Lesson 1 Linear Models

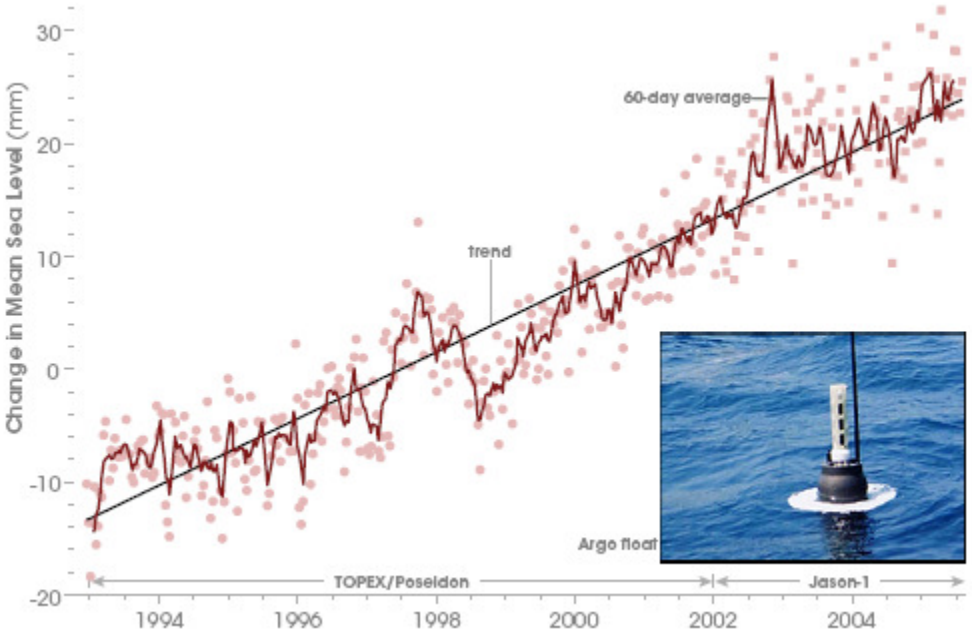
Rising Sea Level

In the debate over global warming, sea level is a reliable indicator of climate change, because it is affected by both melting glaciers and by the thermal expansion of sea water. During the twentieth century, sea level rose by about 15 centimeters, compared to a fairly constant level for the previous 3000 years.

The Intergovernmental Panel on Climate Change (IPCC) predicts that sea level could rise as much as 1 meter during the twentyfirst century. If that happens, low-lying densely populated areas in China, Southeast Asia, and the Nile Delta would become uninhabitable, as well as the Gulf Coast and eastern seaboard of the United States.

Not surprisingly, scientists would like to understand the causes of sea level change as thoroughly as possible, and to measure the rate of sea level rise as accurately as possible. Using data from satellites and floats (mechanical devices drifting in the ocean), oceanographers at NASA's Jet Propulsion Laboratory have calculated that the sea level rose, on average, 3 millimeters (0.1 inches) per year between 1993 and 2005.

The graph below shows the change in mean sea level, measured in millimeters, over that time period.



http://earthobservatory.nasa.gov

In this Lesson we'll see how the rate of increase in sea level is related to the graph. We'll also learn to create simple models to describe changes in a variable such as sea level.

Tables, Graphs, and Equations

A **mathematical model** is a simplified description of reality that helps us understand a system or process.

We can use a model to analyze data, identify trends, and predict the effects of change. The first step in creating a model is to describe relationships between the variables involved.

RQ1 What is the first step in creating a mathematical model?
a. Solve the equation.
b. Analyze data, identify trends, and predict the effects of change.
c. Describe relationships between the variables involved.
d. Make a table of values.

Starting from a description in words, we can represent the relationship by

- a table of values,
- a graph, or by
- an algebraic equation.

Each of these mathematical tools is useful in a different way.

1. **A table of values** lists specific data points with precise numerical values.
2. **A graph** is a visual display of the data. It is easier to spot trends and describe the overall behavior of the variables from a graph.
3. **An algebraic equation** is a compact summary of the model. It can be used to analyze the model and to make predictions.

RQ2 Name three ways to represent a relationship between variables.

In the examples that follow, observe the interplay among the three modeling tools, and how each contributes to the model.

Example 1 In May of 2005, the city of Lyons, France, started a bicycle rental program. Over 3000 bicycles are available at 350 computerized stations around the city. Each of the 52,000 subscribers pays an annual 5 euro fee (about $6.85) and gets a PIN number to access the bicycles. The bicycles rent for 1 euro per hour and can be returned to any station.
Your community decides to set up a similar program, charging a $5 subscription fee and $3 an hour for rental. (A fraction of an hour is charged as the corresponding fraction of $3.)
a. Make a table of values showing the cost, C, of renting a bike for various lengths of time, t.
b. Plot the points on a graph. Draw a curve through the data points.
c. Write an equation for C in terms of t.

Solutions a. There is an initial fee of $5, and a rental fee of $3 per hour. To find the cost of a rental, we multiply the number of hours by $3 per hour, and add the result to the $5 subscription fee. For example, for one hour

$$\text{Cost} = (\$5 \text{ subscription}) + \$3 \times (\text{one hour})$$
$$C = 5 + 3(1) = 8$$

A one-hour bike rental costs \$8.

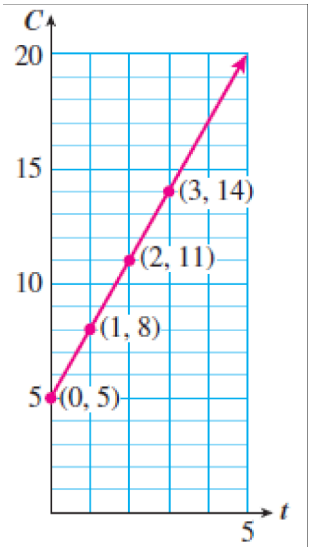
We calculate the cost for other values for t, and record the results in a table, as shown below.

Length of Rental (Hours)	Cost of Rental (Dollars)
0	5
1	8
2	11
3	14

$C = 5 + 3(\mathbf{0})$
$C = 5 + 3(\mathbf{1})$
$C = 5 + 3(\mathbf{2})$
$C = 5 + 3(\mathbf{3})$

(t, C)
(0, 5)
(1, 8)
(2, 11)
(3, 14)

b. Each pair of values represents a point on the graph. The first value gives the horizontal coordinate of the point, and the second value gives the vertical coordinate.
The points lie on a straight line, as shown in the figure. The line extends infinitely in only one direction, because negative values of t do not make sense here.



c. To write an equation, we let C represent the cost of the rental and use t for the number of hours:

$$\text{Cost} = (\$5 \text{ subscription}) + \$3 \times (\text{number of hours})$$
$$C = 5 + 3 \cdot t$$

Exercise 1 Write an equation for the cost C of renting a bicycle for t hours if the subscription fee is \$7 and the rental fee is \$2.50 per hour.

Hint: Fill in the correct numbers:

$$\text{Cost} = (\text{subscription fee}) + (\text{hourly rate}) \times (\text{number of hours})$$
$$C = _____ + _____ \cdot t$$

Equations for Linear Models

The equation we wrote in Example 1 is an example of a **linear model**, which describes a variable that increases or decreases at a constant rate.

Linear Model

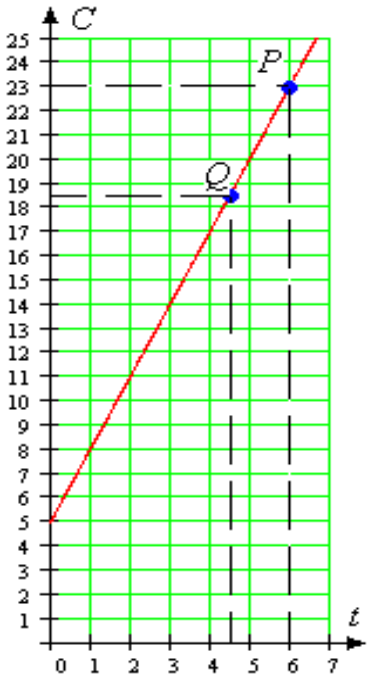
A **linear model** describes a variable that increases or decreases at a constant rate. It has the form

$$y = \textbf{(starting value)} + \textbf{(rate)} \times t$$

RQ3 What sort of variables can be described by a linear model?
a. Increasing variables
b. Variables that change at a constant rate
c. Variables that describe time
d. Variables that can be graphed

In Example 2, we see how a graph and its equation are related.

Example 2 Use the equation $C = 5 + 3t$ you found in Example 1 to answer the following questions. Then show how to find the answers by using the graph.
a. How much will it cost to rent a bicycle for 6 hours?
b. How long can you bicycle for $18.50?



Solutions a. We substitute $t = 6$ into the equation to find

$$C = 5 + 3(6) = 23$$

A 6-hour bike ride will cost $23. The point P on the graph in the figure represents the cost of a 6-hour bike ride. The value on the C-axis at the same height as point P is 23, so a 6-hour bike ride costs $23.

b. We substitute $C = 18.50$ into the equation and solve for t.

$$\begin{aligned} 18.50 &= 5 + 3t \\ 13.50 &= 3t \\ t &= 4.5 \end{aligned}$$

For $18.50, you can bicycle for $4\frac{1}{2}$ hours. The point Q on the graph represents an $18.50 bike ride. The value on the t-axis below point Q is 4.5, so $18.50 will buy a 4.5 hour bike ride.

Exercise 2 In Example 2, how long can you bicycle for $9.50? Show this on the graph.

Hint: Start by finding $9.50 on the Cost (vertical) axis. Then find the point on the graph with C-coordinate $9.50. Finally, find the t-coordinate of that point.

In Example 2, note the different algebraic techniques we used in parts (a) and (b):

In part (a), we were given a value of t and we **evaluated the expression** $5 + 3t$ to find C.
In part (b) we were given a value of C and we **solved the equation** $C = 5 + 3t$ to find t.

RQ4 Consider the expression $C = 5 + 3t$. Finding a value of C when we know t is called the expression. Finding a value of t when we know C is called _____ the equation.

In the examples above, the graph is **increasing** as t increases. In Example 3 we consider a **decreasing** graph.

Example 3 Leon's camper has a 20-gallon fuel tank, and he gets 12 miles to the gallon. (Note that getting 12 miles to the gallon is the same as using $\frac{1}{12}$ gallon of gas per mile.)

a. Write an equation for the amount of gasoline, g, left in Leon's tank after he has driven for d miles.

b. Make a table of values for the equation.

c. Graph the equation.

d. If Leon has less than 5 gallons of gas left, how many miles has he driven since his last fill-up? Illustrate on the graph.

Solutions **a.** We use the form for a linear model,

$$y = \text{(starting value)} + \text{(rate)} \times t$$

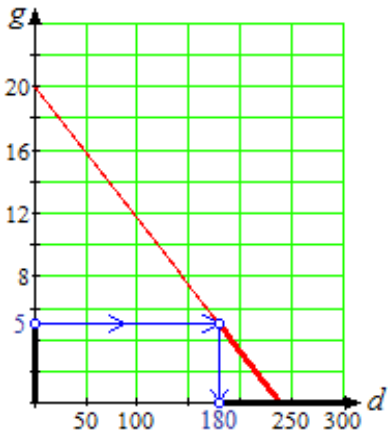
Leon's fuel tank started with 20 gallons, and the amount of gasoline is decreasing at a rate of $\frac{1}{12}$ gallon for every mile that he drives. Thus,

$$g = 20 - \frac{1}{12}d$$

b. Every time Leon drives 12 miles, he uses another gallon of gasoline. So, to make the calculations easier, we choose values of d that are divisible by 12.

d (miles)	0	48	96	144
g (gallons)	20	16	12	8

c. We scale the values of d along the horizontal axis, and the values of g along the vertical axis. Then we plot the points from the table, and connect with a line, as shown in the figure.



d. If Leon has less than 5 gallons of gasoline left, then $g < 5$, as shown on the g-axis. Using our model from part (a), we solve the inequality

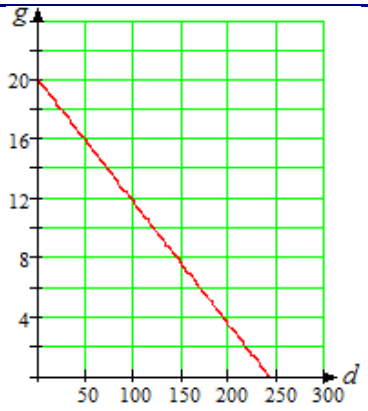
$$\begin{aligned} 20 - \tfrac{1}{12}d &< 5 && \text{Subtract } 20 \text{ from both sides.} \\ \tfrac{-1}{12}d &< -15 && \text{Multiply both sides by } -12. \\ d &> 180 \end{aligned}$$

Leon has driven at least 180 miles. The solution is shown on the graph.

In part (d) of Example 3 we used an **inequality** to answer the question. We use inequalities to model English phrases such as "less than," "more than," "at least," and "at most."

Exercise 3 Leon forgot to reset his odometer after his last fill-up, but he thinks he has driven at least 150 miles. How much gas does he have left? Show this on the graph.

Hint: Locate 150 miles on the d-axis. What part of the axis represents "at least" 150 miles? Find the points on the graph with d-coordinates at least 150. What are the g-coordinates of those points?



Answers to Exercises

1. $C = 7 + 2.50t$ **2.** 1.5 hrs **3.** At most 7.5 gals

Now go to WeBWorK and do the Reading Assignment for this Lesson.

Homework 1

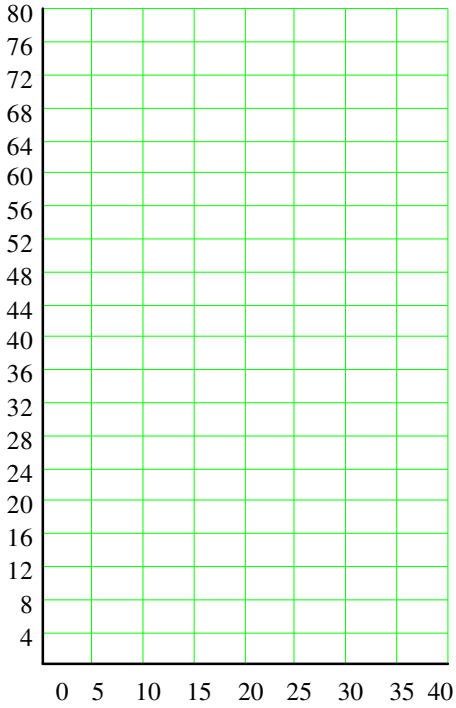
1. Frank plants a dozen corn seedlings, each 6 inches tall. With plenty of water and sunlight they will grow approximately 2 inches per day.

a. Complete the table of values for the height h of the seedlings after t days.

t (days)	0	5	10	15	20
h (inches)					

b. Write an equation for the height, h, of the seedlings in terms of the number of days, t, since they were planted.

c. Graph the equation on the grid. Label the axes with the correct variables.



To answer questions (d) and (e), read the graph. Show your work on the graph:

d. How tall is the corn after 3 weeks?

e. When will the corn grow to 6 feet tall? (How many inches is 6 feet?)

f. Use algebra to find the answers for parts (d) and (e) above.

g. Frank also sets out some tomato plants. The height of the plants in inches after t days is

$$H = 14 + 1.5t$$

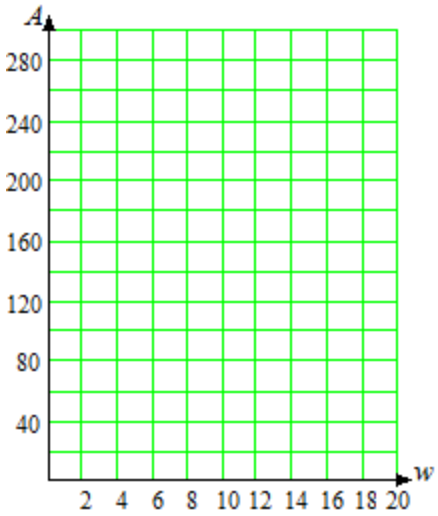
What do the constants in this equation tell us about the tomato plants?

2. On October 31, Betty and Paul fill their 250-gallon oil tank for their heater. Beginning in November they use an average of 15 gallons of oil per week.

a. Complete the table of values for the amount of oil, A, left in the tank after w weeks.

w (weeks)	0	4	8	12	16
A (gallons)					

b. Write an equation that expresses the amount of oil, A, in the tank in terms of the number of weeks, w, since October 31.



c. Graph the equation.

d. How much did the amount of fuel oil in the tank decrease between the third week and the eighth week? Illustrate this amount on the graph.

e. During which weeks will the tank contain more than 175 gallons of fuel oil? Illustrate on the graph.

f. Write and solve an inequality to verify your answer to part (e).

Now go to WeBWorK and do the Skills Practice for this Homework.