

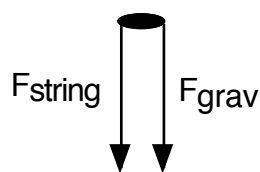
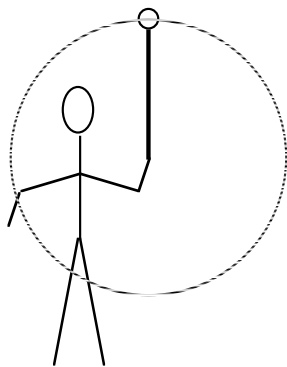
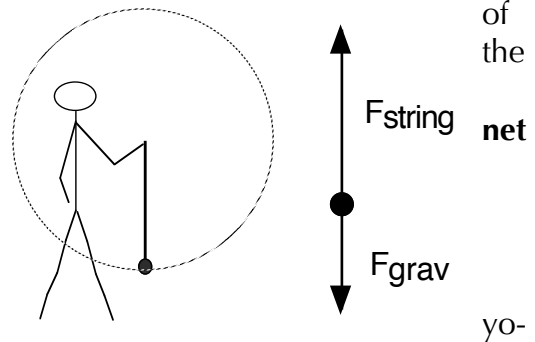


## Background Reading: Moving In Circles

### Other Objects That Move In Circles

The moon maintains its circular orbit because the force of gravity continuously pulls the moon inward away from the straight-line path it would follow if gravity were absent. This is a condition for all objects that move in circles: **a force directed toward the center of the circular path must be present for the circular motion to happen.**

Suppose a child begins to swing his yo-yo vertically in front of himself, as shown in the diagram. A force diagram for the yo-yo at the instant shown (when the yo-yo is at its lowest point) includes the downward force of gravity, as well as an upward force from the string. Since the yo-yo follows a circular path, there must be a **net force directed toward the center of the circular path**. This is only possible if the force of the string, which is directed toward the center of the yo-yo's circular path, is larger than the force of gravity, which is directed away from the center of the circular path. **The net force must be toward the center of the circle.**

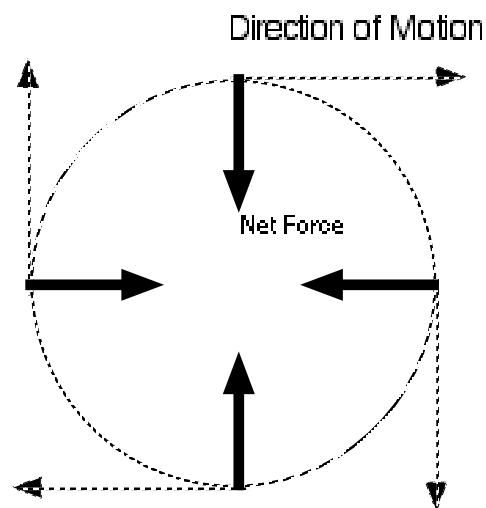


A slightly different situation arises when the yo-yo reaches the top of its circular path. At this point, a force diagram of the yo-yo includes the downward force of gravity and a downward force of the string (the string will always pull the yo-yo toward the child's hand). In this case, both the force of gravity and the force of the string are directed toward the center of the yo-yo's circular path. At this instant, both forces together work to change the direction of the yo-yo. Without these forces pulling on the yo-yo, it would tend to move off in a straight line, tangent to its circular path.

### Important General Ideas

The yo-yo examples help to show an important idea: There may be more than one force acting on an object that is moving in a circle. However, it is only the **NET** force directed toward the center of that circle that is responsible for the circular path the object follows. We refer to this net force toward the center of the circular path as the **centripetal force**. We won't find any forces on a force diagram that are labeled " $F_{\text{centripetal}}$ ". Instead, the centripetal force is the **name** we give to the net force directed toward the center of a circular path of motion. The actual forces present will be many of the forces we have encountered previously: the force of gravity, forces from strings and ropes, forces from surfaces like seats, floors, and the ground, and even friction. Each example of circular motion will involve one or more of these previously encountered forces, but circular motion occurs only if the forces present result in a continuous

NET force directed toward the center of the circular path, or, to be more precise, **a net force directed at a 90 degree angle to the direction of motion**. Any time this combination of force and motion exists, circular motion will result. The moon only experiences one force, but that force is perpendicular to the moon's direction of motion, and a circular orbit results.



### Examples of objects moving along circular paths:

The exact way in which the force or forces combine to generate the centripetal force can be determined by drawing a force diagram.

A circular path is followed as long as an object experiences a net force directed 90 degrees to its direction of motion. This net force does not speed up or slow down the object, **but continuously changes its direction**, resulting in a circular path.

Situation	Force(s) that contribute to centripetal force
Object swung in circle overhead	Force of the String on Object (Tension)
Object swung in circle in front of a person	Force of String on Object (Tension), Force of Gravity on Object (see example in reading)
Car rounding a curve	Friction between tires and road
Student going through loop on a roller coaster	Force of Seat on Student, Force of Gravity on Student
Stunt pilot in airplane flying in a loop	Force of Seat on Student, Force of Gravity on Student

### Next Step: A Little Math

By making careful measurements, or by using geometry and a little bit of trigonometry, physicists determined that the acceleration of objects moving in circular paths is given by the equation

$$a_c = \frac{v^2}{r}$$

where  $a_c$  stands for centripetal acceleration,  $v$  stands for the velocity of the object moving in a circle, and  $r$  represents the radius of the circular path. We do not need to worry about the details of this mathematical derivation. Instead, we will use this equation and Newton's Second Law ( $F_{\text{net}} = ma$ ) to come up with a mathematical equation for the *centripetal force*, the force that causes the centripetal acceleration represented by the equation above. Since  $F_{\text{net}} = \text{mass} \times \text{acceleration}$ , the centripetal force will be equal to mass  $\times$  *centripetal* acceleration, or

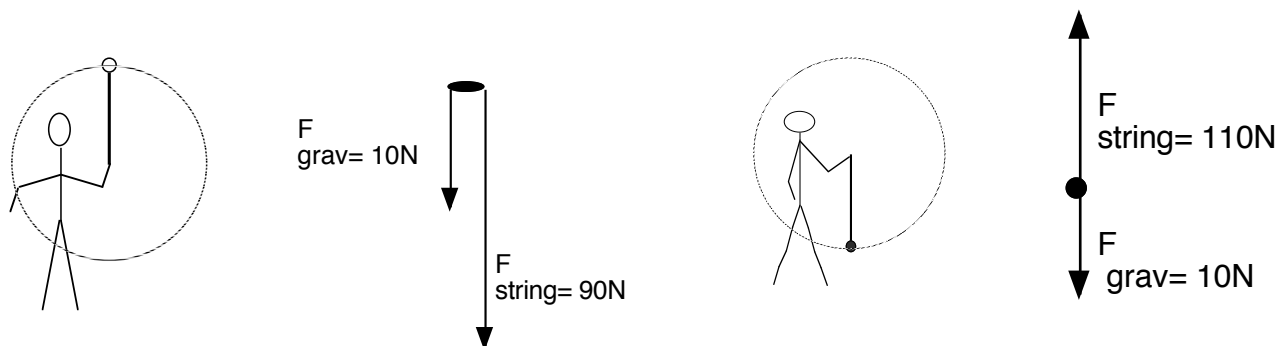
$$F_c = \frac{mv^2}{r}$$

This equation basically tells us that larger masses moving at higher velocities require a greater net force to keep moving along a circular path. The larger the circular path, the smaller the centripetal force needed for a given mass with a given velocity.

Back to the swinging yo-yo. Suppose the yo-yo swung vertically by the child has a mass of 1 Kg and the string to which it is attached is 1.0 meter long (the string defines the radius of the circular path). Suppose also that the child swings the yo-yo so that it travels at 10 meters/sec. We can calculate the force required to keep the yo-yo moving in a circle as follows

$$F_c = \frac{mv^2}{r} = \frac{(1\text{kg})(10\text{m/s})^2}{1\text{m}} = 100\text{N}$$

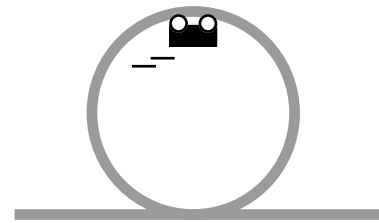
This value can now be used to determine the value of each of the forces acting on the yo-yo. Keep in mind that the centripetal force is just another name for the net force acting on an object moving in a circle. Once this net force is known, the specific values of the *actual* forces that combine to create that net force can be determined. In each of the examples below, the force of gravity is constant at 10 N downward. However, the force of the string varies. In each case, the force of the string is determined by making sure that the net force toward the center of the yo-yo's circular path (the child's hand) is 100N, as the calculation above indicates it should be.



So the tension in the string is higher when the yo-yo is at the bottom of its circular path.

Here is another example:

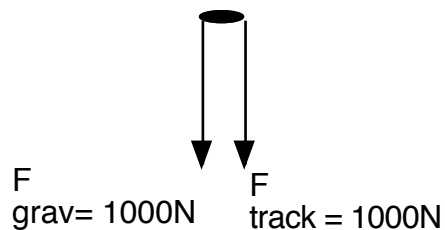
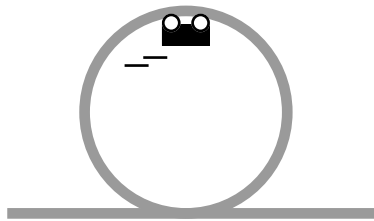
A 100 Kg roller coaster zips through a loop with a radius of 5 meters while traveling at 10 m/sec. What net force is required to keep the cart moving along this circular loop?



Solution:

$$F_c = \frac{mv^2}{r} = \frac{(100\text{Kg})(10\text{m/s})^2}{5\text{m}} = 2000\text{N}$$

Now we can draw a quantitative force diagram for the cart. The two forces acting on the cart—the downward force of gravity and the downward force of the track—**together** create a net force of 2000N directed toward the center of the roller coaster car's circular path.




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### Summary Points:

- An object moves along a circular path when the **net force** acting on the object acts at a **90 degree angle** to the direction of motion. This net force acts continuously toward the center of the circular path and is known as the centripetal force.
- A circular orbit is really just projectile motion that never ends because the curved path of the projectile matches the curvature of the earth's surface.
- Changing directions is a form of accelerating and requires a net force (a centripetal force). Mathematically, centripetal acceleration is calculated by  $a_c = v^2/r$ . The centripetal *force* is given by  $F_c = mv^2/r$ .
- The centripetal force is not a force that actually gets labeled on a force diagram. Instead, the centripetal force is the **net force directed toward the center of the circular path** and may be based on several forces acting on the object at a particular instant.