

Take the A train

Lesson Information
<i>By Ron Lancaster</i>
Subject:
Pre-Calculus Algebra
Topic:
Functions Graphing
Technology:
Graphing Calculator
Level:
Moderate
Activity Structure:
Pooled Data Analysis
Duration of Activity:
Whole Class Period

Overview:

This lesson has students investigate the notion of a limit using a ratio between distances. The lesson introduces the idea of asymptotic behavior and limits at infinity.

Learning Objectives:

- To introduce the concept of a limit
- To find patterns in data
- To use patterns to write functions
- To practice writing ratios

Materials:

- Worksheet
- Graphing Calculator
- Pencil
- Paper

Question 1

The longest subway ride you can take in New York City without a change of train is on the A train, which runs from 207th Street in Inwood to Far Rockaway in Queens. When you are at the 207th Street station, you are 6 miles from the 125th Street station, and 18 miles from High Street Brooklyn Bridge, the first stop in Brooklyn.

(a) The ratio of the distance to High Street-Brooklyn Bridge to the distance to 125th Street is 3:1. Suppose that you get on the train and go towards High Street. After traveling x miles, let $y:1$ represent the ratio of the distance remaining to High Street to the distance remaining to 125th Street. Find the value of y for each of the following cases.

x	y
0	3
.5	
1	
1.5	
2	
2.5	
3	
3.5	
4	
4.5	
5	
5.5	
6	

- (b) Study your results from part(a) and describe how the value of y changed from $x=0$ to $x=6$.
- (c) Develop an equation for y in terms of x and use this equation to sketch the graph of y versus x . Study the behavior of the graph for values of x close to 6 and compare it to your observations in part (b).

- (d) Suppose that it was a Saturday, and out of sheer frustration due to a change-of-service announcement that had the A train running on the F line from West 4th Street to Jay Street, you decided to get out of the station and take a taxi as far away as you possibly can in the other direction. After traveling x miles, let y represent the ratio of the distance to High Street to the distance to 207th Street. Find the value of y for the following cases.

x	y
0	
1	
2	
3	
4	
5	
6	

x	y
7	
8	
9	
10	
11	
12	
13	

x	y
14	
15	
16	
17	
18	
19	
20	

- (e) Study your results from part (d) and predict how the value of y would change if the value of x continues to increase as it has in the chart.
- (f) Develop an equation for y in terms of x and use this equation to sketch the graph of y versus x . Study the behavior of the graph for large values of x and compare it to your observations from part (e).