

Wiggly Paths

Lesson Information
<i>From 1000problems.com</i>
Subject:
Geometry
Topic:
Geometry and Measurement
Technology:
None
Level:
Moderate
Activity Structure:
Self-Guided Problem Solving
Duration of Activity:
Part of a Class

Overview:

Students use the area of a circle to find out the area of a garden path with some surprising properties.

Learning Objectives:

- To practice finding area of a circle
- To apply geometric formulas to non-standard situations
- To look for patterns between cases

Materials:

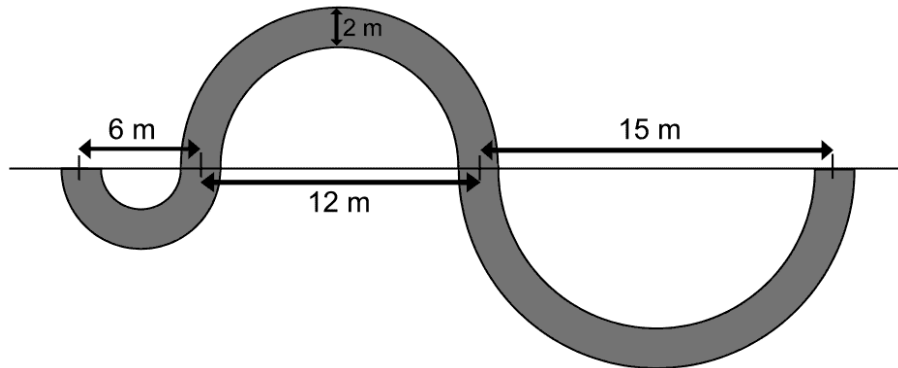
- Pencil
- Paper
- Worksheet

Wiggly Paths

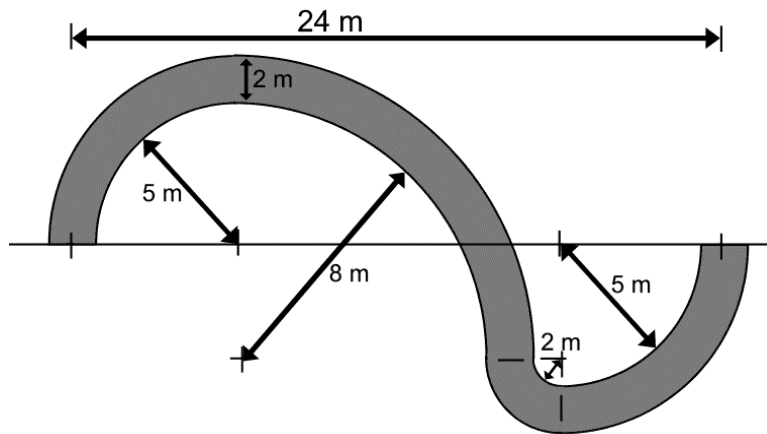
The Problem:

In the picture below, each section of a path is semi-circular and of width 2 meters. The total distance traveled, from left to right, is 33 meters.

1. What is the total area of the path ?



2. Now find the total area of this second path, composed of quarter-circular sections, but travelling a total distance of 24 meters from left to right.



Solutions :

Taking care to correctly identify the radii of the various circles...

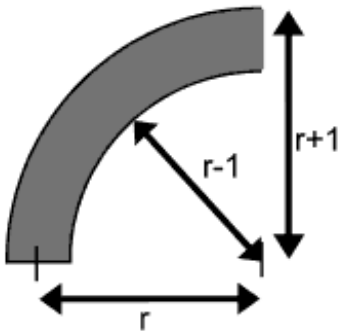
First Path Area

$$\begin{aligned}
 &= \frac{1}{2} \{ \pi[4^2 - 2^2] + \pi[7^2 - 5^2] + \pi[8.5^2 - 6.5^2] \} \\
 &= \frac{1}{2} \{ \pi[12] + \pi[24] + \pi[30] \} \\
 &= \pi \{ 6 + 12 + 15 \} \\
 &= 33\pi
 \end{aligned}$$

Second Path Area

$$\begin{aligned}
 &= \frac{1}{4} \pi \{ [7^2 - 5^2] + [10^2 - 8^2] + [4^2 - 2^2] + [7^2 - 5^2] \} \\
 &= \frac{1}{4} \pi \{ [24] + [36] + [12] + [24] \} \\
 &= \pi \{ 6 + 9 + 3 + 6 \} \\
 &= 24\pi
 \end{aligned}$$

Notice how each quadrant section (and hence semi-circular too) has area equal to π times the distance traveled to the right.



$$\begin{aligned}
 \text{Shaded Area} &= \frac{\pi}{4} \{ (r+1)^2 - (r-1)^2 \} \\
 &= \frac{\pi}{4} \{ (r^2 + 2r + 1) - (r^2 - 2r + 1) \} \\
 &= \frac{\pi}{4} (4r) = \pi r
 \end{aligned}$$

This simple relationship is equivalent to pulling each quadrant section of the path out 'straight', giving a rectangle of length $\frac{1}{4} (2 \pi r)$ and width 2, and hence area πr .