

Point P where you like

Lesson Information
<i>From 1000problems.com</i>
Subject:
Geometry
Topic:
Geometry and Measurement
Technology:
None
Level:
Moderate
Activity Structure:
Self-guided Problem Solving
Duration of Activity:
Part of a Class

Overview:

An interesting property about area and an arbitrary point on a triangle's interior is explored in this activity.

Learning Objectives:

- To practice using area formulas
- To use geometric formulas to in problem solving situations

Materials:

- Worksheet
- Paper
- Pencil

Point P where you like

The Problem:

Draw an equilateral triangle, ABC, of side 10cm.

Choose any point in the interior of the triangle, and call it P.

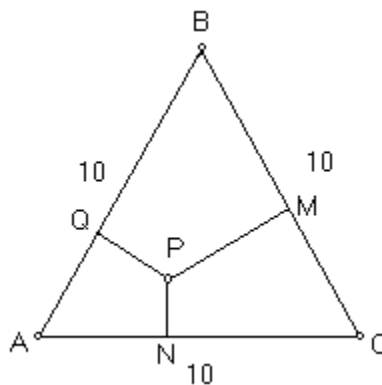
Drop perpendiculars from P to each of the three sides. These points of contact can be called M,N and Q.

Measure the lengths of PM, PN and PQ.

What is the **total** of these three lengths $PM + PN + PQ$?

Try it again with some other point P'.

Can you explain why the total is always the same?

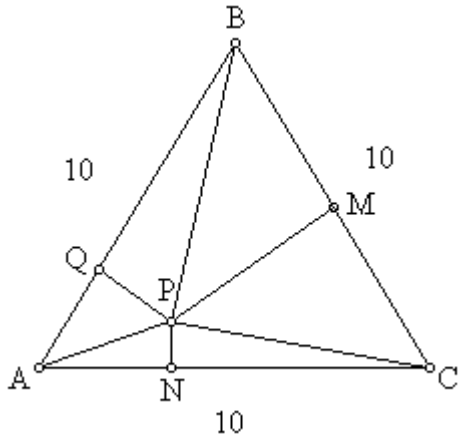


Hint:

Draw some inner triangles, and think about area...

The Solution:

The area of $\triangle ACP$ is $\frac{1}{2} \times 10 \times PN$
 Similarly, $\triangle CBP$ is $\frac{1}{2} \times 10 \times PM$
 $\triangle BAP$ is $\frac{1}{2} \times 10 \times PQ$



Hence, total area of the equilateral triangle = $\frac{1}{2} \times 10 \times (PN + PM + PQ)$

So, $\frac{1}{2} \times 10 \times (PN + PM + PQ) = \frac{1}{2} \times 10 \times \text{height of original triangle,}$

Whence required total $PN + PM + PQ$

equals the height of the original triangle, namely $5\sqrt{3} \approx 8.66\text{cm.}$

In any case, this is the total when P is taken at a vertex itself,

when $PN + PM + PQ = 8.66 + 0 + 0\dots$