

Square Peg, Round Peg

Lesson Information
<i>From 1000problems.com</i>
Subject:
Geometry
Topic:
Geometry and Measurement
Technology:
None
Level:
Easy Moderate
Activity Structure:
Self-Guided Problem Solving
Duration of Activity:
Part of a Class

Overview:

This activity has students investigate which fits better: a square peg in a round hole or a round peg in a square hole.

Learning Objectives:

- Practice working with proportions
- Practice using area formulas for geometric shapes
- Applying geometric formulas to problem solving situations

Materials:

- Worksheet
- Paper
- Pencil

Square Peg, Round Peg ?

The Problem :

Which fits better: a round peg in a square hole, or a square peg in a round hole?
In other words, which fills more of the hole ?

Notes:

This is a pleasing problem, in that it appears to be motivated by the common saying. It is also quite straightforward once the formula for the area of a circle is known. Knowing Pythagoras' theorem helps solve the square peg in the round hole.

This is also a good example where we can express the answer *exactly* in terms of π , but may need to use a calculator to get a sense of just what the fraction is.

The problem can be extended to other regular polygons; the hexagon and equilateral triangle require some exploration of the 30° , 60° , 90° triangle. Others will need trigonometry more explicitly.

Solution :

The proportion of the area taken by a round peg in a square hole is $\pi/4 = A$

The proportion of the area taken by a square peg in a round hole is $2/\pi = B$

Observe that $A > B$, so **the round peg fits more of the hole.**

A good way, in general, of showing $A > B$, hardly necessary here, is to show that $A - B > 0$

So $A - B = \pi/4 - 2/\pi = (\pi^2 - 8) / 4\pi$.

As $\pi > 3$, $\pi^2 - 8 > 0$ so $(\pi^2 - 8) / 4\pi > 0$ and hence $A - B > 0$