CURRIKI ALGEBRA UNIT 5

Quadratic Functions and Modeling

Lesson 5.1: Use Properties of Rational and Irrational Numbers

Track Races Activity

Student Worksheet

These mathematical problems illustrate how rational and irrational numbers are used together. Anytime a problem includes the use of pi, it involves the use on an irrational number. The answer will always be an irrational number.

Speed and distance come together when racing around a track. Many foot races, horse races, dog races, skating races, automobile races, and bicycle races are held on track that have two semicircular turns and two straight sides. Let us first consider a 400-meter running track which has two straight-aways, each 100 meters long, and two equal semicircles measured along the inside rail that are 100 meters long. Such a track is called an **equal-quadrant track**. Suppose that the track has eight lanes, each 1 meter wide.



**Example 1**

If a runner ran one lap of the track in the second lane, how far did she run?

**Solution**: You might think that the answer is 400 meters, since it is a 400-meter track. Both straight-aways are 100 meters but as you go further out from the inside rail on the semicircular turns, you run farther. To calculate the distance around the turns, we use the formula for the circumference (*C*) of a circle with radius (*r*): *C* = 2π*r*. Since the two semicircles on the inside rail form a circle with a circumference of 200 m, we can find the radius of the inside rail semicircles.

(Note: In this section, 3.14 is used as the value of π.)

*C* = 2π*r* 200 = 2π*r* 200 = *r*

2π 31.83 ≈ *r*

Now when you run in the second lane, the semicircle turns have a radius that is one meter longer or 32.83 m. Thus, distance around both turns in the second lane is:

*C* = 2π*r* = 2π(32.83) ≈

**Example 2**

A runner jogs ten laps of the track in the eighth lane. How far does he run a) measured in meters and b) measured in miles?

**Solution**: a) Let us first determine the distance of one lap in the eighth lane. The straight-aways are still 100 m each but the semicircular turns now have a radius that is 8 m more than the radius of the inside rail.

*r* = 31.83 + 8 = 39.83 m

The distance around both turns in the eighth lane is as follows.

*C* = 2π*r* = 2π(39.83) ≈

b) To change 4501 meters into miles we can set up a proportion comparing miles to meters using the conversion, 6.2 mi ≈ 10,000 m.

Let *x* = the distance in miles 6.2 = *x*

10, 000 4501 10, 000 *x* = 27, 906.2

*x*≈

**Example 3**

In the two previous examples, we have seen that the distance you run increases as you are further out from the inside rail. If the lanes are 1 m wide, show that the distance per lap increases by the same amount. Why would this be important in a 400 m race?

**Solution**: The problem is asking us to determine if the distance in the next lane further away from the rail has increased by the same amount over the lane closer to the rail. This can be answered using some algebra.

Let *r* = the radius of the turn in any lane 2π*r* = the circumference of the turns in that lane

*r* + 1 = the radius of the lane that is 1 m further away from the rail 2π(*r* + 1) = the circumference of the turns in that lane



Bodemeister at the Kentucky Derby

In other events such as horse, dog, auto, and skate racing, the tracks are not equal-quadrant tracks. The lengths of the turns and the straight-aways are not equal. Let us examine such a track Churchill Downs, Louisville Kentucky, the home of horse racing’s Kentucky Derby.



PICTURE OF CHURCHHILL DOWNS TRACK http://www.horseparlor.com/churchhill-ky.html

The track is a one mile oval with straight-aways of 1348 feet.

**Example 4**

a) What is the distance around each turn? b) What is the radius of each turn at Churchill Downs?

**Solution:**

a) This oval track consists of two straight-aways and two semi-circles, which total to 1 mile. The distance around the inside rail of that the track is the length of the two 1348 ft straight-aways and a circle.

If *r* = the radius of the circle, the circumference of the circle *C* = 2π*r*. 1348 + 1348 + 2π*r* =

What is the circumference?

b) Solving for *r* in the previous result, we get the following. 2π*r* = 2584

*r* = 2584 ≈

**Example 5**

If a horse runs 10 feet from the inside rail of the Churchill Downs track for two complete turns, how much further does it run?

**Solution**: From Example 4, the distance around the inside for two turns is 2584 feet and the radius of the turns is 411.3 ft. If a horse runs 10 ft from the rail it increases the radius of the turns by 10 feet. The distance around two turns can be found using the formula *C* = 2π*r*.

What is the distance around two turns?

**Example 6**

In the 250 m short track cycling oval described above: a) what the distance around the inside of each turn? b) What is the inside radius of the turns? c) What is the distance of a lap cycled 2 meters from the inside of the oval?

**Solution**:

a) A 250 m lap consists of two 37 m straight-aways and two turns which form a circle.

Let *r* = the radius of the turn

37 + 37 + 2π*r* = 250 74 + 2π*r* = 250 2π*r* = 176

The distance around both turns is the circumference of the circle, *C* = 2π*r* =

What is the circumference?

b) Solving for *r* in the previous result, we get: 2π*r* = 176

c) Two meters from the inside the radius increases by twice the increase of the radius as explained before Example 6, that is 2(0.927). Thus, the length of the turns can be found using *C* = 2π*r*.

Our study of racetracks has shown that the distance around a turn increases as you move from the inside of the turn. The exact amount of increase can be determined by using mathematics.