CURRIKI ALGEBRA UNIT 5

Quadratic Functions and Modeling

Lesson 5.1: Use Properties of Rational and Irrational Numbers

**Unit 5: Quadratic Functions and Modeling**

In preparation for work with quadratic relationships students explore distinctions between rational and irrational numbers. They consider quadratic function, comparing the key characteristics of quadratic functions to those of linear and exponential functions. They select from among these functions to model phenomena. Students learn to anticipate the graph of a quadratic function by interpreting various forms of quadratic expressions. In particular, they identify the real solutions of a quadratic equation as the zeros of a related quadratic function. Students learn that when quadratic equations do not have real solutions the number system must be extended so that a solution exists, analogous to the way in which extending the whole numbers to the negative numbers allows X+1=0 to have a solution. Formal work with complex numbers comes in Algebra II. Students expand their experience with functions to include more specialized functions—absolute value, step, and those that are piecewise defined.

The eight lessons (5.1-5.8) provide the instruction and practice that supports the culminating activity in the final unit project. The lessons in this unit focus on working with quadratic relationships.

**Lesson 5.1: Use Properties of Rational and Irrational Numbers**

In this lesson students will learn why the sum or product of two rational numbers is rational; that the sum of a rational number and an irrational number is irrational and that the product of a nonzero rational number and in irrational number is irrational. Students will address how rational numbers and irrational numbers are alike and how they are different. For example, we use the π in the formula for the area of a circle and the π in the formula of an ellipse as an example of an irrational number. In this lesson we will calculate the area of a typical putting green on a golf course and the area of a track field, which is in the shape of an ellipse.

**Common Core State Standards by Cluster:** (Corresponding Common Core State Standards are listed in parentheses)

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| --- | --- | --- |
| **Grade Level** | **Cluster** | **CCS Standard** |
| 9 | Use properties of rational and irrational numbers | N.RN.3 |

**Lesson Preparation and Resources for Teachers:**

[Video: How to find the Area of a Circle](http://video.about.com/math/How-to-Find-the-Area-of-a-Circle.htm) From About.com

[Video: How to find the Area of an Ellipse](http://video.about.com/math/Find-the-Area-of-an-Ellipse.htm) From About .com

TE\_Track Races Activity Student Worksheet

TE\_Irrational Numbers by Heather McNeill

**Instructional Materials for Students: (print one copy for each student)**

String and paper

[Circle Perimeter and Surface Area Formulas](http://chemistry.about.com/od/chemistry101/ss/2dformulas_6.htm)

[Ellipse Perimeter and Surface Area Formulas](http://chemistry.about.com/od/chemistry101/ss/2dformulas_7.htm)

SE\_Track Races Activity Student Worksheet

SE\_Irrational Numbers

SE\_RationalandIrrationalNumbers Homework sheet

**Time:** 50-minute session

**Lesson Objectives:**

Students will be able to:

* Explain why the sum or product of two rational numbers is rational
* Explain that the sum of a rational number and an irrational number is irrational
* Find the area and radius of a circle
* Find the area of an ellipse

**Lesson Content:**

**1. Background Building Activity for Students (10 minutes)**

1. Vocabulary Building

Ask students to use the Vocabulary Self-Collection Strategy (VSS) to learn math-specific words during the activities in this lesson. This activity is from Ruddell, M. R. (2007) *Teaching content reading and writing*. New Jersey, John Wiley & Sons, Inc.

Teachers initiate VSS by asking each student to note and nominate one math-specific word that they find during class, either in oral discussion or in writing, that they would like to know more about and that they think should appear on a class vocabulary list. The teacher also nominates a word. The students and teacher are required to tell the following about their word:

1. Where they found the word. (Read the sentence if in a text or recall the context if it is from a discussion or activity.)
2. What they think the word means in this context.
3. Why they think the class should learn it (i.e., identify the word’s importance to the content topic).

At the end of the lesson, the teacher writes the words on the board and leads a discussion to define each, first from context as nominations, tell what they think their word means, and then, if needed, from any references available in the room. Discussion should include contributions from other class members.

The list of words will vary, but might include: rational, irrational, perimeter, circle, ellipse, circumference, diameter, radius, and area.

b. Warm Up Problem

In real life, there are many items that illustrate rational and irrational numbers. Rational numbers are numbers that can be written as a quotient of integers (fractions), but numbers that have no whole or end are considered irrational. Irrational numbers cannot be written as a quotient of integers. Examples of irrational numbers are π (pi) and √2. In this lesson we will discuss the irrational number π. We will do this through looking at the area of a circle, πr2.

A circle is an ellipse where the distance from the center to the edge is constant.

* Circumference ( c ) is the distance around the outside of the circle.
* Diameter (d ) is the distance of the line through the center of the circle from edge to edge.
* Radius ( r ) is the distance from the center of the circle to the edge.
* The ratio of the circumference of a circle and its diameter is equal to the number π.

Write these formulas on the board:

d = 2r

c = πd = 2πr

Area = πr2

(From About.com: <http://chemistry.about.com/od/chemistry101/ss/2dformulas_6.htm> )

In the game of golf, many of the putting greens are circular. It could be interesting to a player to calculate the radius and area of the putting greens on a golf course in order to know exactly how far a golf ball needs to go from the edge of the putting green to the hole.

Using the formula for finding the area of a circle, Area = πr2, let’s figure out the area of a putting green. Our putting green has a diameter of 56 feet and it is round. What is the area of our putting green?

Ask students to calculate the radius of the putting green. Point out the formula for finding the diameter of a circle and ask them to write down the formula for finding the radius. Call on a student to write the formula on the board. (r =d/2). The radius is 56 ÷ 2=28.

The whole class should work through the steps in the solution for finding the area of an average size putting green on a golf course. When calculating with pi, the value 3.14159 is often used. Area = πr2 or Area = 3.14 x 282

After this activity, ask students to work in pairs. They will use different size diameters to calculate the radius, then take turns explaining how they arrived at their answers.

The area of our putting green is 3.14 x 784 = 2,463.0065 square feet and rounded off to 2,463 square feet. But as we will find out, Π is an irrational number. This means we can only approximate the actual square footage of a circle. In reality, π= 3.14159 26535 89793 23846 26433 83279 50288 41971 69399 37510 and on to infinity. We use 3.14159 as an approximation of π.

This example introduces the difference between rational (fractional) numbers and irrational (cannot be written as a fraction) number. The radius of the circle is 28 feet and the diameter of the circle is 56 feet, both of which are rational numbers.

In today’s lesson we will solve problems using a combination of rational and irrational numbers.



This is a diagram of one hole on a golf course.

**2. Focus Question: (Write it on the board.)**

What are the dimensions of a running/jogging track?

Most tracks are 400 meters long, 100 meters on each straightaway, and 100 meters on each curve. 4 laps around the track would equal a mile. Ask students to work in pairs to answer this question. Students take turns explaining how they arrived at the answer.

(From:<http://wiki.answers.com/Q/What_are_the_dimensions_of_a_jogging_track#ixzz1taadP382>)

**a. Whole Class Activity:**

Announce to the class that they just learned how to find the area of a putting green. Now they will learn to find the area of an ellipse. Ask them turn to the person next to them and discuss which sports use an ellipse. Some possible sports are: Track and Field, Nascar racing, and Horse racing. There are other examples of an ellipse.

Ellipses show up in any beginning geometry course, but the uses for ellipses extend far beyond the classroom. The shape of an ellipse endows it with a unique ability to reflect various types of waves. For hundreds of years scientists and engineers have taken advantage of these properties and found uses for ellipses in astronomy, optics and even medicine. (By Josh Turner: <http://www.ehow.com/info_8532608_uses-mathematical-ellipse.html> at eHow.com)

**Creating an Ellipse**

While a circle has one center point called a radius, an ellipse has two. Mathematicians call these two points foci. To create an ellipse, take a piece of string and pin down both ends to a piece of paper some distance apart. Pull the center of the string to a point and place it flat on the paper. Line up a pencil with the center point of the string and trace a circle outside of and around the two points. It will form an elongated circle, pointed on both ends that mathematicians call an ellipse.

Ask students to use paper, a string, and follow the directions to create a perfect ellipse.

**Lithotripsy**

Ellipses have reflective traits that make them an invaluable tool in many different fields of science. By making an ellipse out of reflective materials, scientists can shine a beam of light or energy at one focus and it will reflect to the other. This feature has an important use in medical technology. Medical engineers use it as the basis for a machine called a lithotripter. The lithotripter reflects shock waves into the patient's kidneys to break up kidney stones into fragments small enough to pass via their urine.

**Optics**

Ellipses show up in nearly all fields of optics. The two foci of an ellipse allow opticians to accurately predict the path of a beam of light. By manipulating the angle and size of an elliptical lens, they can magnify, refract or reflect light. They then use these properties to create microscopes, telescopes and cameras. Physicists and engineers use the optical properties of ellipses to determine how much light scatters and how much an object absorbs, two important properties of laser mechanics.

**Astronomy**

The ellipse plays an important part in astronomy. Early astronomers believed planets orbited in a perfectly circular pattern, but Johannes Kepler proved that they follow an elliptical orbit and later used the properties of ellipses to create a set of laws about the universe. Using these laws, along with the mathematics of ellipses, astronomers can predict the arrival of comets, planetary orbits, and other physical laws.

**NASA**

Without the ability to actually go to planets and take physical measurements, astrophysicists have to make estimates about their measurements. In 2003, NASA sent remote-controlled land vehicles to Mars. To give the astronauts a visual landing zone they used the shape of an ellipse. The ellipse gave them an area inside of safe terrain and the mathematics of ellipses allowed them to better calculate the chance of landing outside of the predicted zone.

What is the formula for the area of an ellipse of semi-major axis of length A and semi-minor axis of length B? (These semi-major axes are half the lengths of, respectively, the largest and smallest diameters of the ellipse--- see Figure 1.)



For example, the following is a standard equation for such an ellipse centered at the origin:

(x2/A2) + (y2/B2) = 1.

The area of such an ellipse is:

Area = π x A x B

(From [http://www.math.hmc.edu/funfacts/ffiles/10006.3.shtml#](http://www.math.hmc.edu/funfacts/ffiles/10006.3.shtml))

Explain to the class that finding the area of an ellipse involves using a combination of rational and an irrational numbers. The sum and the product of an irrational number and a rational number is an irrational number. Once a product or sum has an infinite number as part of its answer, the answer will always be irrational.

Ask students to work in pairs to determine the area of an ellipse where A= 50 and B= 30. After they determine the area, ask them to work out an explanation for why the area of an ellipse would be an irrational number. Then call on a few pairs to share their explanations with the class. Ask the class to discuss the explanations.

1. **Small Group Practice activity**

Students work in pairs to find the area of these three famous ellipses.

* Olympic size track
* Nascar race track
* Churchill Downs, the Kentucky Derby race track

Student pairs use the Race Track Activity Sheet. Each pair chooses two problems and finds the answers to the questions at the end of each problem. After they complete the activity, ask the pairs to determine an explanation as to why the area of all three tracks is an irrational number.

**3. Whole Class Discussion (10 minutes)**

Student pairs take turns presenting their problems. They first ask the class to think about the solution, then explain the solution and why the areas for all three tracks are irrational numbers.

**4. Assessment (Informal)**

a. Students work in groups of four to create a problem that involves finding the area of a circle or an ellipse. They will solve the problem and write out the solution with an explanation about why the area is rational or irrational.

b. Student groups will exchange their problems with another group. They will solve the other group’s problem.

c. The groups that exchanged the problems will take turns reviewing the problems and solutions. The teacher will rotate around to the groups to check for accuracy and give further assistance.

**5. Extension Activities**

a. The activities described in the Irrational Numbers lesson from Curriki by Heather McNeill are a good way to describe and reinforce learning about rational and irrational numbers. Students should explain why answers are irrational or rational.

b. Students can work in small groups to solve the problems listed on the Irrational Numbers and the Number Line activity sheet by Terrie Teegarden from SAKE on Curriki.

**6. Homework assignment for additional independent practice** (Note: This homework assignment can be done during a subsequent class period if you have the time.)

Rational and Irrational Numbers Homework Sheet Students solve the problems found on the Rational and Irrational Numbers activity sheet by Linda Hires on Curriki.