{2r-\sqrt{2}r}{2}$
5. BFAC = OKFH i.e. areas of A and B regions are equal (**A.S. Posamentier** and **I. Lehmann)**.

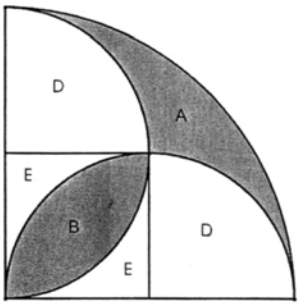


Fig. 6-21

$$Area_B = 2\left(\frac{1}{4}\pi r^2\right)-\left(r^2\right)$$
$$=\frac{\pi}{2}r^2-r^2$$
$$=r^2\left(\frac{\pi}{2}-1\right)$$

So we can clearly see that the two regions *A* and *B* have the same area.

Formula for A and B is $r^2\left(\frac{\pi}{2}-1\right)=\frac{r^2}{2}(\pi-2)$

Here r = radius = 1

From March 1998, there are two π values. The official π value is 3.1415926… and the new π value is $\frac{14-\sqrt{2}}{4}$ = 3.1464466… and which π value is exact and true ?

Let us substitute both the values in $\frac{r^2}{2}(\pi-2)$, then

Official π value $=\frac{r^2}{2}(3.1415926-2)=\frac{r^2}{2}(1.1415926...)$

(It is universally accepted that 3.1415926… is approximate at its **last decimal place** however astronomical it is in its magnitude.)

New π value = $\frac{r^2}{2}(3.\underline{1464466}-2)=\frac{r^2}{2}(1.\underline{1464466}...)$

6. FC = HK (HJ + JK) line segments = $\frac{2r-\sqrt{2}r}{2}$

7. Half of HC and HK are same $\frac{FC}{2}=\frac{HK}{2}=\left(\frac{2r-\sqrt{2}r}{2}\right)\frac{1}{2}=\frac{2r-\sqrt{2}r}{4}$ = **0.1464466…..**

8. Area of A/B region equal to 1.1464466… is similar in **decimal** value of half of FC/HK line segment i.e. 0.1464466…

9. **Conclusion**: Formulae a^2, 4a of square and ½ab of triangle are based on side of the square and altitude, base of triangle, respectively. In this construction, FC and HK are the line segments of A and B regions, respectively.
   As the value 0.1464466… which is half of FC or HK is in **agreement** with the area value of A/B region equal to 1.1464466… in **decimal** part, it is argued that new π value equal to $\frac{1}{4}\left(14-\sqrt{2}\right)$ = 3.1464466… is **exactly** correct.

The decimals 0.1415926… of the official π value 3.1415926… does not tally beyond 3rd decimal with the half the lengths of HK and FC, whose value is 0.1464466, thus, the official π value is **partially** right. This author humbly poses a question to the **individuals** and **mathematical establishment, Is it unmathematical to give significance to line segments FC & HK of A and B regions, in associating with their areas ?** Whereas, FC & HK are **incompatible** with the areas of A & B calculated using official π value. **Then, which π is real, Sirs ?**

This author once again expresses **his gratitude** to Professors **A.S. Posamentier** and **I. Lehmann** for their **ingenious** geometrical analysis. If this author has made any mistake he may be excused.

Ramesh method in the computation of true π value.

1. Square = ABDC

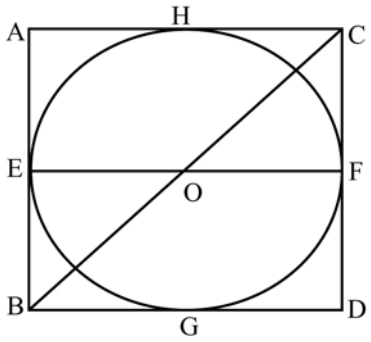
   Side = AB = d

   Diagonal = BC = $\sqrt{2}d$

   Diameter = EF = d

   Circle = = EHFG

   Triangle = BAC



2. An **exact** relationship exists between the inscribed circle and square ABDC/ triangle BAC or BDC.

3. Area of the triangle BAC = $\frac{d^2}{2}$; Perimeter of the triangle = BA + AC + BC = d + d + $\sqrt{2}d$

$$\frac{Area\ of\ the\ triangle}{Perimeter\ of\ the\ triangle} = \frac{d^2/2}{2d+\sqrt{2}d}$$

4. The true value of π can also be obtained from the following relation between triangle and circle.

$$\frac{Area\ of\ the\ triangle}{Perimeter\ of\ the\ triangle} = \text{Circumference} - 3\ \text{diameters}$$

(Where circumference of the circle is unknown)

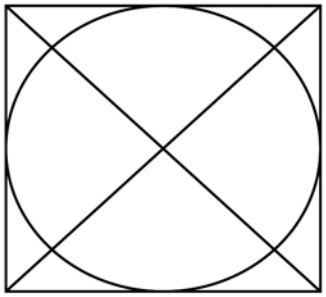
$$\frac{\frac{d^2}{2}}{2d+\sqrt{2}d} = \pi d - 3d \qquad \therefore\ \pi = \frac{1}{2\left(2+\sqrt{2}\right)} + 3 = \frac{14-\sqrt{2}}{4}$$

One day in 1972 at Piler while working as college teacher, this author questioned himself, while going through circle and square, why there is π and its value 22/7. And no such constant is used to calculate area and perimeter of the square. He thought many days. One day he inscribed a circle in a square. The similarity in the values of diameter of circle and side of the square made him to believe that there was answer to his question in this diagram. He strongly opined that there were formulae with radius alone (and without π) if only one studied and thought deeply and understood their interrelation in right perspective. He tried physical methods to understand relation between square and its inscribed circle next 26 years on and off. Meanwhile, as he was a government college lecturer (in Zoology) he was transferred from Piler to Kadapa, Nagari, Anantapur and finally to Chittoor in December 1995 during this period of 26 years.

One Mr. Ramesh Prasad was a neighbor at Chittoor. He was working as a Physics Teacher in a High School. This author discussed his long pending doubt with him. After listening, he questioned this author whether he tried "ratio" between the dimensions of square and its inscribed circle. 'No' was the answer from this author.

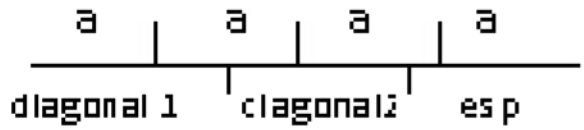
This author finally tried ratio concept as the basic concept. He confirmed that there existed **constancy** between square and circle and became fortunately the first step for his work. This work afterwards has travelled without any break till today i.e., 12-09-2013. Hence this method is named after him as a mark of author's gratitude to Mr. Ramesh Prasad. The formulae are $r\left(\frac{7r}{2}-\frac{\sqrt{2}r}{4}\right)$ for area, and $6r+\frac{2r-\sqrt{2}r}{2}$ for circumference, a fruit of 26 years of on and off association to the problem. Next 15 years of efforts have confirmed the above formulae as correct.

JESUS METHOD TO COMPUTE THE CIRCUMFERENCE OF A CIRCLE



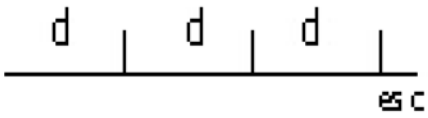
Procedure: Draw a square. Draw two diagonals. Inscribe a circle. Side = a, Diagonal = $\sqrt{2}a$, Diameter is also = a.

1) Straighten the square. Perimeter = 4a



Perimeter – Sum of the lengths of two diagonals = $4a-2\sqrt{2}a$ = esp
esp = end segment of the perimeter of the square.

2) Straighten similarly the inscribed circle



3 diameters plus some length, is equal to the length of the circumference.
Let us say circumference = x.
Circumference – 3 diameters = x – 3a = esc
esc = end segment of the circumference of the circle.

3) When the side of the square is equal to 'a', the radius of the inscribed circle is equal to a/2. So, the radius is $1/8^{th}$ of the perimeter of the square.

4) The above relation also exists between the end segment of the circumference of the circle and the end segment of the perimeter of the square.

Thus as radius $\left(\frac{a}{2}\right)$ of the inscribed circle is to the perimeter of the square (4a), i.e., $1/8^{th}$ of it, so also, is the end segment of the circumference of the circle, to the end segment of the perimeter of the square.

So, the end segment of the circumference = $\frac{end\ segment\ of\ the\ perimeter\ of\ the\ square}{8}$

$$esc = \frac{esp}{8} = x - 3a = \frac{4a - 2\sqrt{2}a}{8} \qquad \therefore x = \frac{14a - \sqrt{2}a}{4}$$

5) Circumference of the circle = πd = πa (where a = d = diameter)

$$\pi a = \frac{14a - \sqrt{2}a}{4} \qquad \therefore \pi = \frac{14 - \sqrt{2}}{4}$$

SQUARING OF FULL CIRCLE AND SEMI CIRCLE OF HIPPOCRATES

INTRODUCTION

By the grace of God the author had already submitted values for squaring one Hippocratean crescent. The present method too supports the proposed π value of the author i.e.**(14 - √2)/4.** Hippoerates had succeeded in squaring lunes. Though the constant π was not in vogue in the Hippocratean period (460 BC) **he foresaw the algebraic nature of π number.** ★

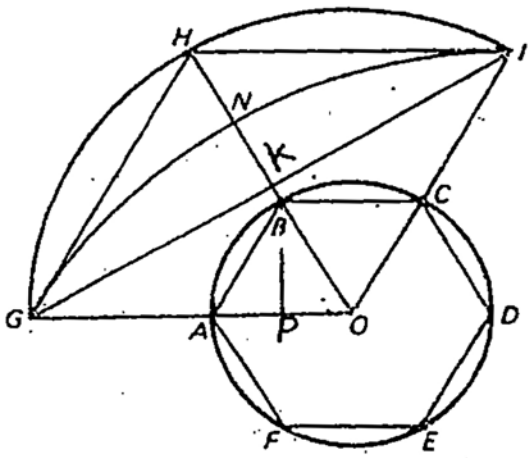


Fig - 23

" Consider two concentric circles with common centre O and radii such that the square of the radius of the larger circle is six times the square of the radius of the smaller one (Fig 7.11). Let us inscribe in the smaller circle the regular hexagon ABCDEF. Let OA cut the larger circle in G, the line OB in H and the line OC in I. On the line GI we construct a circular segment GNI similar to the segment GH. Hippocrates shows that the lune GHIN plus the smaller cirlce is equivalent to the triangle GHI plus the hexagon."

MATHEMATICS AND MATHEMATICIANS
P. Dedron Jitard

1. OA = radius of the smaller circle = a/2, (daimeter = a)

   OH = radius of the larger circle = $\sqrt{6\left(\frac{a}{2}\right)^2}$ = GH = HI.

   OK = OH/2 = HK, GK = $\sqrt{(GH)^2 - (HK)^2}$, GI = 2GK

   Area of the GHI triangle = 1/2 x GI x HK

★The author feels that had the later mathematicians seriously thought about the truth in the Hippocratean Propositions, the π value of Archimedean range 3.1408.... to 3.1428.. would have been corrected in the **3rd decimal place** long back. Next to Archimedes (240 BC) Claudius Ptolemy (150 AD) was the first mathematician who gave π value as 3.1416. In the 17th century curious expressions in the form of infinite series had been proposed. The infinite series is of two categories : arctan series and non-arctan series. As the arctan series is the outcome of right angled triangles inscribed with the circle, the π value derived out of this did support in toto the value of π obtained with the polygon inscribed with the circle of Archimedes. Whereas the non-arctan series is totally unrelated with the parameters of the circle. The number 3.1415926.... (supposed to be equal to π) was the only guiding factor for the expression, ofcourse, for the purpose of arrival of infinite decimal places.

2. OA = a/2 = AB, AP = a/4, PB = $\sqrt{(AB)^2 - (AP)^2}$

Area of the triangle AOB = 1/2 x AO x PB

Area of the hexagon = area the triangle AOB x 6.

3. Area of the smaller Circle = $\frac{\pi d^2}{4} = \frac{\pi a^2}{4} = \left(\frac{14 - \sqrt{2}}{16}\right) a^2$

4. Area of the lune GHIN = Segments of GH + HI + GHN + INH

According to Hippocrates

Area of the lune GHIN + Area of the smaller circle = area of the triangle GHI + area of the hexagon ABCDEF

= segments of GH + HI + GHN + INH + $\frac{\pi a^2}{4}$ = (1/2 x GI x HK) + 6 (1/2 x AO x PB)

$$2\left\{6a^2\left(\frac{14 - (\sqrt{2} + 6\sqrt{3})}{96}\right)\right\} + 2\left\{6a^2\left(\frac{8\sqrt{3} + \sqrt{2} - 14}{64}\right)\right\} + a^2\left(\frac{14 - \sqrt{2}}{16}\right)$$

GH + HI + GHN + INH

$$= \left\{\frac{1}{2} \times 2\left(\sqrt{6\left(\frac{a}{2}\right)^2 - 6\left(\frac{a}{4}\right)^2}\right) \times \sqrt{6\left(\frac{a}{4}\right)^2}\right\} + 6\left\{\frac{1}{2} \times \frac{a}{2} \times \sqrt{\left(\frac{a}{2}\right)^2 - \left(\frac{a}{4}\right)^2}\right\}$$

triangle GHI + hexagon ABCDEF

EXPLAINED ARITHMETICALLY

1. Smaller circle : OA = Radius = $\frac{a}{2}$, AD = Diameter = a

2. Larger Circle : OH = Radius = $\sqrt{6\left(\frac{a}{2}\right)^2} = \left(\frac{\sqrt{6}}{2}\right)a$

3. Third circle : GI = Radius; GI = GK + KI

   $OH = OI = \left(\frac{\sqrt{6}}{2}\right)a$; $OK = \frac{OH}{2} = \left(\frac{\sqrt{6}}{4}\right)a$

   $KI = \sqrt{(OI)^2 - (OK)^2} = \left(\frac{3\sqrt{2}}{4}\right)a$

   Radius of the third circle GI = 2 x KI = $\left(\frac{3\sqrt{2}}{2}\right)a$

4. Area of the GHI triangle = $\frac{1}{2} \times GI \times HK$; $HK = \frac{OH}{2} = \left(\frac{\sqrt{6}}{4}\right)a$

   $= \frac{1}{2} \times \left(\frac{3\sqrt{2}}{2}\right)a \times \left(\frac{\sqrt{6}}{4}\right)a = \left(\frac{3\sqrt{3}}{8}\right)a^2$

5. Area of the AOB triangle

   $OA = AB = \frac{a}{2}$; $AP = \frac{a}{4}$; $PB = \sqrt{(AB)^2 - (AP)^2} = \left(\frac{\sqrt{3}}{4}\right)a$

   Area of the AOB triangle = $\frac{1}{2} \times OA \times PB = \frac{1}{2} \times \frac{a}{2} \times \left(\frac{\sqrt{3}}{4}\right)a = \left(\frac{\sqrt{3}}{16}\right)a^2$

6. Area of the hexagon = Area of the triangle AOB x 6

   $= \left(\frac{\sqrt{3}}{16}\right)a \times 6 = \left(\frac{6\sqrt{3}}{16}\right)a = \left(\frac{3\sqrt{3}}{8}\right)a$

7. Area of the smaller circle = $\frac{\pi d^2}{4} = \frac{\pi a^2}{4} = \left(\frac{14 - \sqrt{2}}{16}\right)a^2$

8. Area of the segment GH = Segment HI

   Area of the larger circle = $\frac{\pi d^2}{4} = \frac{\pi a^2}{4} = \frac{14 - \sqrt{2}}{4} \times \frac{1}{4} 6a^2$

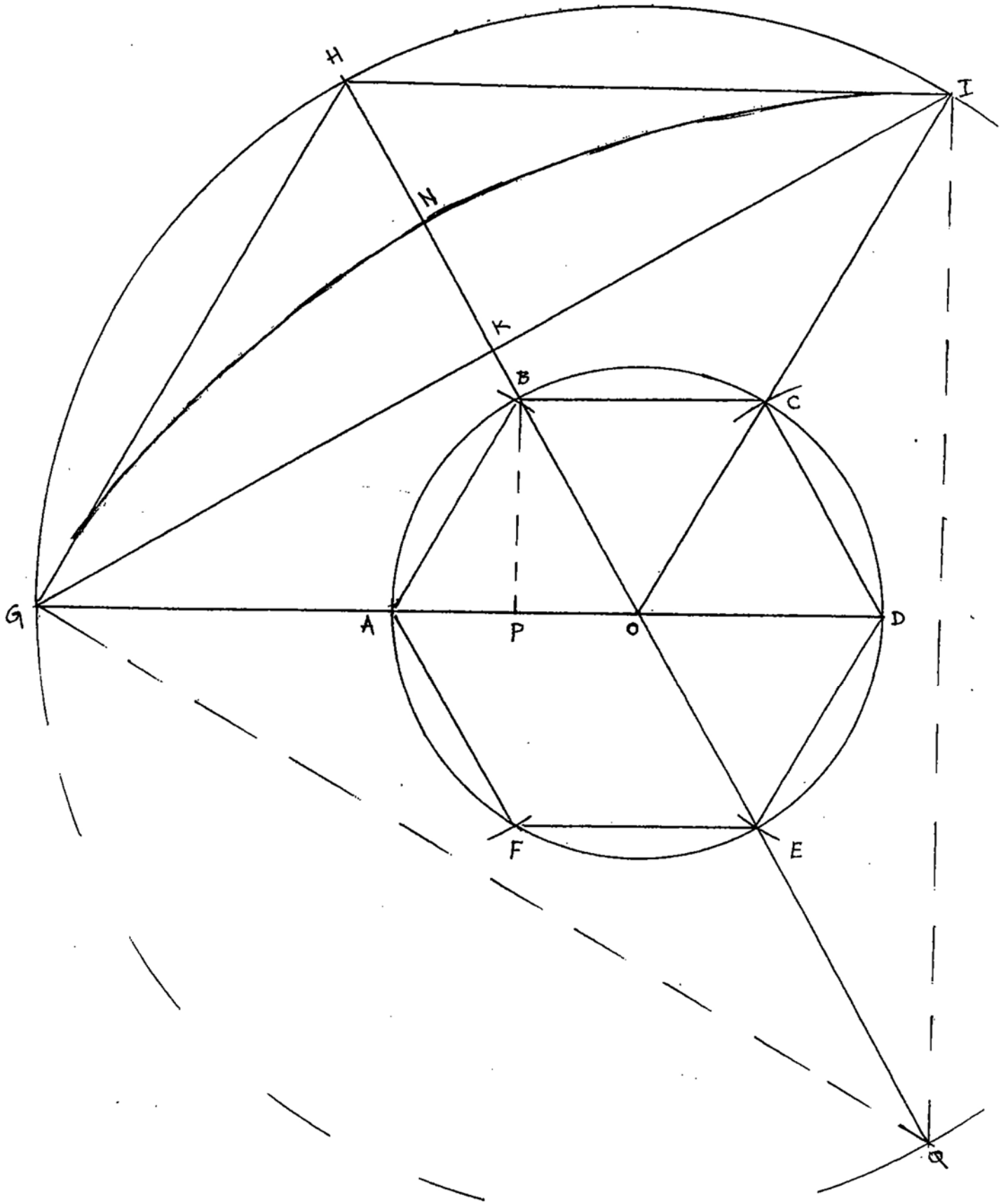


Fig. 2

Area is divided into 6 sectors $= \left(\frac{14-\sqrt{2}}{16}\right)6a^2 \times \frac{1}{6} = \left(\frac{14-\sqrt{2}}{16}\right)a^2 = \frac{\pi}{4}$

Area of the triangle OGH = OHI = GHI $= \left(\frac{3\sqrt{3}}{8}\right)a^2$

S. No. 4

Area of the GH segment = HI segment = Sector – triangle

$= \left(\frac{14-\sqrt{2}}{16}\right)a^2 - \left(\frac{3\sqrt{3}}{8}\right)a^2 = \left(\frac{\pi}{4} - \frac{6\sqrt{3}}{16}\right)a^2$

There are two segments GH and HI $= 2\left(\frac{\pi}{4} - \frac{6\sqrt{3}}{16}\right) = \frac{2\pi - 3\sqrt{3}}{4}$

9. Similarly, GNIK is also a segment which is the part of the sector GNIQ of Fig.2. It consists of the triangle GIQ and GNIK segment.

10. To find out the area of the sector GNIQ, first let us find out the area of the circle whose diameter is equal to that of the **third circle**.

Diameter of the third circle (S. No. 3) $= \left(3\sqrt{2}\right)a$

$$\frac{\pi d^2}{4} = \frac{\pi a^2}{4} = \frac{14-\sqrt{2}}{4} \times \frac{1}{4} \times \left\{\left(3\sqrt{2}\right)a\right\}^2 = \left(\frac{9\pi}{2}\right)a^2$$

So, the area of the third circle $= \left(\frac{9\pi}{2}\right)a^2$.

Then let us find out the area of the sector $= \left(\frac{1}{6}\text{th}\right)$

$$= \left\{\left(\frac{9\pi}{2}\right)a^2\right\}\frac{1}{6} = \left(\frac{9\pi}{12}\right)a^2 = \left(\frac{3\pi}{4}\right)a^2$$

11. Now let us find out the area of the triangle GIQ $= \frac{1}{2}ab = \frac{1}{2} \times KQ \times GI$

GI = Radius of the third circle = QI = GQ

$KI = \frac{GI}{2}$ = S. No. 3 $= \left(\frac{3\sqrt{2}}{4}\right)a$

$$KQ = \sqrt{(QI)^2 - (KI)^2} = \sqrt{\left\{\left(\frac{3\sqrt{2}}{2}\right)a\right\}^2 - \left\{\left(\frac{3\sqrt{2}}{4}\right)a\right\}^2} = \left(\frac{3\sqrt{6}}{4}\right)a$$

Area of the GIQ triangle $= \frac{1}{2} \times KQ \times GI = \frac{1}{2} \times \left(\frac{3\sqrt{6}}{4}\right)a \times \left(\frac{3\sqrt{2}}{2}\right)a$

$$= \left(\frac{9\sqrt{3}}{8}\right)a^2$$

12. Area of the segment GNIK = Sector – Triangle

$$\left(\frac{3\pi}{4}\right)a^2 - \left(\frac{9\sqrt{3}}{8}\right)a^2 = \left(\frac{6\pi - 9\sqrt{3}}{8}\right)a^2$$

Area of the GNIK segment $= \left(\frac{6\pi - 9\sqrt{3}}{8}\right)a^2$

13. Now it has become possible to calculate the area of GHIN segment

Triangle GHI – Segment GNIK

(S. No. 4) (S. No. 12)

$$= \left(\frac{3\sqrt{3}}{8}\right)a^2 - \left(\frac{6\pi - 9\sqrt{3}}{8}\right)a^2 = \left(\frac{6\sqrt{3} - 3\pi}{4}\right)a^2$$

So, area of the GHIN segment $= \left(\frac{6\sqrt{3} - 3\pi}{4}\right)a^2$

14. Area of the lune GHIN

Segments + Segment + Circle $= \frac{2\pi - 3\sqrt{3}}{4} + \left(\frac{6\sqrt{3} - 3\pi}{4}\right) + \frac{\pi}{4} = \frac{3\sqrt{3}}{4}$

GH & HI GHIN

(S. No. 8) (S. No.13)

15. Area of the triangle GHI + Area of the hexagon ABCDEF $= \frac{3\sqrt{3}}{8} + \frac{3\sqrt{3}}{8} = \frac{3\sqrt{3}}{4}$

(S. No.4) (S. No.6)

So, sum of the areas of the lune and **circle** = sum of the areas of triangle and hexagon

Thus squaring of lune and **circle** put together was done successfully, exactly, **by all the time the great mathematician Hippocrates of Chios.**

HIPPOCRATEAN CRESENT No.2

"Hippocrates next inscribed half a regular hexagon ABCD in a semicircle whose centre was O, and on OA, AB, BC, and CD as diameters described semicircles of which those on OA and AB are drawn in the figure. The AD is double any of the lines OA, AB, BC and CD,

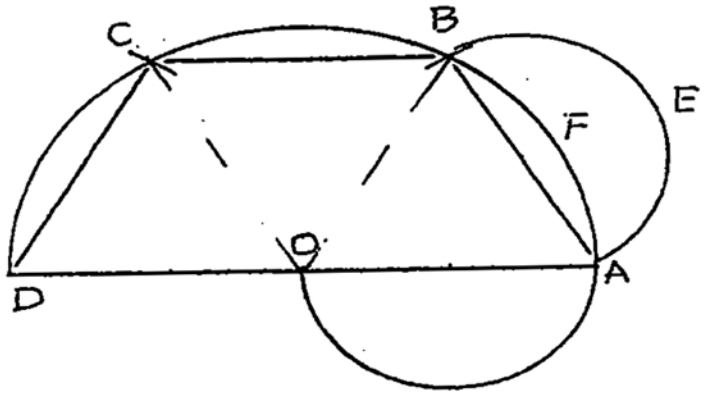


Fig - 25

∴ Sq. on AD = sum of sqs. on OA, AB, BC and CD,

∴ area semicircle ABCD = sum of areas of semicircles on OA, AB, BC and CD.

Take away the common parts.

∴ area trapezium ABCD = 3 lune AEFB + semicircle on OA".

ABCD = ½ hexagon.

OA = AB = BC = CD = a/2 (diameters for OA, AB, BC, and CD)

1. Area of semicircle on AD = $\frac{\pi a^2}{8}$
2. Area of semicircles on OA, AB, BC and CD = $\frac{\pi\left(\frac{a}{2}\right)^2}{8} = \frac{\left(\frac{14-\sqrt{2}}{4}\right)\left(\frac{a}{2}\right)^2}{8}$
3. Area of semicircle on AD = sum of sqs. on OA, AB, BC and CD.

∴ area semicircle ABCD = sum of areas of semicircles on OA, AB, BC, and CD.

Take away the common parts.

4. ∴ area trapezium ABCD=3 lune AEBF + semicircle on OA
5. Lune AEBF = semicircle on AB – AFB segment.

$$= \left\{\frac{\left(\frac{14-\sqrt{2}}{4}\right)\left(\frac{a}{2}\right)^2}{8}\right\} - \left\{a^2\left(\frac{14-(\sqrt{2}+6\sqrt{3})}{96}\right)\right\} = X$$

So X = area of the lune.

6. Area of the trapezium ABCD = 3X + semicircle on OA

$$= 3X + \left\{ \frac{\left(\frac{14 - \sqrt{2}}{4}\right)\left(\frac{a}{2}\right)^2}{8} \right\}$$

by conventional method

Area of the trapezium = areas of 3 triangles

Area of each triangle (ODC or COB or OBA) = 1/2 x base x altitude

$$= \frac{1}{2} \times \frac{a}{2} \times \sqrt{\left(\frac{a}{2}\right)^2 - \left(\frac{a}{4}\right)^2}$$

area of 3 triangles $= 3 \times \frac{\sqrt{3}a^2}{16}$ or $\frac{1}{2} h (a + b) = \frac{1}{2} \times \frac{\sqrt{3}a}{4}\left(a + \frac{a}{2}\right)$

$$\left(\frac{3\sqrt{3}}{16}\right)a^2 = 3a^2 \left\{ \frac{24\sqrt{3} + \sqrt{2} - 14)}{384} \right\} + \left\{ \frac{\left(\frac{14 - \sqrt{2}}{4}\right)\left(\frac{a}{2}\right)^2}{8} \right\}$$

3x

The geometrical constructions of Hippocrates could be possible only when value of π is **$(14-\sqrt{2})/4$.**

Reference:

W.W.Rouse Ball (1960), **A Short Account of the History of Mathematics**, Dover Publications, New York.

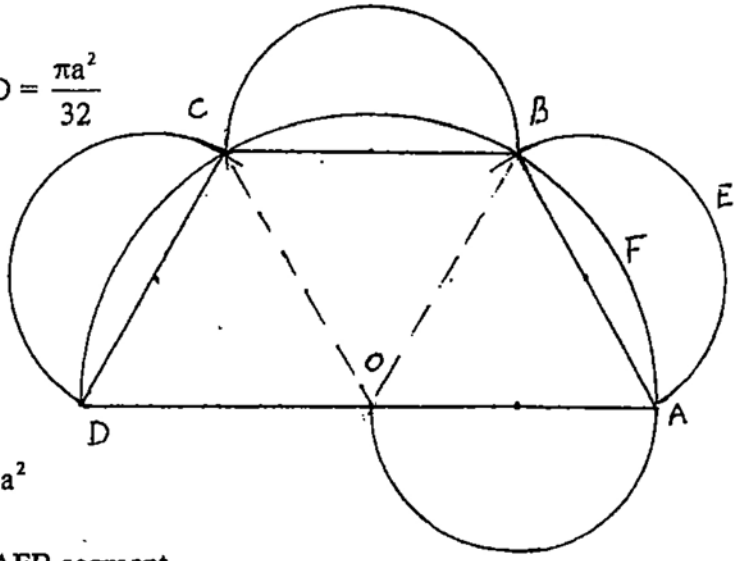
EXPLAINED ARITHMETICALLY

"∴ Sq. on AD = sum of Sqs. on OA, AB, BC and CD

∴ Area semicircle ABCD = Sum of areas of semicircles on OA, AB, BC and CD.

Take away the common parts

∴ Area trapezium ABCD = 3 lune AEFB + Semicircle on OA"



1. Area of semicircle on AD = $\frac{\pi a^2}{8}$

2. Area of semicircles on OA, AB, BC and CD = $\frac{\pi a^2}{32}$

3. Area of sector OAFB = $\left(\frac{\pi a^2}{4}\right)\frac{1}{6} = \frac{\pi a^2}{24}$

4. Area of triangle OAB = $\left(\frac{\sqrt{3}}{16}\right)a^2$

5. Area of segment AFB = Sector – Triangle

$$= \frac{\pi a^2}{24} - \left(\frac{\sqrt{3}}{16}\right)a^2 = \left(\frac{2\pi - 3\sqrt{3}}{48}\right)a^2$$

6. Area of lune AEBF = Semicircle on AB – AFB segment

$$= \frac{\pi a^2}{32} - \left(\frac{2\pi - 3\sqrt{3}}{48}\right)a^2 = \left\{\frac{\left(6\sqrt{3} + 3\pi\right) - 4\pi}{96}\right\}a^2 = x$$

x = area of the lune

7. Area of the trapezium ABCD = 3x + semicircle OA

$$= 3x + \frac{\pi a^2}{32} = 3\left\{\frac{\left(6\sqrt{3} + 3\pi\right) - 4\pi}{96}\right\}a^2 + \frac{\pi a^2}{32} = \left\{\frac{\left\{\left(6\sqrt{3} + 3\pi\right) - 4\pi\right\} + \pi}{32}\right\}a^2$$

8. By conventional method

Area of the trapezium = Area of the 3 triangles = $\left(\frac{3\sqrt{3}}{16}\right)a^2$

9. Area of 3 lunes = Area of the trapezium $\left\{\frac{\left\{\left(6\sqrt{3} + 3\pi\right) - 4\pi\right\} + \pi}{32}\right\}a^2 = \left(\frac{3\sqrt{3}}{16}\right)a^2$

$$= \left(\frac{6\sqrt{3}}{32}\right)a^2 = \left(\frac{3\sqrt{3}}{16}\right)a^2$$

The above construction of Hippocrates clearly showed that he had squared exactly lunes and a semicircle.

A humble submission to the mathematics community

The demarcation of a line segment equal to 3.14159 ..... in the length of the circumference of the circle

Sir, Namasthe !

When my work on deriving the new value of π equal to $\frac{14-\sqrt{2}}{4} = 3.1464466....$ was submitted, I was advised by many to think how to derive the present π value 3.1415926.... cautioning me as the new one was wrong.

In obedience, I hereby submit the following formula giving the number exact upto five decimal places.

$$3+\frac{9\pi}{200} = 3.14159.... \text{ where } \pi = \frac{14-\sqrt{2}}{4}$$

To get the same value (3.14159...) from Classical method of Archimedes, we have to travel a long way using the following general formula to calculate the length of the perimeter of the inscribed polygon of 1536 sides starting from 6 sides, in a circle

$$\text{Side} = s_{2n} = \sqrt{\left[2r^2 - r\sqrt{\left(4r^2 - s_n^{\,2}\right)}\right]}$$

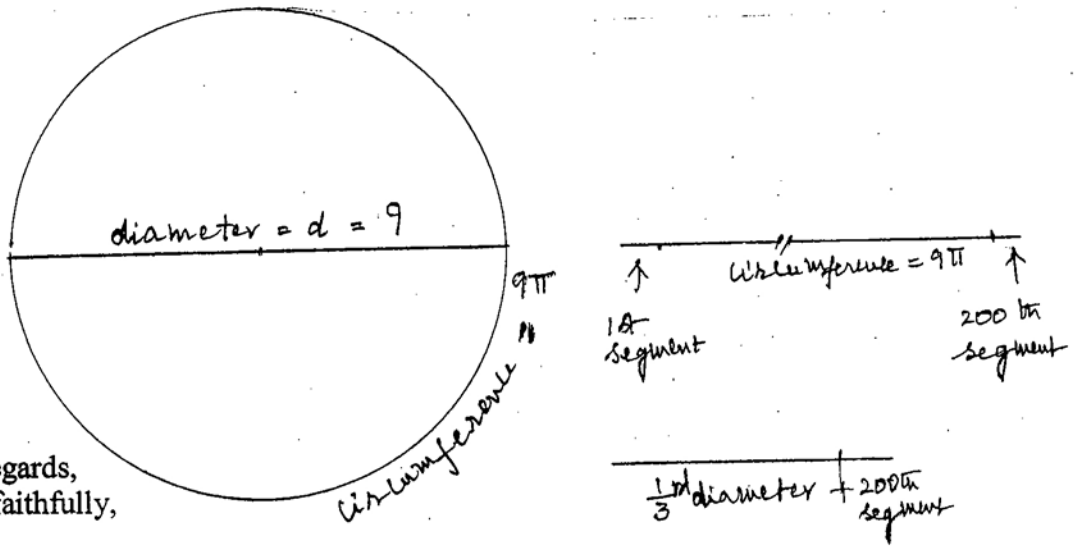
A geometrical line segment for 3.14159....

Let us draw a circle with diameter 9 and radius 9/2. Cut the circumference at one point and straighten it and further divide it into 200 equal segments.

Similarly, divided the diameter into 3 equal segments.

$\frac{1}{3}^{rd}$ diameter + one segment length of circumference

$$\left(\frac{9}{3}\right) + \left(\frac{9\pi}{200} = \frac{9\left(\frac{14-\sqrt{2}}{4}\right)}{200}\right) = 3.14159 ....$$



With regards,
Yours faithfully,

R.D. Sarva Jagannadha Reddy, Retd. Zoology Lecturer, 19-9-73/D3, Jayalakshmi Colony, S.T.V. Nagar, TIRUPATI – 517 501, INDIA.