

[Space Sierpinski Triangle Fractal by titoinou](#)

The Art of Triangles Project Teacher Edition

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Our Team

Curriki Geometry would not be possible if not for the tremendous contributions of the content contributors, editors, and reviewing team.

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Table of Contents

Section 1: Introduction and Project Resources

Introduction to Curriki Geometry: The Art of Triangles	6
What Is Project-based Learning (PBL)?	9
Tools and Resources	10
The Art of Triangles Additional Resources	12
Overview of Project Management	13
Overview of Assessment	14
Protocol for Refining the Driving Question	16
Curriki Geometry Glossary	18

Section 2: Rubrics

Mathematical Practices Rubric	24
Presentation and Performance Rubric	26
Teamwork Rubric	28
The Art of Triangles Rubric	30

Section 3: The Art of Triangles Project

The Art of Triangles Overview	32
The Art of Triangles Suggested Pacing Guide	37

Section 4: Reflection Tools

Critical Friends Protocol	49
Teacher’s Post-project Review	50
Reflection Matrix	54
Methods for Reflection	56

Section 5: Appendices

Appendix A: Logs

Daily Learning Log: Individual	60
Daily Learning Log: Team	61
Project Milestones Checklist	62

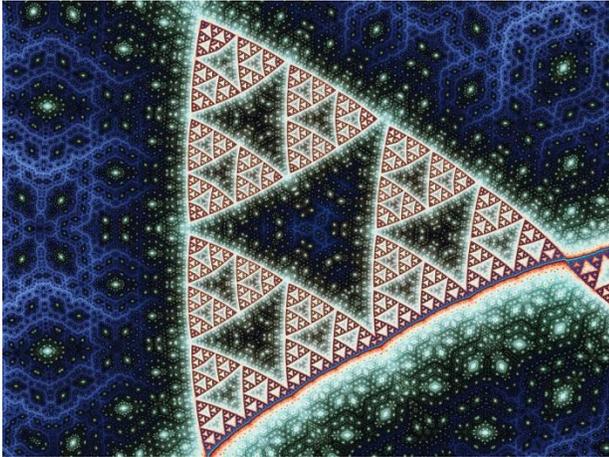
Appendix B: Teams and Grouping

Teams and Grouping Introduction	64
Team Roles	65
Designing Team Contracts	68
Student Contract Sample	70
Rules for High Performance Collaboration	73
Teambuilding Exercises	74

<u>Communication Worksheet</u>	78
<u>Conflict Resolution Speak</u>	79
<u>Problem/Compromise Card Instructions</u>	81
<u>Problem/Compromise Cards</u>	82

Appendix C: Visible Thinking Routines

<u>Visible Thinking Routines</u>	85
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Section 1: Introduction and Project Resources

Introduction to Curriki Geometry: The Art of Triangles Project

Welcome!

Welcome to Curriki Geometry, a project-based geometry course. The Art of Triangles Project the fifth of six complete projects. All the projects are designed in a project-based learning (PBL) format.

This and all Curriki Geometry projects have been created with several goals in mind: accessibility, customization, and student engagement—all while encouraging students toward high levels of academic achievement. In addition to specific Common Core State Standards (CCSS) high school geometry standards, the projects and activities are designed to address the Standards for Mathematical Practice, which describe types of expertise that mathematics educators at all levels should seek to develop in their students. These practices rest on important “processes and proficiencies” with established importance in mathematics education. Furthermore, all projects have been designed to address 21st century skills: the knowledge and skills students need in 21st century communities and workplaces.

How to Use Curriki Geometry

Curriki Geometry is designed with flexibility in mind. You may choose to teach all or only some of the projects. In addition, the projects can be taught in any order. You can customize Curriki Geometry in a manner that works best for you.

Standards

The projects are designed to meet Common Core State Standards for both Traditional and Integrated Math pathways in the following Units:

Project	Traditional	Integrated
1 Selling Geometry	Unit 1	Unit 5 Year 1
2 Designing a Winner	Unit 2	Unit 5 Year 2
3 What’s Your Angle, Pythagoras?	Unit 3	Unit 6 Year 2 (partial)
4 TED Talk: House of the Future	Unit 4	Unit 6 Year 1
5 The Art of Triangles	Unit 5	Unit 6 Year 2
6 How Random Is My Life?	Unit 6	Unit 4 Year 2

Rubrics and Overview of Assessment

This project includes a [Teamwork rubric](#), a [Presentation and Performance rubric](#), and a suggested [overall rubric](#) for the project. All rubrics can be used to generate a score point for grading.

Each project focuses on two or more of the eight Mathematical Practices recommended in the Common Core State Standards. A rubric for assessing [Mathematical Practices](#) is included in the project resources. It is important that teachers use the Mathematical Practices to encourage inquiry, problem solving, and mastery of mathematical methods.

Suggested Pacing Guide

The Suggested Pacing Guide offers day-by-day suggestions and guidance for completing the project. Remember, this is only a suggestion, so as the educator, you can adapt the schedule to your needs. As much as practical throughout each project, intersperse instruction with team discussions rather than whole class discussion.

Differentiating Instruction

The Pacing Guide is designed for a typical student.

For students who may struggle in a project, consider:

- Scaffolding skills by breaking them down into steps, such as the ability to listen or to resolve conflicts as steps to becoming a better team member. The scaffolds can be taught prior to the project.
- Adjusting the complexity of the task to fit the ability level or background knowledge of the student.
- Setting up a buddy system in which an advanced student on the team assists his or her teammate. This is particularly useful if students are struggling with language acquisition challenges.
- Offering more ongoing coaching support. In PBL, individual student needs will vary.

For advanced students, consider:

- Encouraging students to develop a set of personal goals that exceed the normal demands of the project or that meet college and career readiness goals.
- Putting advanced students in leadership roles in the team.
- Discussing performance that meets the criteria for breakthrough, the category on the rubrics that exceeds mastery of academic content and indicates a high level of critical thinking or creativity.

Tools and Resources

Resources for use in the project include viewables such as videos, documents, web pages, and dynamic geometry constructions, quizzes and exam suggestions for assessment, and other tools related to the project.

Additional Resources

The appendices include logs for daily learning and project milestones, as well as information on teams and [Visible Thinking Routines](#).

What Is Project-based Learning?

“In PBL, students go through an extended process of inquiry in response to a complex question, problem, or challenge. While allowing for some degree of student “voice and choice,” rigorous projects are carefully planned, managed, and assessed to help students learn key academic content, practice 21st Century Skills (such as collaboration, communication & critical thinking), and create high-quality, authentic products & presentations.” – [BIE, The Buck Institute for Education](#)

For more information on PBL:

- Read about [PBL and Thom Markham’s PBL Design and Coaching Guide](#).
- Read an [article](#) on PBL.
- See examples of PBL.
 - [Wing Project Part 1](#) (video; 5 minutes)
 - [Wing Project Part 2](#) (video; 5 minutes)
 - [PBL at High Tech High](#) (video; 15 minutes)

Tools and Resources

[These resources](#) may be useful for all of the Curriki Geometry projects. There are also additional resources useful for specific projects.

General Sites

Resource	Description	Best Use
Curriki	Curriki has over 1600 searchable resources for geometry.	You can search by topic to find problems, exams, or lesson plans that fit your needs with each project. In most cases, each project identifies the Curriki resources that will be the most helpful to you.
Khan Academy	The Khan Academy offers a one-semester course in geometry.	The course is searchable on Curriki . Find topics and lessons that fit your needs.
Math for America	Math for America offers resources and links to high quality math instruction and sample lessons in geometry and all other math subjects.	You can easily search lesson plans by grade and subject on this site.
Analyze Math	This site has thousands of math problems. Solutions and detailed explanations are included. Free math tutorials and problems help your students explore and gain a deep understanding of geometry and other math topics.	Use this site for finding geometry problems that make your teaching more active and engaging to students.
Quizpoo	This is an online quiz creator for teachers.	Use this site when you need to create quizzes.
Quizlet	This is an online quiz creator for teachers and students.	This is a student-friendly tool that helps students design their own quizzes as study tools.
Visuwords	This is an online tool for vocabulary building.	This is an excellent tool for extending students' knowledge of origins and relationships between words.
Cloze Notes	The eHow.com website offers and explanation of cloze notes, a method for helping students learn and retain key ideas and vocabulary.	Cloze is useful for ensuring student participation in videos as well as shared reading.
Three Ring	Three Ring allows teachers to photograph any work and record presentations or discussion. It also allows students to upload their work.	This supports project-based learning.

Presentation and Communication Tools

Resource	Description	Best Use
<u>Prezi</u>	This is a flexible, creative visual presentation tool that allows students to tell detailed stories about their work.	Prezi is free, but requires students to create an account.
<u>Doceri</u>	This is an iPad interactive whiteboard and screencast recorder with sophisticated tools for hand-drawn graphics and built-in remote desktop control.	Doceri works well in a 1-to-1 classroom.
<u>Padlet</u>	Padlet is an Internet application that works like an online sheet of paper where students can put any content (e.g. images, videos, documents, text) anywhere on the page, together with anyone, from any device.	Think of it like a multimedia friendly, free-form, real-time wiki.
<u>Haiku Deck</u>	Haiku Deck is a free presentation software tool that makes it easy to integrate graphics and pictures into creative presentations.	Haiku Deck requires an iPad.
<u>Blogster</u>	This is a mainstream blogging site that students can use to create and post blogs.	This site may require monitoring.
<u>Google SketchUp</u>	This is free, downloadable software for creating 3D presentations.	This is easy to learn and is integrated with Google Maps.
<u>Movenote</u>	Movenote works with Google Drive, Microsoft mail, Gmail, and Google Docs and on any device. Students open Movenote inside their email program, prompting their camera to automatically be activated. They then add their content (from any document type) and begin recording themselves talking through the information. Free registration with an email account is required. If you are a Google Apps for Education district, there is a simple add-on available in Google Drive. There is also an app for iOS devices.	This is a wonderful tool for the flipped classroom and for having students showcase what they've learned in a particular unit of study.
<u>Trello</u>	This is the perfect tool for organizing anything, including a group project. Working on iOS, Android, Windows 8, and the web, it allows you to brainstorm ideas, set to-do lists, add photos, monitor progress, and keep everyone informed with the latest details.	It works especially well with project-based learning activities and group work.

The Art of Triangles Additional Resources

LINK	DESCRIPTION
Triangles and Art	
Samples of triangles in artwork	This Google search contains many examples of artwork that incorporates triangles.
Creating art from triangles	These are samples of triangle art posted on Tumblr.
A triangles and art project	This Prezi on art and triangles was created by a high school student.
Using triangles as a tool for composition in artwork	This site shows how various types of art use triangles to compose paintings.
An exercise in art and triangles	This exercise helps students identify the use of triangles in a piece of fine art.
Triangles and fingernails	This is a geometric challenge on how to put triangular art on fingernails.
Geometry Resources for Triangles	
Unit and problem sets on triangles and congruence	These are problem sets from Curriki that may be used for student practice on problems.
Getting started – Congruence and similarity	This mini-lesson from Curriki takes students through the basics of congruence and similarity.
Introduction to Triangles – video	These Khan Academy videos on Curriki helps students work out problems on triangles. (4 videos, each approximately 11 minutes)
Solving Problems Using Triangles	This YouTube video on Curriki shows students how to solve problems using isosceles and equilateral triangles. (9 minutes)

Overview of Project Management

Managing a project differs from classroom management. Primarily, you will need to give equal attention to the *process* of learning as well as to the delivery of content. You become a “people manager” as well as a classroom teacher. You will be expected to facilitate student problem solving by helping them ask questions, formulate solutions, test hypotheses, and design products and presentations. This may be one of the challenges that teachers face when implementing the Common Core State Standards. It is true of project-based learning as well.

Project management can be broken down into five categories. For each category, we offer resources to meet the challenge. These resources represent a set of best practices for PBL that have been tested and refined by PBL teachers.

- 1. *Forming and Managing Effective Student Teams.*** The resources in Appendix B: [Teams and Grouping](#) provide tools such as conflict resolution assistance and teambuilding activities.
- 2. *Maintaining quality throughout the project.*** Appendix C: [Visible Thinking Routines](#) offers links to dozens of student-to-student protocols for effective interaction between students.
- 3. *Supporting different ability levels of students.*** Appendix B: [Teams and Grouping](#) contains resources and activities for coaching team members. These methods allow teachers to provide individual support to ELL or Special Education students.
- 4. *Completing the project in a positive manner.*** This project includes a Pacing Guide to allow sufficient time for students to solve problems, create the final product, and prepare for presentations. Use rubrics often as feedback tools on performance throughout the project. Plan to re-teach key concepts and standards, if necessary.
- 5. *Reflecting on the project to anchor learning and retention.*** At the end of each project see the [Reflection resources](#) for tools such as lists for project milestones or methods for reflection. The [Reflection Matrix](#) is your key tool.

Overview of Assessment

Both project-based learning (PBL) and the Common Core State Standards (CCSS) assume that teachers will use multiple assessments, including tests and performance rubrics. You may choose your own assessments, but this project also includes built-in assessments, suggestions for making your own assessments, as well as links to the [Khan Academy](#) geometry assessments, geometry assessments on [Curriki](#), and other resources.

In addition to the core content of this project, it is designed to teach and assess 21st century skills, mathematical practices, vocabulary, and end-of-project presentations and products. This project has a rubric that may be used for assessment. You may also choose to assess the two critical 21st century skills, teamwork and presentations, using the [Teamwork Rubric](#) or the [Presentation and Performance Rubric](#).

Project Rubrics versus Individual Skills Rubrics

The individual rubrics (Mathematical Practices, Teamwork, Presentation and Performance) and the Project Rubric can be used separately or in tandem. This may vary with the project and time of year. For example, early in the academic year, you may wish to use the Teamwork or Presentation Rubric separately as a method for training students on a detailed performance rubric. Later in the year, the Project Rubric, which condenses the individual rubrics and provides a holistic assessment of the overall project, may be more appropriate. It will be helpful to review all rubrics before starting the project to find the best combination for your students.

The Mathematical Practices Rubric

The [Mathematical Practices Rubric](#) is key to project success and student learning. The CCSS for Mathematics include eight mathematical practices that accompany the content standards. Use of the rubric is included in the [Suggested Pacing Guide](#). It will be helpful to highlight the Mathematical Practices Rubric early in the course and before the project begins.

Encouraging Critical Thinking and Creativity

With the exception of the Mathematical Practices Rubric, all the rubrics for this project contain a Breakthrough option. This is designed to capture exceptional performance, unusual insights, or creative thinking. It is not the 'A' part of the rubric; it signifies that the student has gone beyond the standards and achieved an unexpected result.

Grading and Rubrics

An important note about grading: Below each element and section of the rubrics, you can locate a point scale that allows you to transfer the rubric evaluations into a point-based grade book. Since grading varies among teachers, the suggested point scales can be modified to meet your classroom, school, or project guidelines.

Using Additional Rubrics

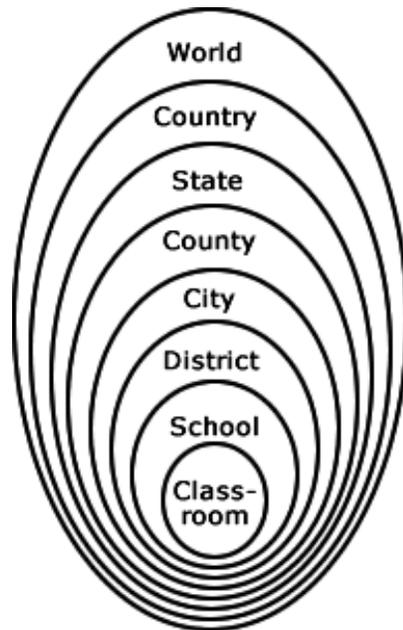
You may also choose to use your own existing rubrics, develop additional assessments, or change the rubric language. Students can also help design language to help evaluate the presentation.

Protocol for Refining the Driving Question

1. Can my students read and comprehend the Driving Question (DQ)? Or will it be fun for them to have some need-to-know words in the DQ?
2. Is the DQ open-ended or can it be answered with a *yes* or *no*?
3. Would the DQ benefit from a local context to either narrow the scope, or increase the scope, and most importantly to allow interaction with local adults and to have power over their local environment? (How can we change the district lunch menu so that it is more nutritious and more appealing?)
4. Does the DQ offer opportunities for students to express voice and choice?
5. Does the DQ ask students to engage in an inquiry that is both rigorous (challenging and full of critical thinking opportunities) and relevant?
6. Does the DQ sound like a traditional teacher/academic question, or does it sound like something exciting and different?
7. Does the DQ show that the students will be doing something relevant and exciting to them (that some might do even if it wasn't required), or are they just going to be doing it for the teacher?
8. Does the DQ allow me to design both individual and collaborative learning tasks that require higher-level thinking skills, or is it make a list and collect some facts?
9. Does the DQ require students to learn new skills and knowledge and to demonstrate higher-level understandings or applications?
10. Does the DQ encourage students to perform tasks and applications that adults would possibly do? Can it result in an action plan presented to outside adults?
11. Will the DQ benefit from referring to the future to make it more open-ended? (How do we build a school in the parking lot in the year 2050?)
12. Should I change the implied audience in the question to increase the challenge? (How do we write a book on global warming for 4th graders?)
13. Should I go up or down the Concentric Circles of Scope (see below)?

14. If I think what the ideal student team would be working on and wrestling with three-quarters of the way through the project (to focus what I am most passionate about in this project), can I change the DQ to more directly get all students to that place?

15. Does the DQ allow students to have experiences that will help them determine their proclivities and interests for certain careers versus other careers?



Curriki Geometry Glossary

The following terms are used throughout the Curriki Geometry projects and represent the core vocabulary and concepts that students should know to meet Common Core State Standards.

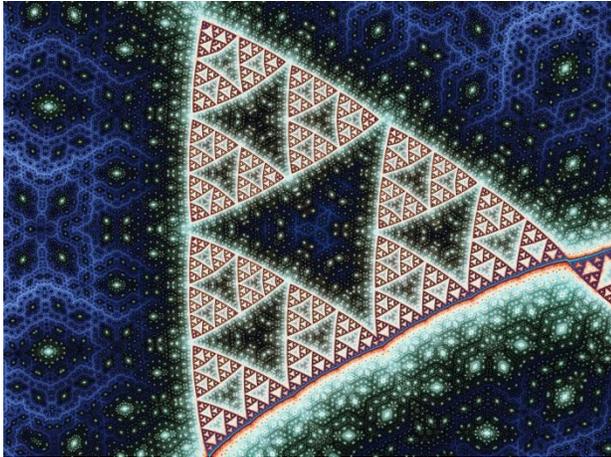
Term	Definition
Acute triangle	A triangle for which all interior angles are acute, or less than 90 degrees. (see also <i>obtuse triangle, right triangle</i>)
Alternate exterior angles	Exterior angles on alternate sides of the transversal (not on the same parallel line)
Alternate interior angles	Interior angles on alternate sides of the transversal (not on the same parallel line)
Altitude of a triangle	A straight line through a vertex and perpendicular to (i.e. forming a right angle with) a line containing the base (the opposite side) of a triangle
Angle bisector theorem	Concerned with the relative lengths of the two segments that a triangle's side is divided into by a line that bisects the opposite angle. It equates their relative lengths to the relative lengths of the other two sides of the triangle.
Angle bisector theorem, converse	If a point is equidistant from the sides of an angle, then it is on the angle bisector.
Angle	Formed by two rays, called the sides of the angle, sharing a common endpoint, called the vertex of the angle
Arc	A closed segment (symbol: \frown) of a differentiable curve in the two-dimensional plane
Area	Any particular extent of space or surface
Bisector	A line that divides something into two equal parts
Center of a polygon	In a rotation, the point that does not move. The rest of the plane rotates around this one fixed point.
Centroid of a triangle	The point where the three medians of the triangle intersect
Circumcenter of a triangle	The point where the three perpendicular bisectors of a triangle meet
Circumference (circles)	A complete circular arc; also the distance around the outside of a circle
Circumscribed	A geometric figure that is drawn around another geometric figure so as to touch all its vertices
Combination	A way of selecting several things out of a larger group, where (unlike permutations) order does not matter.
Common parts	Informal language that describes similarities, differences, parts (e.g., number of sides and vertices/"corners") and other attributes (e.g., having sides of equal length) of two- and three-dimensional shapes, in different sizes and orientations
Compass	An instrument for drawing circles and arcs and measuring distances between points, consisting of two arms linked by a movable joint.
Complement probability	In probability theory, the complement of any event A is the event [not A], i.e. the event that A does not occur. (see also <i>conditional probability, experimental probability, probability, theoretical probability</i>)
Complementary angle	Two angles that add up to 90 degrees (ie they form a right angle)
Composition	The combining of distinct parts or elements to form a whole
Compound event	An event whose probability of occurrence depends upon the probability of

	occurrence of two or more independent events
Compression	To reduce a shape in size while retaining proportions
Conditional probability	The probability that an event will occur, when another event is known to occur or to have occurred (see also <i>complement probability, experimental probability, probability, theoretical probability</i>)
Conditional probability formula	The conditional probability of A given B is denoted by $P(A B)$ and defined by the formula $P(A B) = \frac{P(AB)}{P(B)}$, provided $P(B) > 0$. (see also <i>probability formula</i>)
Congruency by AAS, ASA, SAS, SSS	Triangles are congruent if any pair of corresponding sides and their included angles are equal in both triangles.
Congruent	Identical in form; coinciding exactly when superimposed
Construction	The drawing of various shapes using only a compass and straightedge or ruler. No measurement of lengths or angles is allowed.
Coordinates	On the coordinate plane, the pair of numbers giving the location of a point (ordered pair). In three-dimensional coordinates, the triple of numbers giving the location of a point (ordered triple). In n -dimensional space, a sequence of n numbers written in parentheses.
Corresponding angles	The angles in matching corners when two lines are crossed by another line (which is called the transversal)
Dependent events	When the outcome of one event affects the outcome of another (see also <i>independent events, mutually exclusive events</i>)
Dilation	A transformation that grows or shrinks a polygon by a given proportion about a center point
Equidistant	Distant by equal amounts from two or more places
Endpoint	Either of two points marking the end of a line segment (see also <i>midpoint</i>)
Events	A set of outcomes of an experiment (a subset of the sample space) to which a probability is assigned
Experimental probability	The ratio of the number of times the event occurs to the total number of trials (see also <i>complement probability, conditional probability, probability, theoretical probability</i>)
Exterior angle	The angle between any side of a polygon and an extended adjacent side (see also <i>interior angle</i>)
Frequency table	Lists items and uses tally marks to record and show the number of times they occur
Fundamental counting principle	When there are m ways to do one thing, and n ways to do another, then there are $m \times n$ ways of doing both.
Glide reflection	A transformation in which a graph or geometric figure is picked up and moved to another location without any change in size or orientation (see also <i>reflection</i>).
Glide reflectional symmetry	The symmetry that a figure has if it can be made to fit exactly onto the original when it is translated a given distance at a given direction and then reflected over a line. (see also <i>reflectional symmetry, rotational symmetry, symmetry</i>)
Horizontal line	A constructive line, either drawn or imagined, which passes through the point of sight, and is the chief line in the projection upon which all verticals are fixed, and upon which all vanishing points are found
Hypotenuse	The longest side of a right-angled triangle; the side opposite of the right angle
Hypotenuse-leg theorem (HL theorem)	If the hypotenuse and leg of a right triangle are congruent to the hypotenuse and leg of another right triangle, then the triangles are congruent.
Image	An optically formed duplicate, counterpart, or other representative reproduction of an object, especially an optical reproduction formed by a lens or mirror

Incenter of a triangle	The point where the three angle bisectors of a triangle meet
Included angle	The angle made by two lines with a common vertex
Independent events	When the outcome of one event does not influence the outcome of the second event (see also <i>dependent events</i> , <i>mutually exclusive events</i>)
Inscribed in (the triangle)	Drawing one shape inside a triangle so that it just touches the sides of the triangle
Interior angle	Any of the four angles formed between two straight lines intersected by a third straight line (see also <i>exterior angle</i>)
Intersection	The probability that events A and B both will occur
Isometry	A transformation that is invariant with respect to distance. That is, the distance between any two points in the pre-image must be the same as the distance between the images of the two points.
Isosceles triangle theorem	The angles opposite the two equal sides of an isosceles triangle are equal.
Isosceles triangle theorem, converse	If two angles of an isosceles triangle are congruent, the sides opposite them are congruent.
Leg	Either of the sides in a right triangle opposite an acute angle
Line of symmetry	The line of symmetry of a two-dimensional figure is a line such that, for each perpendicular constructed, if the perpendicular intersects the figure at a distance d from the axis along the perpendicular, then there exists another intersection of the figure and the perpendicular, at the same distance d from the axis, in the opposite direction along the perpendicular. (see also <i>point of symmetry</i>)
Median of a triangle	A line segment joining a vertex of a triangle to the midpoint of the opposing side
Midpoint	A point at or near the middle of, or equidistant from, both ends, as of a line, the midpoint of a boundary (see also <i>endpoint</i>)
Midpoint formula in the coordinate plane	The point halfway between the endpoints of a line segment is called the midpoint. A midpoint divides a line segment into two equal segments.
Midsegment of a triangle	The segment joining the midpoints of two sides of a triangle
Mutually exclusive events	Two events that cannot occur at the same time (see also <i>dependent events</i> , <i>independent events</i>)
n factorial	The factorial of a natural number n is the product of the positive integers less than or equal to n .
Non-included angle	The side of a triangle that is not included by two given angles
Obtuse triangle	A triangle which has an obtuse angle (an angle greater than 90 degrees but less than 180 degrees) as one of its interior angles (see also <i>acute triangle</i> , <i>right triangle</i>)
Ordered pair	Two numbers written in the form (x, y) (see also <i>ordered triple</i> , <i>n-tuple</i>)
Ordered triple	Three numbers written in the form (x, y, z) (see also <i>ordered pair</i> , <i>n-tuple</i>)
n -tuple	n numbers written in the form $(x_1, x_2, x_3, \dots, x_n)$ (see also <i>ordered pair</i> , <i>ordered triple</i>)
Orthocenter of a triangle	The point where the three altitudes of a triangle intersect
Outcome	The result of an experiment in probability theory
Overlap	Similar triangles in which one triangle is on top of (overlapping) another triangle
Parallel	Two lines on a plane that never meet. They are always the same distance apart.
Permutation	All possible arrangements of a collection of things, where the order is important
Perpendicular	A straight line at an angle of 90 degrees to a given line, plane, or surface

Perpendicular bisector	A line or a ray that cuts another line segment into two equal parts at 90 degrees
Perpendicular bisector theorem	The perpendicular bisector of a line segment is the locus of all points that are equidistant from its endpoints.
Perpendicular bisector theorem, converse	If a point is equidistant from the endpoints of a segment, then it is on the perpendicular bisector of the segment.
Point of concurrency	The point where three or more lines intersect
Point of symmetry	A special center point for certain kinds of symmetric figures or graphs. If a figure or graph can be rotated 180° about a point P and end up looking identical to the original, then P is a point of symmetry. (see also <i>line of symmetry</i>)
Polygon angle-sum theorem	The sum of the measures of the angles of an n -gon is $(n-2)180$
Polygon angle-sum theorem, corollary	The measure of each interior angle of a regular n -gon is $180 \cdot (n-2)/n$
Polygon exterior angle-sum theorem	If a polygon is convex, then the sum of the measures of the exterior angles, one at each vertex, is 360 .
Preimage	The original figure prior to a transformation.
Probability	The chance that something will happen or how likely it is that some event will happen (see also <i>complement probability, conditional probability, experimental probability, theoretical probability</i>)
Probability distribution	A graph, table, or formula that gives the probability for each value of the random variable
Probability formula	The number of ways an event can occur divided by the total number of possible outcomes (see also <i>conditional probability formula</i>)
Isosceles triangle	A triangle with two equal sides and two equal angles
Proportions	Comparative relation between things or magnitudes as to size, quantity, number, etc.
Pythagorean theorem	An equation relating the lengths of the sides of a right triangle. The sum of the squares of the legs of a right triangle is equal to the square of the hypotenuse. The formula is $a^2 + b^2 = c^2$.
Ratios	The result of dividing one number or expression by another. Sometimes a ratio is written as a proportion, such as 3:2 (three to two). More often, though, ratios are simplified according to the standard rules for simplifying fractions or rational expressions. Note: The word <i>rational</i> indicates that a ratio (in the second sense) is involved. The word <i>rate</i> also indicates a ratio is involved, as in instantaneous rate of change or average rate of change.
Ray	A part of a line that begins at a particular point (called the endpoint) and extends endlessly in one direction
Reflection	A transformation that creates a mirror image of a shape (see also <i>glide reflection</i>).
Reflectional symmetry	The descriptive term for an object or figure that is indistinguishable from its transformed image (see also <i>glide reflectional symmetry, rotational symmetry, symmetry</i>)
Reflexive property of equality	Anything is equal to itself
Relative frequency	The ratio of the actual number of favorable events to the total possible number of events; often taken as an estimate of probability
Remote interior angles	The two angles of a triangle that are not adjacent to the exterior angle, which is drawn by extending one of the sides.
Right triangle	A triangle which has a right (90 degree) interior angle (see also <i>acute triangle, obtuse triangle</i>)

Rigid motion	The variance in position and orientation when a rigid body moves
Rotation	A transformation in which a plane figure turns around a fixed center point. In other words, one point on the plane, the center of rotation, is fixed and everything else on the plane rotates about that point by a given angle.
Rotational symmetry	When an object that looks the same after a certain amount of rotation (see also <i>glide reflectional symmetry</i> , <i>reflectional symmetry</i> , <i>symmetry</i>)
Same-side exterior angles	Exterior angles are created where a transversal crosses two (usually parallel) lines. Each pair of these angles is outside the parallel lines, and on the same side of the transversal.
Same-side interior angles	When two parallel lines are intersected by a transversal, one type of angle formed is same-side interior angles. Same side interior angles are pairs of angles that are found on the same side of the transversal.
Sample space	In probability theory, the set of all possible outcomes or results of an experiment
Segments	A line segment is a part of a line that is bounded by two distinct end points, and contains every point on the line between its end points.
Slope	The tangent of the angle between a given straight line and the x-axis of a system of Cartesian coordinates
Straightedge	A bar or piece of material (wood, metal, plastic, etc) with a straight edge for testing straight lines and surfaces or for cutting along or drawing straight lines
Supplementary angles	Two angles that add up to 180 degrees
Symmetry	Illustrated by a geometric figure or a graph consisting of two parts that are congruent to each other (see also <i>glide reflectional symmetry</i> , <i>reflectional symmetry</i> , <i>rotational symmetry</i>)
Tessellation	A plane with identically shaped pieces that do not overlap or leave blank spaces. The pieces do not have to be oriented identically. A tessellation may use tiles of one, two, three, or any finite number of shapes.
Theoretical probability	The likelihood of an event happening based on all the possible outcomes (see also <i>complement probability</i> , <i>conditional probability</i> , <i>experimental probability</i> , <i>probability</i>)
Transformation	Operations that alter the form of a figure. The standard transformations are translations, reflections, dilations (stretches), compressions (contractions or shrinks), and rotations.
Translation	A transformation in which a graph or geometric figure is picked up and moved to another location without any change in size or orientation
Transversal	A line intersecting two or more lines
Tree diagram	A representation of a tree structure in which the probability of each branch is written on the branch and the outcome is written at the end of the branch
Triangles	A closed plane figure having three sides and three angles
Vertex	The point about which an angle is measured
Vertical angles	One of two opposite and equal angles formed by the intersection of two lines
Volume (prisms, cylinders, pyramids, cones, spheres)	The total amount of space enclosed in a solid



Section 2: Rubrics

Student: _____

Project/Unit: _____

Date: _____

Mathematical Practices Rubric

CRITERIA	WEIGHT	Emerging (Below Standards)	Proficient (Meets Standards)	Mastery (Exceptional Performance)
1. Make sense of problems and persevere in solving them.	15%	<ul style="list-style-type: none"> Student cannot recognize givens, constraints, relationships, and goals of a problem. Student does not monitor progress or adjust approach to problem. Student does not check solutions for errors. 	<ul style="list-style-type: none"> Student analyzes givens, constraints, relationships, and goals of a problem. Student monitors and evaluates progress and changes course if necessary. Student checks solutions for errors. Student asks continually: "Does this make sense?" 	In addition to meeting the PROFICIENT criteria <ul style="list-style-type: none"> Student quickly analyzes key aspects of a problem. Student easily monitors progress and adjusts approach to problem. Student routinely checks solutions for errors. Student is able to fully explain solution to others.
		1-----3-----5	6-----8-----12	13-----14-----15
2. Reason abstractly and quantitatively.	15%	<ul style="list-style-type: none"> Student cannot represent problem symbolically. Student shows limited ability to contextualize the problem. 	<ul style="list-style-type: none"> Student abstracts a given situation and represents it symbolically. Student manipulates the representing symbols and shows ability to contextualize the problem. Student creates a coherent representation of the problem. 	In addition to meeting the PROFICIENT criteria <ul style="list-style-type: none"> Student represents a problem symbolically in ways that show thorough understanding. Student manipulates the representing symbols in ways that clearly contextualize the problem. Student can explain the representation of the problem to others.
		1-----3-----5	6-----8-----12	13-----14-----15
3. Construct viable arguments and critique the reasoning of others.	15%	<ul style="list-style-type: none"> Student cannot state assumptions, definitions, and results in constructing arguments. Student makes limited conjectures or builds an illogical progression of statements to explore conjectures. Student shows limited ability to reason inductively or use logic. 	<ul style="list-style-type: none"> Student understands and uses stated assumptions, definitions, and results in constructing arguments. Student makes conjectures and builds a logical progression of statements to explore conjectures. Student analyzes situations and can recognize and use counterexamples. Student reasons inductively and uses logic and reasoning. 	In addition to meeting the PROFICIENT criteria <ul style="list-style-type: none"> Student can construct argument using stated assumptions and definitions, and results in constructing arguments. Student makes original conjectures and builds an elegant progression of statements to explore conjectures. Student quickly uses original counterexamples to explain or construct problem.
		1-----3-----5	6-----8-----12	13-----14-----15

4. Model with mathematics.	15%	<ul style="list-style-type: none"> • Student cannot link important quantities in a practical situation with use of tools such as diagrams, two-way tables, graphs, flowcharts, and formulas. • Student shows limited knowledge of how mathematics applies to problems arising in everyday life, society, and the workplace. 	<ul style="list-style-type: none"> • Student identifies important quantities in a practical situation and maps his or her relationships using such tools as diagrams, two-way tables, graphs, flowcharts and formulas. • Student applies mathematics to solve problems arising in everyday life, society, and the workplace. 	<p>In addition to meeting the PROFICIENT criteria</p> <ul style="list-style-type: none"> • Student maps complex practical situations with such tools as diagrams, two-way tables, graphs, flowcharts, and formulas. • Student has unusual insight into how mathematics applies to solving problems arising in everyday life, society, and the workplace.
		1-----3-----5	6-----8-----12	13-----14-----15
5. Use appropriate tools strategically.	10%	<ul style="list-style-type: none"> • Student does not use the available tools when solving a mathematical problem. • Student has limited ability to use technological tools to explore and deepen his or her understanding of concepts. 	<ul style="list-style-type: none"> • Student considers the available tools when solving a mathematical problem. • Student is able to use technological tools to explore and deepen his or her understanding of concepts. 	<p>In addition to meeting the PROFICIENT criteria</p> <ul style="list-style-type: none"> • Student chooses the best available tools when solving a mathematical problem. • Student is expert at using technological tools to explore and deepen his or her understanding of concepts.
		1-----3-----5	6-----8-----12	13-----14-----15
6. Attend to precision.	10%	<ul style="list-style-type: none"> • Student does not use definitions in discussion with others and in his or her own reasoning. • Student calculates accurately and efficiently, and expresses numerical answers with a minimal of precision appropriate for the problem context. 	<ul style="list-style-type: none"> • Student uses clear definitions in discussion with others and in his or her own reasoning. • Student calculates accurately and efficiently, and expresses numerical answers with a degree of precision appropriate for the problem context. 	<p>In addition to meeting the PROFICIENT criteria</p> <ul style="list-style-type: none"> • Student uses clear definitions in a variety of ways in discussion that helps others clarify their own reasoning. • Student calculates accurately and efficiently, and expresses numerical answers with the exact degree of precision appropriate for the problem context.
		1-----3-----5	6-----8-----12	13-----14-----15
7. Look for and make use of structure.	10%	<ul style="list-style-type: none"> • Student cannot discern a pattern or structure without assistance. 	<ul style="list-style-type: none"> • Student looks closely to discern a pattern or structure. 	<p>In addition to meeting the PROFICIENT criteria</p> <ul style="list-style-type: none"> • Student can easily identify a pattern or structure in a wide range of natural settings or practical ways.
		1-----3-----5	6-----8-----12	13-----14-----15
8. Look for and express regularity in repeated reasoning.	10%	<ul style="list-style-type: none"> • Student overlooks calculations that are repeated, and does not look for general methods or for shortcuts. 	<ul style="list-style-type: none"> • Student notices if calculations are repeated, and looks both for general methods and for shortcuts. 	<p>In addition to meeting the PROFICIENT criteria</p> <ul style="list-style-type: none"> • Student rarely repeats calculations, and first looks both for general methods and for shortcuts when problem solving.
		1-----3-----5	6-----8-----12	13-----14-----15

Student: _____

Project/Unit: _____

Date: _____

Presentation and Performance Rubric

CRITERIA	WEIGHT	Emerging (Below Standards)	Proficient (Meets Standards)	Mastery (Exceptional Performance)
Structure and Organization <ul style="list-style-type: none"> Intro Main ideas Supporting Materials Conclusion Length requirement TIME:	30%	<ul style="list-style-type: none"> No formal introduction or introduction had no clear thesis statement nor offered any preview of topics to be discussed. Main ideas were not separated into a logical progression. Important ideas were not supported with references or data. No conclusion or conclusion did not adequately summarize presentation Presentation did not use time allotted. 	<ul style="list-style-type: none"> Introduction had clear thesis statement and a preview of topics to be discussed. Main ideas were separated into a logical progression Speaker supported important ideas and viewpoints through accurate and detailed references to text or other works. Conclusion restated thesis statement and summarized the ideas presented Time requirement was met for specific assignment (neither too long nor too short). 	In addition to meeting the PROFICIENT criteria ... <ul style="list-style-type: none"> Clever attention-getting introduction or an imaginative thesis and preview. Ideas connected by original transitions, logical throughout; creative pattern. Conclusion tied speech together and left audience with memorable message Speaker used logical, ethical, and emotional appeals that enhanced a specific tone and purpose.
		1-----3-----5	6-----8-----12	13-----14-----15
Vocal Expression <ul style="list-style-type: none"> Rate and Volume of Speech Pitch, Articulation, and Pronunciation Memorization Performance 	25%	<ul style="list-style-type: none"> Speaker was hard to hear or understand. Voice or tone distracted from purpose of presentation. Excessive use of verbal fillers Did not memorize lines. Acting lacked expression. 	<ul style="list-style-type: none"> Speaker was easy to hear and understand. Tone was conversational, but with purpose. Voice sounded natural, neither patterned nor monotone. Speaker pronounced words clearly, correctly, and without verbal fillers. Had lines memorized. Expressive acting. 	In addition to meeting the PROFICIENT criteria ... <ul style="list-style-type: none"> Speaker was enjoyable to hear; used expression and emphasis. Speaker used voice to create an emotional response in audience. Knew all your lines and possessed dramatic flair.
		1-----3-----5	6-----8-----12	13-----14-----15
Physical Characteristics <ul style="list-style-type: none"> Eye Contact Posture Gestures Movement Attire 	15%	<ul style="list-style-type: none"> Little eye contact with audience. Poor or slouchy posture. Movements were stiff or unnatural. Attire was inappropriate for audience . 	<ul style="list-style-type: none"> Strong eye contact with entire audience. Posture conveyed confidence. Gestures and movements were natural and effective. Attire was appropriate for audience and purpose. 	In addition to meeting the PROFICIENT criteria ... <ul style="list-style-type: none"> Posture was commanding and purposeful. Attire was chosen to enhance presentation.
		1-----3-----5	6-----8-----12	13-----14-----15

Appropriateness of Content and Language For Audience, purpose, and assignment.	10%	<ul style="list-style-type: none"> • Speaker used inappropriate language, content, or examples for this audience. • Speaker did not demonstrate a clear understanding of the assignment or purpose of presentation. 	<ul style="list-style-type: none"> • Speaker obviously considered the audience and used appropriate language and examples. • Speaker displayed a clear understanding of assignment requirements and content. • Speaker understood purpose of presentation. 	In addition to meeting the PROFICIENT criteria ... <ul style="list-style-type: none"> • Examples and words were creative and well chosen for target audience.
		1-----3-----5	6-----8-----12	13-----14-----15
Overall Impact <ul style="list-style-type: none"> • Energy • Enthusiasm • Sincerity • Originality/ Creativity 	10%	<ul style="list-style-type: none"> • Speaker appeared bored by the message or presented without conviction. 	<ul style="list-style-type: none"> • Speaker appeared to believe strongly in message and demonstrated desire to have audience listen, understand, and remember. 	In addition to meeting the PROFICIENT criteria ... <ul style="list-style-type: none"> • Overall presentation was creative and exciting.
		1-----3-----5	6-----8-----12	13-----14-----15
Features <ul style="list-style-type: none"> • Multimedia • Visuals • Audio 	10%	<ul style="list-style-type: none"> • Materials detracted from content or purpose of presentation or were of such low quality as to discredit speaker. 	<ul style="list-style-type: none"> • Materials added, did not detract from presentation • Materials used were quality products; easy to see and hear. 	In addition to meeting the PROFICIENT criteria <ul style="list-style-type: none"> • Speaker creatively integrated a variety of objects, charts, and graphs to amplify the message.
		1-----3-----5	6-----8-----12	13-----14-----15
BREAKTHROUGH Evidence for exceptional or creative performance beyond mastery.	The evidence for breakthrough is:			

Student: _____

Project/Unit: _____

Date: _____

Teamwork Rubric

CRITERIA	Emerging (Below Standards)	Proficient (Meets Standards)	Mastery (Exceptional Performance)
Cohesion and Purpose	<ul style="list-style-type: none"> Team establishes no norms or weak norms. Team does not delegate or assign tasks. Majority of team members disengaged. Team shows little engagement in problem solving Team is unable to resolve conflicts. <p>1-----3-----5</p>	<ul style="list-style-type: none"> Team quickly establishes norms. Team assigns and delegates tasks. Team engages all members. Team shows purposeful engagement. Team listens and plans together. Team members show attentive posture and eye contact. Team demonstrates ability to resolve conflicts. Team demonstrates ability to look at issues from multiple points of view. <p>6-----8-----12</p>	<ul style="list-style-type: none"> Team quickly establishes thoughtful norms. Team assigns and delegates tasks based on thorough discussion of team strengths. Team immediately and deeply engaged in problem solving. Team overcomes conflicts immediately. Team uses listening and communication strategies to include all viewpoints. <p>13-----14-----15</p>
Organization	<ul style="list-style-type: none"> Team unable to generate ideas. Team does not focus on task or cannot prioritize. Team misses or does not establish deadlines. Team does not develop a realistic project schedule. Team cannot find resources. <p>1-----3-----5</p>	<ul style="list-style-type: none"> Team easily generates ideas. Team focuses on task and develops agenda. Team operates on time and meets deadlines. Team develops a project schedule. Team monitors their progress. Team accepts changes in task or focus. Team shows resourcefulness in finding information. <p>6-----8-----12</p>	<ul style="list-style-type: none"> Team uses techniques to brainstorm high quality ideas. Team develops a comprehensive, realistic agenda. Team meets or exceeds deadlines. Team monitors progress and adapts new schedules based on feedback. Team welcomes and feels challenged by changes in task focus. Team is unusually resourceful <p>13-----14-----15</p>
Inquiry and Problem Solving	<ul style="list-style-type: none"> Team cannot brainstorm solutions. Team does not use thinking strategies. Team does not pursue or demonstrate background knowledge for problem solving. Team shows little conceptual understanding of the problem or question. Team does not use the conventions and terms of the discipline in discussions. <p>1-----3-----5</p>	<ul style="list-style-type: none"> Team often brainstorms solutions. Team demonstrates use of thinking strategies. Team demonstrates use of background knowledge. Team shows conceptual understanding of problem. Team demonstrates knowledge of criteria for judgment of solutions. Team uses conventions and terms of the discipline. <p>6-----8-----12</p>	<ul style="list-style-type: none"> Team brainstorming leads to new solutions or fresh thinking. Team routinely uses thinking strategies. Team shows deep background knowledge. Team shows deep conceptual understanding of the problem or question. Team able to easily use the conventions and terms of the discipline. <p>13-----14-----15</p>

BREAKTHROUGH
Evidence for
exceptional or creative
performance beyond
mastery.

The evidence for breakthrough is:

The Art of Triangles Rubric

Student: _____

Project/Unit: _____

Date: _____

CRITERIA	Emerging (Below Standards)	Proficient (Meets Standards)	Mastery (Exceptional Performance)
Geometric Practices	<ul style="list-style-type: none"> Student demonstrates proficiency below 70% on tests and quizzes throughout the project. <p>1-----3-----5</p>	<ul style="list-style-type: none"> Student demonstrates proficiency at 80% level in tests and quizzes throughout the project. <p>6-----8-----12</p>	<ul style="list-style-type: none"> Student demonstrates proficiency at 90% level in tests and quizzes throughout the project. <p>13-----14-----15</p>
Mathematical Practices	<ul style="list-style-type: none"> Student does not demonstrate proficiency on elements 3, 5, or 7 of the Mathematical Practices Rubric. <p>1-----3-----5</p>	<ul style="list-style-type: none"> Student demonstrates proficiency on either element 3, 5, or 7 of the Mathematical Practices Rubric. <p>6-----8-----12</p>	<ul style="list-style-type: none"> Student demonstrates proficiency on elements 3, 5, and 7 of the Mathematical Practices Rubric. <p>13-----14-----15</p>
Vocabulary, Terms, and Definitions	<ul style="list-style-type: none"> Student demonstrates minimum proficiency in use of vocabulary and definitions throughout the project. <p>1-----3-----5</p>	<ul style="list-style-type: none"> Student demonstrates proficient use of vocabulary and definitions throughout the project. <p>6-----8-----12</p>	<ul style="list-style-type: none"> Student demonstrates exceptional mastery of vocabulary and definitions throughout the project. <p>13-----14-----15</p>
Teamwork and Collaboration	<ul style="list-style-type: none"> Student does not meet the criteria for an effective team member. Student scores on the Emerging level on the Teamwork Rubric. <p>1-----3-----5</p>	<ul style="list-style-type: none"> Student meets the criteria for an effective team member. Student scores on the Proficient level on the Teamwork Rubric. <p>6-----8-----12</p>	<ul style="list-style-type: none"> Student meets or exceeds the criteria for a highly effective team member. Student scores on the Mastery level on the Teamwork Rubric. <p>13-----14-----15</p>
Critical Inquiry and Artwork	<ul style="list-style-type: none"> Fewer than two triangles in artwork. No geometric information included. No evidence of creative use of triangles to create an appealing design. <p>1-----3-----5</p>	<ul style="list-style-type: none"> Two or more different types of triangles in artwork. Accurate geometric information included. Use of triangles shows thoughtful approach to creating artwork. <p>6-----8-----12</p>	<ul style="list-style-type: none"> Four types of triangles in artwork. Unusual use of triangles contributes to innovative appealing design. Accurate geometric information included and shows strong command of geometric knowledge. <p>13-----14-----15</p>
BREAKTHROUGH Evidence for exceptional or creative performance beyond mastery.	The evidence for breakthrough is:		



Section 3: The Art of Triangles Project

The Art of Triangles Overview

Unit Focus

The Art of Triangles focuses on the special qualities of the four types of triangles:

1. Right triangles
2. Isosceles triangles
3. Obtuse triangles
4. Acute triangles

The project addresses priority Common Core Standards for Units 1 and 2, Traditional Geometry, and Unit 5, Integrated Geometry.

It also focuses students on Mathematical Practice 3 (Construct viable arguments and critique the reasoning of others), Mathematical Practice 5, Use appropriate tools strategically, and Mathematical Practice 7 (Look for and make use of structure).

Timeline & Duration

3 weeks, based on approximately
5 hours of class time per week

Project Rating: Intermediate

You can explore the CCSS [here](#).

In addition, students will practice using online technologies and other resources for geometry. If you want to review the project outcomes again, go back to the [Overview of Assessment](#).

Critical Area

In previous grades, students were asked to draw triangles based on given measurements. They also have prior experience with rigid motions: translations, reflections, and rotations, and have used these to develop notions about what it means for two objects to be congruent. In this unit, students establish triangle congruence criteria, based on analyses of rigid motions and formal constructions. They use triangle congruence as a familiar foundation for the development of formal proof. Students prove theorems—using a variety of formats—and solve problems about triangles, quadrilaterals, and other polygons. They apply reasoning to complete geometric constructions and explain why they work.

Overview of The Art of Triangles

This project teaches students the basics of triangles, such as types, congruence, proofs, and similarity, by asking each student to create a poster, drawing, or personal adornment such as a fingernail design, piece of jewelry, or tattoo that uses at least two different triangular

shapes. The product must meet the criteria for ‘beautiful’—that is, it must display symmetry or other design elements that make it attractive.

Students will create an individual artwork product, but work in small cohorts to receive feedback and increase project quality.

Throughout the project, there will be an emphasis on reflection, using Learning Logs. Students should make regular entries and respond frequently to the Driving Question. At the end of the project, the products are compiled into a digital collage. All students then view the collage and use it as a basis for a group reflection on the Driving Question. Does their work meet the criteria for beautiful?



The Scenario and Challenge

The challenge in this project is for students to use triangular shapes to create a piece of art that they can defend as ‘beautiful.’ They must craft and think through their criteria for their artwork, rather than merely create it, and be able to explain the criteria to peers. In the early stages of the project, students will explore triangles in art to learn how this shape has influenced artistic design.

Students should choose their own art piece. The choices will vary with classes and teacher discretion. Possibilities include: a poster for a room; a piece of fine art; an image of a person represented in triangles; a drawing of a mandala; fingernail art; or a tattoo. Each piece will be individually designed. However, students will work throughout the project in small cohorts as they proceed on their design.

Students must also be able to answer the Driving Question by demonstrating that their art piece is ‘beautiful,’ according to criteria they will develop with their cohort. They must also be able to explain the kinds of triangles and how they were used in one other piece of art created by another student.

The Art of Triangles Project is designed to engage students in the study of geometry by having them use their knowledge of congruence and similarity in various types of triangles to create artwork. Students will be expected to know how triangles have influenced art and symmetry, and how they can use triangles in their own designs. They must be able to discern and describe triangles in a piece of artwork, and explain the properties of the four types of triangles. They must also know how to bisect triangles and prove theorems for triangles.

The project will conclude with an exam and a gallery of creations. Students will view the gallery of designs and reflect on them.

The crucial aspect of the project is the final product. Students will be expected to use their knowledge and skill to create a poster, drawing, or personal adornment that employs triangles to make it ‘beautiful.’ Students should be able to describe, in their own words why their product is ‘beautiful.’ Other students must be able to identify and explain the use of triangles in the artwork.

The Driving Question

The following has been written for the student audience.

How can I use a variety of triangles to create a poster, drawing, or personal adornment that meets the criteria for ‘beautiful’ artwork?

Getting to Know the Project: Note to Teachers

Welcome to the Art of Triangles Project. The project will require approximately three-plus weeks to complete and will address the key CCSS for geometry in the following clusters:

- Prove geometric theorems.
- Make geometric constructions.
- Prove theorems involving similarity.
- Apply geometric concepts in modeling situations.

Before beginning the project, take time to review all the documents provided for this project.

Project-based learning is designed to focus students on solving an open-ended problem and providing a solution to the problem through working in teams. Keeping this perspective in mind will be helpful. The new CCSS stress application and inquiry. Your goal is to have students learn the fundamentals of geometry, as defined by Unit 1 Traditional Geometry or Unit 5 Integrated Pathways standards, but also to coach them through a process of thinking *about* geometry.

Common Core State Standards for The Art of Triangles

The Art of Triangles addresses the following [Common Core State Standards](#) in High School Geometry.

Priority Standards

G.CO.10	Prove theorems about triangles. <i>Theorems include: measures of interior angles of a triangle sum to 180°; base angles of isosceles triangles are congruent; the segment joining midpoints of two sides of a triangle is parallel to the third side and half the length; the medians of a triangle meet at a point.</i>
G.CO.11	Prove theorems about parallelograms. <i>Theorems include: opposite sides are congruent, the diagonals of a parallelogram bisect each other, and conversely, rectangles are parallelograms with congruent diagonals.</i>

Supporting Standards

G.CO.9	Prove theorems about lines and angles. <i>Theorems include: vertical angles are congruent; when a transversal crosses parallel lines, alternate interior angles are congruent and corresponding angles are congruent; points on a perpendicular bisector of a line segment are exactly those equidistant from the segment's endpoints.</i>
G.CO.12	Make formal geometric constructions with a variety of tools and methods (compass and straightedge, string, reflective devices, paper folding, dynamic software, etc.) <i>Copying a segment; copying an angle; bisecting a segment; bisecting an angle; constructing perpendicular lines, including the perpendicular bisector of a line segment; and constructing a line parallel to a given line through a point not on the line.</i>
G.SRT.4	Prove theorems about triangles. <i>Theorems include: a line parallel to one side of a triangle divides the other two proportionally, and conversely; the Pythagorean Theorem proved using triangle similarity.</i>
G.SRT.5	Use congruence and similarity criteria for triangles to solve problems and to prove relationships in geometric figures.

Note to Students

Also available in Student Project Packet

Getting to Know the Project

Welcome to the Art of Triangles Project. The project will require approximately three-plus weeks to complete and will address the key CCSS for geometry in the following clusters:

- Prove geometric theorems.
- Make geometric constructions.
- Prove theorems involving similarity.
- Apply geometric concepts in modeling situations.

The Art of Triangles Project is a fun, creative project that will help you learn the four types of triangles, and how to use proofs and similarity to manipulate triangles. You will also learn how triangles have been a major influence on art throughout history, and how you can use triangular shapes to create a poster, drawing, or personal adornment (e.g., fingernail art or a tattoo) that is beautiful and appealing. You will be expected to know how to identify the types of triangles, bisect them, and use proofs.

A crucial aspect of this project is your final creation. You will create a poster, drawing, or personal adornment that employs triangles to make it 'beautiful.' But be prepared, in your own words, to describe to other students why your product is 'beautiful.' You must also be able to find and explain the use of triangles in other students' artwork.

Project-based learning is designed to focus you on solving an open-ended problem and providing a solution to the problem through working in teams. In this project, you will work with a *cohort*. This is a group of two or three other students who will act as your support team as you learn about triangles and create your art work. This is a common way that people work together to improve the quality of their work.

Suggested Pacing Guide

Over a three-week period, this project mixes team-based student inquiry, direct instruction, and teacher- or student-directed activities. Curriki Geometry offers a suggested sequence for The Art of Triangles, but the final calendar and sequence remain up to you. With project-based work, you often find the initial timeline will need to be modified. The important goal is not to rush through the curriculum, but to see it as a problem-solving process that requires time. This is especially true when working with student teams. Teams may require extra days during the project to share and reflect on their draft products. As the facilitator, your primary goal is allow teams sufficient time to produce quality work.

It may be necessary to modify the project based on your circumstances, the needs of students, or the tools and technology available to you. The project can easily be done with paper, pencil, basic geometry tools, and posters. However, this guide contains many suggestions for using the wide array of digital resources available online including on [Curriki](#).

Sequence of The Art of Triangles Project

Preparing for the Project

0 Teacher prepares for successful project implementation.

Launching the Project

1 Introduce the project and generate interest.

Managing the Project

2 Introduce the four kinds of triangles and key concepts and vocabulary.

3 Continue the four kinds of triangles. Begin initial research.

4 Continue initial research and have students decide on their product.

5 Introduce key concepts and vocabulary.

6 Continue key concepts and vocabulary.

7 Quiz Day. Begin design work.

8 Continue design work.

9 Complete design work. Introduce triangles. Check progress on cohorts.

10 Introduce key concepts and vocabulary.

11 Teach and review for exam.

12 Exam and Summation Day

13 Share and evaluate artwork.

14 Post or create a gallery.

Assessing the Project

15 Gallery Walk & Reflection Day

Teaching Guide

Preparing for the Project

0 Teacher Preparation

Prior to beginning the project with students, review this entire document.

Launching the Project

1 Introduce the project and generate interest.

If your students have no or little experience with PBL, you may have to begin the first day by explaining the process. They can also watch the [video](#) Project Based Learning: Explained (approximately 4 minutes) or a [video](#) on PBL and mathematics (approximately 30 minutes).

1. Introduce the challenge. This project teaches students the basics of triangles, such as types, congruence, proofs, and similarity, by asking each student to create a poster, drawing, or personal adornment such as a fingernail design, piece of jewelry, or tattoo that uses at least two different triangular shapes. The product must meet the criteria for ‘beautiful’—that is, it must display symmetry or other design elements that make it attractive.

2. Describe the scenario and the task. Clarify, if necessary

The challenge in this project is for students to use triangular shapes to create a piece of art that they can defend as ‘beautiful.’ They must craft and think through their criteria for their artwork, rather than merely create it, and be able to explain the criteria to peers. In the early stages of the project, students will explore triangles in art to learn how this shape has influenced artistic design.

Students should choose their own art piece. The choices will vary with classes and teacher discretion. Possibilities include: a poster for a room; a piece of fine art; an image of a person represented in triangles; a drawing of a mandala; fingernail art; or a tattoo. Each piece will be individually designed. However, students will work throughout the project in small cohorts as they proceed on their design.

Students must also be able to answer the Driving Question by demonstrating that their art piece is ‘beautiful,’ according to criteria they will develop with their cohort. They must also be able to explain the kinds of triangles and how they were used in one other piece of art created by another student.

The Art of Triangles Project is designed to engage students in the study of geometry by having them use their knowledge of congruence and similarity in various types of triangles to create artwork. Students will be expected to know how triangles have influenced art and symmetry, and how they can use triangles in their own designs. They must be able to discern and describe triangles in a piece of artwork, and explain the properties of the four types of triangles. They must also know how to bisect triangles and prove theorems for triangles.

The project will conclude with an exam and a gallery of creations. Students will view the gallery of designs and reflect on them.

The crucial aspect of the project is the final product. Students will be expected to use their knowledge and skill to create a poster, drawing, or personal adornment that employs triangles to make it ‘beautiful.’ Students should be able to describe, in their own words why their product is ‘beautiful.’ Other students must be able to identify and explain the use of triangles in the artwork.

3. Frame the task with the Driving Question. How can I use a variety of triangles to create a poster, drawing, or personal adornment that meets the criteria for ‘beautiful’ artwork?
4. Describe the concepts that students will learn as they complete the project. The project will teach the four kinds of triangles and the theorems and procedures associated with triangles, including bisectors, similarities, and proofs.
5. Distribute The Art of Triangles Project Packets to students.
7. [Anchor rubrics](#). Students should understand project grading and rubric language.

Tips and Tools

Generate interest in the project by using an Entry Event. Examples:

- Have students view [Cezanne’s The Card Players](#) and identify the [triangular shapes](#) in the painting.
- Show and discuss [examples](#) of triangles in art. Students may also view these on their own.

If you or students would like to change or refine the Driving Question, use this protocol for refining the [Driving Question](#).

It’s critical to a good start that students understand they will be assessed on (1) geometry content; (2) mathematical practices; (3) teamwork; (4) presentations.

The [Project Rubric](#) includes a ‘Breakthrough’ category at the bottom of the rubric, which has been left blank to recognize unusual creativity and design. This section can be filled in by the teacher, or by students after discussion with cohorts.

Managing the Project

2 Introduce the four kinds of triangles and key concepts and vocabulary.

1. Introduce the isosceles triangle theorem and its converse. Show students theorems support one another.
2. Introduce equilateral triangles and their properties.
3. Introduce key [vocabulary terms](#). The vocabulary list for this project is extensive. Students will be familiar with some terms, while other will need to be taught or reviewed.

- Acute triangle
- Altitude of a triangle
- Congruent

- Corresponding angle
- Isosceles triangle
- Median
- Midsegment
- Obtuse triangle
- Right triangle

4. Emphasize that students must be able to identify the types of triangles and explain them in artwork created by peers.

Tips and Tools

Use an online tool for vocabulary building such as [Visuwords](#). You can also have students download [AcceleStudy](#) to a smart phone or tablet to build on the key terms and vocabulary.

Highlight Mathematical Practice 3, Construct viable arguments and critique reasoning of others, Mathematical Practice 5, Use appropriate tools strategically, and Mathematical Practice 7, Look for and make use of structure. Have students discuss the criteria on the rubric and how the criteria may apply to their final products.

3 Continue the four kinds of triangles. Begin initial research.

1. Continue instruction on triangles as necessary.
2. Begin research. Initial research should be done individually. This can be done as assigned homework or in class. The purpose of initial research is to give students the opportunity to view examples of triangles and art.
3. Start [Daily Learning Logs for individuals](#) or [teams](#). If the logs will be graded, announce grading standards and expectations.

Tips and Tools

See this project's [Additional Resources](#) for websites to research.

Use [this](#) Khan Academy assessment on finding values in an isosceles triangle.

Have students enter their observations (what they learned, what they need to learn, etc) in their [Daily Learning Logs](#).

4 Continue initial research and have students decide on their product.

1. Each student chooses a final product.
2. Group students by product type and form cohorts. Create cohorts of 3 to 5 students. Explain the purpose of cohorts: To provide support and feedback during a process of learning. Cite examples of cohorts in school and the workplace.
3. Have teams review and discuss the [Teamwork Rubric](#).
4. Design an initial task for the cohorts. Each student in the cohort should present a draft idea for his or her product. Cohort members should respond using a simple protocol using I like, I wonder, and I suggest. Have cohorts review their process at the end of the period. Were they effective and did they listen to one another?
5. Plan for resources. Students should create a list of materials or resources they will need for creating the product. This is due on Day 7.

Tips and Tools

If necessary, use the team-building resources in Appendix B to establish effective working teams. Suggested:

- [Team Roles](#)
- [Conflict Resolution Speak](#)
- [Rules for High Performance Collaboration](#)

Share the initial brainstorm on the Driving Question using [Gallery Walks](#) (link takes you to an explanation of a Gallery Walk) or [Padlet](#) (a site where you can add anything to your blank “wall”).

Have students enter their observations (what they learned, what they need to learn, etc) in their [Daily Learning Logs](#).

5 Introduce key concepts and vocabulary.

1. Introduce students to midsegments and theorems. Allow students to find the parallel side and the lengths of the midsegment as well as the third side when asked.
2. Introduce students to the median of a triangle and show that all the medians of a triangle intersect at the centroid. Introduce centroid properties with respect to distance. Have students experiment with the median and finding lengths.
3. Show students how to identify the altitude of a triangle and how to identify altitudes given multiple triangles. Introduce and have the students identify the orthocenter.

Tips and Tools

Have students enter their observations (what they learned, what they need to learn, etc) in their [Daily Learning Logs](#).

6 Continue key concepts and vocabulary.

1. Introduce students to the perpendicular bisector theorem and the definition of a point of concurrency.
2. Introduce students to the circumcenter of the triangle and how to circumscribe a triangle. Have the students find the circumcenter of a triangle and notice the three different locations of the circumcenter: inside (acute triangle); outside (obtuse triangle); and on the triangle (right triangle).
3. Introduce the angle bisector theorem and the point of concurrency, showing the incenter as well as the center of the circle that is inscribed in the triangle.
4. Have students confirm whether the point in the triangle is the incenter or not. Have them give reasoning (mathematical proof/reasoning).
5. Prepare a quiz. You can use this [visual quiz](#) in which students identify triangle properties.

Tips and Tools

Use [this](#) Khan Academy video on understanding centroids and medians. (9 minutes)

Highlight Mathematical Practice 7, Look for and make use of structure.

If appropriate, use [Khan Academy videos](#) for visual instruction on triangles.

7 Quiz Day. Begin design work.

1. Begin the day with a quiz.
2. Students sketch an initial design.
3. Students meet briefly in cohorts to review initial designs.
4. Assign homework. What are three criteria you will use to judge the 'beauty' of your artwork?

Tips and Tools

Have cohort members track their progress on the [Project Milestones form](#).

Have students enter their observations (what they learned, what they need to learn, etc) in their [Daily Learning Logs](#).

Collect assignments from Day 4 for review and/or grading.

Highlight Mathematical Practice 5, Use appropriate tools strategically.

8 Continue design work.

1. Students continue artwork.
2. Students present three criteria for their artwork to cohort and receive feedback. When criteria are finalized, they can be added to the Critical Inquiry and Artwork portion of the [Project Rubric](#).
3. Collect or view criteria for feedback and/or grading.

Tips and Tools

This is first critical milestone in the project. Have students offer their criteria for their artwork to cohorts and receive feedback. Are the criteria clear, compelling, and convincing?

Have cohorts enter their progress on the [Project Milestones form](#).

Have students enter their observations (what they learned, what they need to learn, etc) in their [Daily Learning Logs](#).

Collect and review logs for grading or feedback.

Highlight Mathematical Practice 3, Construct viable arguments and critique reasoning of others.

9 Complete design work. Introduce triangles. Check progress on cohorts.

1. Introduce SSS and SAS congruency and have students recognize these by giving multiple triangles that show one or none of these properties.
2. Introduce ASA and AAS congruency and have students recognize these by giving multiple triangles that show one or none of these properties.
3. Show the students that when two triangles are congruent, each part of the triangles is congruent to one another.

4. Have students individually self-evaluate their progress on the Mathematical Practices Rubric by highlighting one strength and one area of challenge on Mathematical Practice 6. Are they incorporating what they have learned about triangles into their artwork? Is there room for deeper incorporation?
5. Collect design work for review and feedback with student.
6. Have cohorts evaluate themselves on the [Teamwork Rubric](#).

Tips and Tools

Students can post results on [Blogster](#) or Google Docs. Guide the critique with appropriate questions or directions. These will depend on the status and progress of your students.

Teams should enter progress on [Project Milestones form](#).

Have students enter their observations (what they learned, what they need to learn, etc) in their [Daily Learning Logs](#).

10 Introduce key concepts and vocabulary.

1. Review key terms.
 - Hypotenuse
 - Leg
2. Introduce the hypotenuse-leg theorem (HL theorem). Have students find measures of segments, algebraically, given the triangles are congruent by the HL theorem.
3. Show students multiple sets of triangles where at least two overlap and show the students how to separate and identify parts that they share (have in common).
4. Allow additional time for design, if necessary.

Tips and Tools

Use [this](#) Khan Academy activity on measuring segments algebraically.

11 Teach and review for exam.

1. Review topics as needed.

2. Prepare an exam for Day 12. You can use [this](#) Khan Academy assessment on congruent triangles.

Tips and Tools

If available, use a Smarter Balance Assessment that incorporates problem solving.

12 Exam and Summation Day

1. Exam.

13 Share and evaluate artwork.

1. Explain the peer evaluation system and criteria. Collect evaluations after groups complete the first step of the evaluation exercise.

2. The second phase is more objective. It may be used as an assessment tool or as a discussion tool.

Tips and Tools

The evaluation proceeds in three steps:

1. Form new groups of three, with students (A), (B), and (C). (A) presents artwork to (B) and (C). (B) explains the triangles in the artwork to (C) while (A) listens. (A) responds and marks rubric. Continue until each student has completed a round.
2. (A) states criteria for judging his or her artwork. (B) and (C) listen and respond. Continue until each student has completed a round.
3. Each student writes an individual response to discussion using the prompt: Did my artwork meet the criteria for beautiful?

14 Post or create a gallery.

1. Students post their artwork for viewing, along with their response to the prompt: Did my artwork meet the criteria for beautiful?

Tips and Tools

The gallery can be created either through posters, photos, or posts on [Tumblr](#) or similar sites.

Assessing the Project

15 Gallery Walk & Reflection Day

1. Conduct a Digital Gallery Walk by having students view each other's products.
2. Use your preferred method from the [Reflection Matrix](#) to review the project.
3. Reflect and discuss. What did we learn about art, triangles, and beauty?
4. Offer ideas/additions on this project on [Curriki](#).

Tips and Tools

Students can also reflect online using blog posts or other collaborative tools.

Use volunteers or extra credit to have students post materials if time is limited.



Section 4: Reflection Tools

Critical Friends Protocol

Who	What	How Long
Presenters	Presentation: Project Title and Idea, Driving Question, Culminating Products or Performances, Entry Event, and any concerns you'd like feedback about	7 minutes
Everyone	Clarification: Audience asks short clarifying questions	3 minutes
Audience	Good Stuff: Audience shares what they liked about the project	4 minutes
	Wondering Stuff: Audience shares their concerns and probing questions	4 minutes
	Next Stuff: Audience shares their thoughts on resources and improvements	4 minutes
Presenters	Reflection: Group reflects on useful feedback and next steps	3 minutes
		TOTAL TIME: 25 minutes

Teacher's Post-project Review

Project: _____

Teacher: _____

Date: _____

Select the items upon which you would like to reflect. Ignore the rows that you do not intend to discuss during the reflection.

Project Idea, Design, and Implementation Considerations	Sample Discussion Questions	Reflections
Student engagement	<ul style="list-style-type: none"> • What was the level of student engagement during the project? • What key aspects of the project design or delivery contributed to the level of student engagement? 	
Overall idea for the project	<ul style="list-style-type: none"> • What reflections do you have about the overall idea for the project? 	
Overall results for student learning	<ul style="list-style-type: none"> • What goals did you establish regarding student performance on the culminating products? • To what degree did you achieve your goals? • What key aspects of the project design or delivery contributed to the overall results for student learning? 	
Authenticity of project tasks and products	<ul style="list-style-type: none"> • Did the project focus on actual community issues? If so, what issues? If it did not, could it be refined to focus on actual issues? • Was the project useful to an outside audience? Did it meet an authentic need? If it did not, could it be refined to meet an authentic need? • Did students present to an audience beyond families or the classroom teacher? If not, would this enhance the project? • Were students required to consider 	

	<p>multiple perspectives on issues? If not, could the project be refined to enable them to do so?</p> <ul style="list-style-type: none"> ● Were experts from the community utilized to help students critique their work against professional standards? If not, would this enhance the project? 	
Quality and use of Driving Question	<ul style="list-style-type: none"> ● Were your students able to easily read and comprehend the Driving Question? ● The driving question should clearly state the purpose of the project and give a focus to all of the tasks students do. To what degree did your Driving Question accomplish this? ● How effectively did your students answer the Driving Question in their products and performances? ● What strategies did you use to maintain a focus on the Driving Question throughout the project? ● Do you plan to refine the Driving Question before launching this project in the future? 	
Scope: - Length of time - Complexity - Number of subjects/people/organizations involved - Use of technology	<ul style="list-style-type: none"> ● Would you adjust the scope of the project before launching this project in the future? <ul style="list-style-type: none"> ○ Duration? ○ Complexity? ○ Interdisciplinary planning? ○ Involvement with local experts? ○ Use of technology? 	
Selection of content standards	<ul style="list-style-type: none"> ● To what degree did the project demand breadth and depth of specific knowledge of central concepts? ● To what degree were the Driving Question, end products, rubrics, and key learning experiences aligned and linked to the selected standards? 	
Selection of appropriate 21st century skills	<ul style="list-style-type: none"> ● What 21st century skills did you teach and assess? ● Would you add any additional 21st century skills in subsequent projects? 	
Selection of culminating products and	<ul style="list-style-type: none"> ● How well did your culminating products and performances align with your intended outcomes? ● How authentic were the products and 	

performances	performances? (How similar was the student work to the work in the professional world?)	
Effectiveness of Entry Event	<ul style="list-style-type: none"> ● How effective was your Entry Event in sparking student interest and igniting their curiosity? ● Did your Entry Event cause your students to pose questions about the topic and tasks? ● Would you make any revisions to your Entry Event or adjust your approach when launching the project in the future? 	
Quality of rubrics	<ul style="list-style-type: none"> ● Ideally, your rubrics clearly articulate the criteria for success to enable students to easily use the rubric to evaluate their own work during the revision process. To what degree did your rubrics accomplish this? ● Did you use your rubric as a teaching tool? 	
Quantity and mix of scaffolding and learning activities	<ul style="list-style-type: none"> ● How well did you build your students' background knowledge for inquiry? ● How effective was your project scaffolding. (To what degree did you provide the necessary amount of skill instruction, mini-lessons, and use of models and drafts to build the understanding and skills needed to produce high quality work?) 	
Ability of students to work well in teams	<ul style="list-style-type: none"> ● How well did the students work in groups? ● How did your selected project management strategies/structures contribute to your students' level of success with team work? 	
Ability of students to work independently	<ul style="list-style-type: none"> ● How well did the students work independently? ● How did your selected project management strategies/structures contribute to your students' level of success in working independently? 	
Ability of students to use inquiry skills and think deeply	<ul style="list-style-type: none"> ● What did you observe about your students' ability to use inquiry skills and think deeply about the topic of study? 	
My management of the process, coaching of	<ul style="list-style-type: none"> ● What role did you play while students were engaged in group work? ● How did you respond when students needed support? ● To what degree did you promote a 	

students, and provision of support	culture of independence?	
Involvement of other adults	<ul style="list-style-type: none"> • What role did adults from the community, or other places beyond the community, play in your project? • Were they involved in helping you design the project? Teaching the students? Critiquing work against professional standards? • Were they helpful? Would you make any adjustments in the way in which you used these resources in the future? 	
Adequacy of resources	<ul style="list-style-type: none"> • Would you make any adjustments to the resources that you provided students to help them engage in inquiry? 	

Notes:

Reflection Matrix

Goal	Student	Team	Teacher
Encourage Deep Learning, Collective Knowledge Building, and Retention	<p>What do I know?</p> <p>Why is it important?</p> <p>Where is this knowledge used in the world?</p> <p>How engaged was I?</p> <p>What did I like?</p> <p>What do I wonder?</p>	<p>What do we know?</p> <p>What did we learn from each other?</p> <p>How did we help each other?</p> <p>How engaged were we?</p> <p>What do we wonder about?</p>	<p>What have they learned?</p> <p>Why is it important?</p> <p>How meaningful was the project?</p> <p>How clear were the goals and instructions?</p> <p>Is there evidence of new knowledge creation?</p>
Focus on Excellence	<p>How did I perform?</p> <p>Did my skills improve?</p> <p>Did my habits of learning improve?</p>	<p>How did we perform?</p> <p>Did we improve our teamwork skills?</p>	<p>Did I balance content and process?</p> <p>Did I provide ongoing assessment and interventions?</p> <p>Was the project design adequate?</p> <p>Were the evaluations fair and accurate?</p>
Develop a 'Growth Mindset'	<p>What new questions do I have?</p> <p>What are my next steps to becoming a better learner?</p> <p>What action can I take?</p> <p>How am I different after this</p>	<p>What new questions do we have?</p> <p>How can we use what we learned to do better next time?</p> <p>Where can we apply our knowledge?</p>	<p>Did I differentiate?</p> <p>Were my teaching and learning strategies effective?</p> <p>What new projects can I design based on what has been learned?</p>

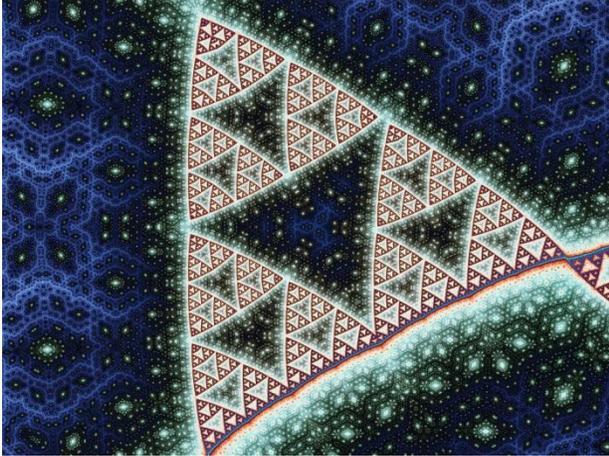
	<i>project?</i>	<i>What else do we want to explore?</i> <i>How are we different after this project?</i>	
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Methods for Reflection

Method	Notes
	<i>Reflection is an integral part of the project. Prepare for reflection by using Gallery Walks, portfolios, and performance-based rubrics throughout the project. Reflection is both an individual and collective exercise. For teams, begin by reviewing norms for listening, attentiveness, and contribution. Encourage 'I like, I wonder,' format. Provide prompts from the Reflection Matrix.</i>
Think-Pair-Share	Brainstorm knowledge gained from the project individually and then share with a partner. Create a joint list. Post lists. Give all students five colored dots and have them walk the room to look at lists. Place dots on five ideas that most interest them. Discuss the reason that the lists appealed.
Four Corners	Post one interesting outcome of the project in each corner of the room. Have students move to the corner that interests them. Discuss issues in each corner. Move to other corners if appropriate. Use chart paper divided into quadrants for students to defend their corner.
Expert Line	Reflect on the research stage of the project by dividing the class into researchers and consultants. Have students face each other in a line. Use a timed consultation to both receive problems and consultation. Switch roles. Debrief as a group.
Think Tank	Individual students complete a self-assessment on presentations. Discuss in teams. As a whole, create a presentation tip sheet.
Question Builder Chart	After culminating presentations, use teams to build a chart of new questions to ask. Use <i>Who, What, When, Where, How, Why, and Which</i> paired to <i>is, did, can, would, will, might</i> verbs.
Question Storming	After presentations, have students record 5 to 6 questions on sticky notes. Share question in teams. Post in categories. Complete a Gallery Walk. Discuss.
Self Question	Create a self-assessment form for students. Share in teams or in group.
Six Thinking Hats	Reflect on the project results by having students choose one of the six thinking hats (<i>blue—global thinking; back—critic; red—hunches, intuition; white—facts and figures; green—creative ideas; yellow—logical, positive</i>). Analyze the results through the lens of each hat.
Stretch Your Thinking	Set up a reflection activity based on Bloom's Taxonomy, or similar comparisons of levels of learning.
Interact with an Expert	Use individuals or teams to interview or videoconference with an expert. Review and debrief project. Share findings with class.
That's Good, That's bad	Use a Good-Bad organizer to review project tasks, environment, methods, teams, etc.
Pose a New Problem	Have students take findings from the project and create a new problem using the findings.
Web Excursion	Have students use sticky notes to create a wall web of all resources, personnel, tools, and technologies they used to complete the project. Discuss the wall web.
Begin with a Poll	Use Wiggio or Poll Everywhere to use email or cell phones for instant polling on questions about the project.
Writing	Create a class wiki or blog. Have students post, then share as a whole.

Create Visuals	Give students specific guidelines for creating a visual to ensure they create a particular message and supporting ideas. Have them create a timeline, chart, flowchart, graph, collage, or cartoon to depict the project results.
Create a Skit	Have students select a key artifact from the project and create a short dramatization around it.
Use Videos and Photos	Use video or photos to broadcast the main message of the project.
Play in the Sandbox	Use play by having students imagine they a reign over a sandbox. What does the learning smell like? Taste like? Are there colors?
Critical Friends Protocol	For a more formal debrief on a project plan, put students in a critical friends circle and facilitate a protocol on the project. See the Critical Friends Protocol .
Fishbowl	Use a simple fishbowl format to discuss the project. Use a small circle with an empty seat. Circle the remaining students in a circle outside the fishbowl and allow one student at a time to join the inner circle for a comment.

* Thanks to *Big Think*, by David Loertscher, Carol Koechlin, and Sandi Zwaan, and other sources.



Section 5: Appendices

Appendix A: Logs

Daily Learning Log: Individual

Name:			
Project Name:		Date:	

Today I had the following <u>goals</u> for project work.	1	
	2	
	3	
	4	
	5	

Today I accomplished...	1	
	2	
	3	
	4	
	5	

My next steps are...	1	
	2	
	3	
	4	
	5	

My most important concerns, problems, or questions are...	1	
	2	
	3	
	4	
	5	

Daily Learning Log: Team

Names:			
Project Name:		Date:	

Today we had the following <u>goals</u> for project work.	1	
	2	
	3	
	4	
	5	

Today we accomplished...	1	
	2	
	3	
	4	
	5	

Our next steps are...	1	
	2	
	3	
	4	
	5	

Our most important concerns, problems, or questions are...	1	
	2	
	3	
	4	
	5	

Appendix B: Teams and Grouping

Teams and Grouping Introduction

This appendix contains tools useful for teachers and students to help teams operate successfully and achieve their goals. Four documents are designed to aid teachers in assigning students to teams and having teams agree on norms and objectives. These are:

- 1. Team Roles*
- 2. Designing Team Contracts*
- 3. Student Contract Sample*
- 4. Rules for High Performance Collaboration.*

Four other documents are samples and suggestions for exercises that help students learn to operate as a team, resolve conflicts, and communicate effectively with one another. These are:

1. Team Building Exercises
2. Communication Worksheet
3. Conflict Resolution Speak
4. Problem/Compromise Cards

Team Roles

Task Roles

Initiator suggests or proposes new ideas to the team. He or she offers a novel point of view concerning problems, procedures, goals, or solutions.

Information seeker asks for clarification of suggestions made in terms of their factual adequacy, for authoritative information and facts pertinent to the problem being discussed.

Opinion seeker asks primarily for a clarification of values pertinent to what the team is undertaking or values involved in various suggestions that have been made.

Information giver offers facts or generalizations that are "authoritative" or relates his or her own experience pertinent to the team problem.

Opinion giver states his or her belief pertinent to a suggestion made. The emphasis is on what he or she believes should be the team's view of pertinent values, not primarily upon relevant facts or information.

Elaborator spells out suggestions in terms of examples or developed meanings, offers a rationale for suggestions previously made, and tries to deduce how an idea or suggestion would work out if adopted primarily upon relevant facts or information.

Coordinator shows or clarifies the relationships among various ideas and suggestions, tries to pull ideas and suggestions together, or tries to coordinate the activities of various members of sub-teams.

Energizer prods the team to action or decision, attempts to stimulate or arouse the team to "greater" or "higher quality" activity.

Procedural technician expedites team movement by doing things for the team, e.g. passing out materials or setting up chairs.

Recorder writes down suggestions, makes a record of team decisions, or writes down the product of discussion. The recorder fills the role of "team memory."

Maintenance Roles

Encourager praises, agrees with, and accepts the contribution of the others. He or she indicates warmth and solidarity in his or her attitude toward other team members, offers commendation and praise, and in various ways indicates understanding and acceptance of other points of view, ideas, and suggestions.

Harmonizer mediates the differences between other members, attempts to reconcile disagreements, relieves tension in conflict situations through good-hearted jokes, a soothing attitude, etc.

Compromiser operates from within a conflict in which his or her idea or position is involved. He or she may offer a compromise by yielding status, admitting his or her error, by disciplining him- or herself to maintain team harmony, or by "coming halfway" in moving along with the team.

Gate-keeper expedites attempts to keep communication channels open by encouraging or facilitating the participation of others ("we haven't gotten the ideas of Mr. X yet," etc.) or by proposing regulation of the flow of communication ("why don't we limit the length of our contributions so that everyone will have a chance to contribute?" etc.)

Standard setter expresses standards for the team. These standards apply to the quality of the team process, or set limits on acceptable individual behavior within the team.

Team observer keeps records of various aspects of team process and feeds such data with proposed interpretations into the team's evaluation of its own procedures.

Summarizer defines the position of the team with respect to its goals by summarizing what has occurred, points to departures from agreed upon directions or goals, or raises questions about the direction which the team discussion is taking.

Reality tester subjects the accomplishment of the team to some standard or set of standards of team functioning in the context of the team task. Thus, he or she may evaluate or question the "practicality," the "logic," the "facts," or the "procedure" of the suggestion or of some unit of team discussion.

Blocking Roles

Dominator tries to assert authority or superiority in manipulating the team or certain members of the team. This domination may take the form of flattery, of asserting a superior status or right to attention, giving directions authoritatively, interrupting the contributions of others, etc.

Aggressor may work in many ways: deflating the status of others, expressing disapproval of the values, acts, or feelings of others, attacking the team or the problem it is working on, joking aggressively, showing envy toward another's contribution by trying to take credit for it, etc.

Blocker tends to be negativistic and stubbornly resists, disagreeing and opposing without or beyond "reason" and attempting to maintain or bring back an issue after the team has rejected or bypassed it.

Recognition-seeker works in various ways to call attention to him- or herself, whether through boasting, reporting on personal achievements, acting in unusual ways, struggling to prevent being placed in an "inferior" position, etc.

Anecdoter uses the audience opportunity, which the team setting provides, to share stories or feelings unrelated to the task at hand.

Distractor makes display of his or her lack of involvement in the team's processes. This may take the form of cynicism, nonchalance, horseplay, and other more or less studied forms of "out-of-field" behavior.

Help-seeker attempts to call forth a "sympathy" response from other team members or from the whole team, whether through expressions of insecurity, personal confusion, or depreciation of him- or herself beyond "reason."

Designing Team Contracts

You may find it helpful to have your students sign team contracts. Follow these guidelines to include students in developing the contracts. You can find a [sample contract here](#).

Sample Contract Format Options

1. Give the students a nearly completed contract. Have teams fill in their task list and norms.
2. Using the guiding questions listed here facilitate a class discussion that leads to the development of a contract.
3. Give students a form with open-ended questions. Have teams fill in the form.

Sample Guiding Questions

1. Who is on the team?
2. What will we create?
3. What is each person's task?
4. When are the items due?
5. What is each person's role within the team?
6. How will we assess our team's performance during the project?
7. What are the team norms?
8. What happens when some doesn't do their work?
9. How will we choose a leader?
10. What is the leader's role?
11. Does the leader receive additional points for playing that role?
12. Is there a reward for doing your share of the work?
13. Will points be deducted for being off task?
14. What are the expectations regarding task completion?
15. What happens if I don't do the work on time?

- 16.** What happens if I'm absent?
- 17.** Can a student elect to work alone? If so, what is his or her responsibility?
- 18.** How will the team handle problems within the team?
- 19.** Can a member request team counseling?
- 20.** What happens when a new student joins the team when the project is in progress?
- 21.** How will we be graded?

Student Contract Sample

This also appears in the Project Packet for students.

Members:

Project:

Task List

Using the task list, assign all work to individual team members. An assignment may be assigned to more than one team member.

Team Constitution

Foreword

This contract is a binding document and governs the team until the assigned project deadline. If the team separates, or a member is fired, the basic contract laws remain intact for both parties. However, being fired may cause work responsibilities to shift.

Article I: Absence Policy

- a. If a team member will be absent on a day in which work is due, he or she must tell another team member a day in advance and have all work that he or she is responsible for turned in. All team members must stick to the provided agenda to have the assignments completed on time. If there will be an unexpected absence, the team member is to complete the work from home and email another team member to let them know he or she is gone for the day.
- b. Team members will contact one another if they are absent for any amount of period during the time allotted for working on the projects.
- c. Contact must be made by phone, email, or other acceptable method.

Article II: Work Policy

- a. Any member that is mentally or physically disabled and can prove that they cannot complete the work assigned to him or her alone may acquire assistance from other team

members to help complete it. This will only apply for work that is team work and not individual work, and work will only be finished by that team member; the assisting team member will not write it.

b. Each team member will work to the best of his or her ability, making sure to complete the work is up to standard, and that her or she completes it with punctuality.

c. If a team member commits plagiarism, he or she is solely responsible and will incur the punishment on his or her own.

Article III: Leadership

a. At the beginning of the project, a leader will be voted upon democratically. If a team member is absent at the time of voting, he or she waives his or her right to participate in voting. The member who wins the most votes becomes the leader. If there is an unclear outcome (same number of votes for different members), the team will have no leader until one can be chosen by a revote.

b. By being elected leader, the member must perform the following duties:

- Organize team meetings.
- Create and enforce a team agenda to govern team progress.
- Organize any out-of-school project efforts.
- Provide communication between team members in order to help them work toward the project goal.

c. If the team leader fails to perform these duties, or another member is also carrying them out, a revote may be taken to determine whether to obtain a new leader.

Article IV: Work Ethics

a. If a team member does not complete work he or she was assigned, the punishment for the infringement will be of detriment solely to the team member at fault. No negative grading shall be given to any other team members.

b. At the end of the project, 'hard workers' will be designated by means of a democratic vote. The people voted as the top two will each receive the ten bonus points. If one candidate is voted as hard worker by a margin of 75%, he or she will receive 20 points. If there is a tie, the team will discuss and come to resolution or else no points will be granted to the disputed members.

Article V: Member Dismissal

a. The following conducts will result in a team member being able to be dismissed;

- Incomplete or missing team work (This is non-negotiable and will be enforced by the teacher.)
- Plagiarism or any form of cheating
- If a team member decides to leave under his or her own will

b. Any team member leaving under his or her own will be able to submit all his or her own work, while the other team members may not. Any team member fired for breaking any of the conducts under Article V-a will have his or her work taken from his or her possession to be used at the discretion of the original team, but not for the member being fired. In addition, any fired member may not use any work completed by other team members, subject to punishment under Article II-c.

c. If a team member leaves under the stipulation of Article V-a, he or she retains all the work he or she has already provided for the team. The original team cannot use this work or it is subject to punishment under Article II-c.

Article VI: Signature

By signing this contract, the following team members abide to the articles listed here. If any member fails to abide by the articles of this contract, he or she may be fired from the team given at least a 50% vote in favor of firing the member.

Project Leader

The team has elected _____ as the project leader under Article III.

Signature

Rules for High Performance Collaboration

1. Each team member is responsible for his or her own behavior and learning.
2. Each team member should be willing to help any other team member who asks for help.
3. You can only ask me for help when all members of the team have the same question.
4. It is critical to work, think about, and engage in the work that you are asked to do. Any time that you think your team is done with the task you were asked to do, immediately find me wherever I am in the classroom and send a member of your team over to tell me that you think you are done with the task. I will come over and confirm that you have completed it.
5. You will often be asked to work quietly by yourself on a problem in preparation for working with your team. This is so that you can develop confidence in your ability to tackle problems alone. Whenever possible, this will be followed by an opportunity to share your work with your teammates and to get help from them where you need it.

Words we can use to make sure everyone on our team feels valued and involved:

- “Can we make a plan so everybody has something to do?”
- “_____ has a good idea.”
- “What do you think, _____?”
- “Does everybody agree with that?”
- “_____, we need your help.”
- “I don’t think we heard _____’s idea.”

General Hint: How you try to say something is every bit important as what you say. Did you say it with a smile?

Teambuilding Exercises

Puzzle

Give each member of a 4-person team roughly a fourth of the pieces of a 100 piece puzzle. When you divide the puzzle up, state firmly “this is yours, this is yours....etc.”

The objective: Complete the puzzle.

The parameters: 1. No talking. 2. No taking. If students want you to explain, don't. Only restate: No talking, no taking.

This exercise takes between 20 and 40 minutes.

Sarong

Place a sarong on the ground strategically next to a chair and/or a table. Ask 4 to 6 students to stand on the sarong.

The objective: Flip a sarong over completely.

The parameters: No using your hands, no touching the floor. I model this by placing my hands behind my back or in my pockets. If students want you to explain, don't. Only restate: No using your hands, no touching the floor. Students will begin to “think outside the box” and “take risks” like standing on the table or sitting on each others' laps...etc. You can discuss the importance of this later.

Comic Strip

Find a 4-section comic strip, print it out, black out all of the dialog and cut each segment apart from the others. Give one segment to each student in a team of four. They are only allowed to see their segment. The four students have to communicate descriptively to determine the order of the comic strip and the storyline without showing each other their respective segments. They must reach consensus. The exercise is not about getting the correct answer but about communicating descriptively, listening actively, and reaching agreement. At the end, they tell their version of the story and the order and then lay them down. It helps to select a comic strip that has multiple characters or at least one change of scene.

Once you rotate all participants through 2 to 3 activities, you prompt reflection saying, “you had to think critically, you had to problem solve, you had to work around parameters and

limits, you had to think strategically, and you had to collaborate to complete these activities. What did collaboration in your team look like, sound like, and feel like?” They reflect together and then after about 5 to 10 minutes, you pull the team back together to share out. You synthesize what is said and write it down. Here is the tricky part; you have to deliberately change the grammar before you write it on the chart paper so that it will be correct after the words “we will” (which are not yet present). The participants won’t realize this until after you have scribed the entire list and you write the words “we will…” at the top and demonstrate that we can replicate this in the classroom as a way of cultivating student-driven, student-experienced collaboration norms. I have seen teachers hold discussions with students to create student-driven collaboration norms. This is good (and better than many classrooms), but using team-builders is even better because students identify the behaviors from this experience not from a concept of what one “should do” in school.

[Helium Stick](#)

This is a simple exercise in teamwork. Have the students form two lines, facing each other, and point their index fingers and hold their arms out. Lay a long, thin, lightweight rod on the team’s fingers. The students must lower the stick to the ground—easier said than done!

The objective: Lower the stick to the ground.

The parameters: All the students’ fingers must be in contact with the stick at all times. Students may not pinch or grab the stick.

This exercise takes approximately 20 minutes, including set up and discussion.

[Toxic Waste](#)

This exercise works best for more established teams, not newly formed ones. The setup and instructions are quite detailed, so refer to the link for complete information.

The objective: Teams must figure out how to transfer a bucket of “toxic waste” to a “neutralization” bucket.

The parameters: Teams must figure out how to move the waste safely while avoiding spillage and radiation zones.

This exercise takes about 30 to 45 minutes.

[Mine Field](#)

This exercise requires pair work. One team member is blindfolded and the other team member must successfully guide his or her partner through a “mine field” using only verbal instruction.

The objective: To safely guide a team member through a mine field.

The parameters: The walking member must remain blindfolded.

This exercise takes up to an hour including set up and debrief.

[Pipeline](#)

This exercise is fast paced and requires both problem solving skills and strong communication skills among team members.

The objective: To maneuver a marble from one end of pipe to another.

The parameters: Each team member only has a short piece of pipe. Other rules can include members being required to carry the marble at least once, or in different orders, or members must have both feet on the floor at all times.

This exercise takes about 30 minutes.

[Keypunch](#)

This exercise is not only about completion, but improvement. Working together, the team tries to better their timing as they complete the task. See the link for full details on setup and instruction.

The objective: In sequence, team members must touch randomly placed numbers within a set time frame.

The parameters: The randomly placed numbers are not clearly visible to the team. Within five attempts, the team must complete the task or improve on their time if they have solved the challenge.

This exercise takes about 40 minutes.

[Great Egg Drop](#)

This small team exercise is one that has been used widely. It is especially good for teaching project management.

The objective: To drop an egg from 8 feet high without breaking it.

The parameters: Teams can use any materials to build their container.

This exercise takes about 2 hours from start to finish.

You can find some of these and several other teambuilding exercises [here](#).

Communication Worksheet

Combining “I Feel” Statements and Positive Requests

To encourage teams to work effectively together, pose these questions to students and have them discuss outcomes. How would they solve these interpersonal issues?

- 1.** One member in your group is doing all the building on the project. Every time you try to make a suggestion, you are ignored. When you pick something up to try to help, it is taken away from you.
- 2.** One member of your group has been wandering around visiting friends while the rest of you worked on the poster for your presentation. Now just as you are about to finish, he or she bumps into your desk causing you to make an ugly mark all the way down the page.
- 3.** There are only three people in you group. The other two are good friends but you don't know them very well. They are sitting close together and acting as if you don't exist.
- 4.** All the other members in your group are actively discussing the questions for your activity. You would like to say something too, but every time they ask you for your opinion, they move on to someone else before you've had a chance to put your thoughts into words.
- 5.** One member of your team always gets good grades on all of his or her regular class work, but in the group he or she never contributes. You suspect that he or she knows the way to solve the problem you're all working on.

Conflict Resolution Speak

Most of the time when people feel hurt and become angry because of something someone else has done, they confront each other with accusations in ways that only escalate conflict. Blaming statements usually begin with “You” and focus on the other person in a highly judgmental and negative way. “I feel” statements focus on our own feelings in response to the other person’s behavior.

Try using this formula for phrasing “I feel” statements:

When you... (State problem behavior)

I feel... (Express feeling)

Because... (State reasons for you feeling)

Situation #1: A member of your team interrupts you constantly when you are talking.

“You” Statement: *“You’re so rude! You never let me say anything!”*

“I feel” Statement: *“When you interrupt me, I feel really hurt because I think that what I have to say is important too.”*

Situation #2: Two members of the team are holding the instruction sheet so you can’t see what it says.

“You” Statement: *“You guys are always hogging everything!”*

“I feel” Statement: *“I feel left out when you guys have the instruction sheet between you because I can’t follow what’s going on.”*

Situation #3: A member of your team is busy throwing paper wads and talking to members of another other team.

“You” Statement: *“You’re such a goof-off. You never help.”*

“I feel” Statement: *“When you start doing things with people in other teams, I feel really upset because we need everyone’s help to get this project done on time.”*

Positive Requests

Positive requests usually begin with “I want you to,” “I would,” “I’d like you to,” and “I need you to.”

Negative: “Stop interrupting me.”

Positive and specific: “I want you to wait until I’m finished before you start talking.”

Negative: “Stop hogging all the cards.”

Positive and specific: “I need you to put the cards in the middle of the table so I can see them.”

Negative: “Stop messing around.”

Positive and specific: “I would like you to fill in the chart with the information from our notes.”

Problem/Compromise Card Instructions

Use these graphics when conflict arises within a group.

Step 1: Complaining party identifies problem *and* his or her proposed solution.

Step 2: He or she gives the problem card to the other party.

Step 3: The other party has 24 hours to think, reflect, and respond in writing.

Choice 1: He or she agrees with the solution and states how he or she will comply with/adopt the solution using the “I will...” prompt.

Step 4: The other party returns the card to the complaining party and they consider the issue resolved.

Choice 2: He or she disagrees with the solution and proposes a counter-solution.

Step 4: Complaining party fills out a compromise card. A conference with the students may be advisable at this point.

Problem Card

Problem:

Solution:

I agree

I disagree

I will...

My counter-solution is:

Problem Card

Problem:

Solution:

I agree

I disagree

I will...

My counter-solution is:

Compromise Card

Your counter-solution is:

I agree

We/I will...

I disagree

I propose a
compromise:

Compromise Card

Your counter-solution is:

I agree

We/I will...

I disagree

I propose a
compromise:

Appendix C: Visible Thinking Routines

Visible Thinking Routines

The [Visible Thinking](#) website is an excellent way to learn about Visible Thinking and how it can become routine in the classroom.

You are encouraged to visit all areas of the site, including

[Visible Thinking in Action](#)

[Getting Started](#)

[Thinking Routines](#) (some projects will reference specific routines from this part of the site; links are included in the Suggested Pacing Guide)

[Thinking Ideals](#)

[School Wide Culture of Thinking](#)