

Factor Monster

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Factor Monster is our name for a classic game about natural numbers, factors, proper factors, prime numbers, composite numbers, and related algebraic alakazams. A computer version called **Taxman** appeared in *People's Computer Company* in 1972. It was also known as **The Factor Game** and described in the article "The Factor Game" by J.B. Harkin and D.S. Martin in the *Arithmetic Teacher*, volume 20, pages 580-582 (1973).

How to Play

Start with a list of **natural numbers** 1 to n . For example, start with a list of numbers 1 to 6.

- List of natural numbers 1 to 6: **1 2 3 4 5 6**

You may take any number that has one or more **proper factors** in the list. A proper factor of a number is a **factor** that is less than the number. In your first move, you cannot take 1 because there is no proper factor of 1 in the list. Indeed, 1 does not have a proper factor. In the list of natural numbers 1 to 6, for your first move you may take 2 or 3 or 4 or 5 or 6.

After you select your number, alas, alack, and oh heck, greedy Factor Monster gets **all of the proper factors of the number you took**. If you take 6, then Factor Monster gets 1, 2, and 3.

More bad news! If the list contains only numbers that have **no** proper factors remaining in the list, Factor Monster gets those numbers and the game is over. The player and Factor Monster add the numbers they have collected. The winner is the one who has the larger total.

Sample Game #1. Here's a game that Bob played with a list of natural numbers 1 to 6.			
	The List	Bob	Factor Monster
Starting list of natural numbers 1 to 6:	1 2 3 4 5 6		
Bob takes 6.	1 2 3 4 5	6	
Factor Monster gleefully takes all of the proper factors of 6.	4 5		1, 2, 3
Oops! There are no proper factors of 4 or 5 in the list, so Factor Monster gets 4 and 5.			4, 5
Bob and Factor Monster add their numbers.		6	15

Bob did not play well! He can do better. In fact, he can play so cleverly that he gets 15 and Factor Monster gets only 6. We encourage you to play with a list of natural numbers 1 to 6 and get a better total than did bumbling Bob. Remember:

- You may take any number in the list that has a proper factor in the list.
- You may not take a number that has no proper factor in the list.
- After you take a number, Factor Monster gets all of the proper factors of the number you took. Gobble, gobble!
- If the numbers that remain in the list have no proper factors in the list, Factor Monster get them. Munch, munch!

Make a list of natural numbers 1 to 6. Play well and beat Factor Monster!

Sample Game #2. Another badly-played-by-Bob game with natural numbers 1 to 8.			
	The list of numbers	Bob	Factor Monster
Starting list of natural numbers 1 to 8:	1 2 3 4 5 6 7 8		
Bob bodaciously takes 8.	1 2 3 4 5 6 7	8	
Factor Monster chomps the proper factors of 8.	3 5 6 7		1, 2, 4
Bob mumbles, grumbles, and takes 6.	3 5 7	6	
Factor Monster slurps the proper factor of 6.	5 7		3
Oops! There are no proper factors of 5 and 7 in the list, so Factor Monster gobbles up 5 and 7.			5, 7
Bob and Factor Monster add their numbers.		14	22

Bob can do much better. In fact, he can play so cleverly that he gets 21 and Factor Monster gets only 15. You can play with a list of natural numbers 1 to 8 and get a better total than did Bob.

- You may take any number in the list that has a proper factor in the list.
- You may not take a number that has no proper factor in the list.
- After you take a number, Factor Monster gets all of the proper factors of the number you took.
- If the numbers that remain in the list have no proper factors in the list, Factor Monster get the remaining numbers.

Make a list of natural numbers 1 to 8. Play well and beat Factor Monster!

Your Turn

Make a list of natural numbers 1 to n (you choose n) and play Factor Monster. Maximize your total and minimize Factor Monster's total. For numbers 1 to n , what is the maximum total the player can get? What is the corresponding minimum total for Factor Monster? What is the sum of the natural numbers 1 to n ? What is the sum of the player's and Factor Monster's score?

Complete the **Player** and **Factor Monster** columns in the table below. For each list of natural numbers, the table shows the maximum total the Player can get and the corresponding minimum total for Factor Monster.

List of natural numbers	Maximize Player's total		Sum of scores
	Player	Factor Monster	
1	0	1	1
1 2	2	1	3
1 2 3	3	3	6
1 2 3 4	7	3	10
1 2 3 4 5	9	6	15
1 2 3 4 5 6	15	6	21
1 2 3 4 5 6 7	17	11	28
1 2 3 4 5 6 7 8	21	15	36
1 2 3 4 5 6 7 8 9			
1 2 3 4 5 6 7 8 9 10			
1 2 3 4 5 6 7 8 9 10 11			
1 2 3 4 5 6 7 8 9 10 11 12			
Et cetera, et cetera, et cetera			

Internet Resources

- NCTM : Illuminations (http://illuminations.nctm.org/index_o.aspx?id=130). This site describes an intimidating version of The Factor Game with natural numbers 1 to 30 that you can play online. We wonder why they don't let the player choose the number of numbers.
- ENC Online: Curriculum Resources: Games and puzzles for elementary and middle school math (<http://www.enc.org/resources/records/0,1240,004619,00.shtm>). Describes a book of games, including the Factor Game, from the *Arithmetic Teacher* from 1956 to 1974.

Tiny Tutorial: Numbers, Factors, Proper Factors, et cetera, et cetera

In the game of **Factor Monster**, you play with **natural numbers**, **composite numbers**, **prime numbers**, **factors**, and **proper factors**. In this tiny tutorial, you'll find definitions and instructional stuff about numbers from composite number to zillion.

composite number, n . A composite number is a **natural number** that has **factors** other than 1 and the number itself. A composite number has three or more *different* factors. If a natural number is not 1 and is not a **prime number**, then it is a **composite number**.

- **1** is not a composite number. The one and only factor of 1 is 1.
- **2** is not a composite number. It has exactly two different factors, 1 and 2. **2** is a **prime number**.
- **3** is not a composite number. It has exactly two different factors, 1 and 3. **3** is a **prime number**.
- **4** is a composite number. It has three different factors: 1, 2, and 4.
- **6** is composite number. It has four different factors: 1, 2, 3, and 6.

counting number, n . Counting numbers are the numbers 1, 2, 3, and so on. A counting number is a **natural number** or **positive integer**. Confusion, bogglement! Some definitions include zero as a counting number. We recommend a visit to Counting Number – from Eric Weisstein's World of Mathematics (<http://mathworld.wolfram.com/CountingNumber.html>)

factor, n . If you multiply two **natural numbers**, the product is a natural number. The numbers you multiplied to obtain the product are factors of the product. If $a \cdot b = c$, then a and b are factors of c . For example, the factors of 6 are 1, 2, 3, and 6:

- Multiply 1 and 6. The product is 6 ($1 \times 6 = 6$). 1 and 6 are factors of 6.
- Multiply 2 and 3; the product is 6 ($2 \times 3 = 6$). 2 and 3 are factors of 6.

Here are some amazing alakazams about natural numbers and factors:

1 has only one factor. The only factor of 1 is 1. **1** is the only natural number that has exactly one factor and 1 is a factor of every natural number. Below: "onederful" words from an old folk song.

I'll sing you one-o
 Green grow the rushes-o.
 What is your one-o?
 One is one and all alone
 And evermore shall be so.

2 has two different factors. The factors of 2 are 1 and 2. A natural number that has exactly two different factors is a **prime number**. **2** is the smallest prime number and the only even prime number.

3 has two factors. The factors of 3 are 1 and 3. **3** is a **prime number**. It has exactly two different factors. It is the smallest odd prime number.

4 has three factors. The factors of 4 are 1, 2, and 4. If a natural number is not 1 and is not a prime number, then it is a **composite number**. **4** is the smallest composite number and the smallest natural number that has exactly three factors.

6 has four factors. The factors of 6 are 1, 2, 3, and 6. **6** is a **composite number** and the smallest natural number that has exactly four factors.

integer, n . An integer is a **natural number**, zero, or the negative (opposite) of a natural number. You might see the integers described in a sentence like this:

- The integers are the numbers 0, ± 1 , ± 2 , ± 3 ,

The set of three dots (...) following " ± 3 ," is an *ellipsis* that means "and so on." The fourth dot is the period that ends the sentence. Read the sentence as, "The integers are the numbers 0, ± 1 , ± 2 , ± 3 , and so on."

You might see the integers described as shown below with a left-hand ellipsis (...) indicating that the numbers go on in the negative direction and a right-hand ellipsis (...) indicating that the numbers go on in the positive direction:

- The integers are the numbers ..., -3, -2, -1, 0, 1, 2, 3,

One way to read the sentence is, "The integers are the numbers -1, -2, -3, and so on, 0, and 1, 2, 3, and so on."

Here's another definition of an integer: An integer is any number that can be obtained by adding or subtracting two **natural numbers**.

- How do you get 0? Pick a natural number and subtract it from itself: $1 - 1 = 0$.
- How about -1? Easy: subtract 2 from 1, like this: $1 - 2 = -1$.
- To get 2, add 1 to 1: $1 + 1 = 2$.
- Now that we have 2, add 1 to get 3: $2 + 1 = 3$.
- And so on, and so on, and so on (...).

Integers come in three flavors:

- Positive integers (1, 2, 3, ...), also called **natural numbers** or **counting numbers**.
- Zero (0).
- Negative integers (... , -3, -2, -1). Negative integers are the opposites (negatives) of the positive integers.

natural number, n . The natural numbers are 1, 2, 3, and so on forever. They keep going and going. If you pick any natural number, we can pick a larger natural number. Easy – we pick the natural number that is 1 more than the natural number you picked. If you pick 1, we pick

2. If you pick 2, we pick 3. If you grab 100, we latch on to 101. If you bodaciously choose a *secret* natural number, we admire your cleverness and say, "Our natural number is your secret natural number plus 1." Natural numbers are also called **counting numbers** and positive **integers**.

In a math book, you might see the natural numbers described in a sentence like this:

- The natural numbers are 1, 2, 3,

The set of three dots (...) following "3," is an *ellipsis*. It means "and so on." The fourth dot is the period that ends the sentence. Read the sentence as, "The natural numbers are 1, 2, 3, and so on."

positive integer, *n*. The positive integers are the numbers 1, 2, 3, and so on. They are also called **natural numbers** or **counting numbers**.

prime number, *n*. 1. A prime number is a **natural number** that has exactly two different **factors**. 2. A prime number is a natural number greater than 1 whose only factors are 1 and the number itself.

- **1** is not a prime number. It has only one factor. The one and only factor of 1 is 1.
- **2** is a prime number. It has exactly two different factors, 1 and 2. It is the smallest prime number and the only even prime number.
- **3** is a prime number. It has exactly two different factors, 1 and 3. It is the smallest odd prime number.
- A prime number has exactly one **proper factor**. The one and only proper factor of a prime number is 1.
- If a **natural number** is greater than 1 and is not a prime number, then it is a **composite number**.

proper factor, *n*. A proper factor of a **natural number** is a **factor** of the number other than the number itself. A proper factor of a number is a factor that is less than the number.

- **1** doesn't have a proper factor. The only factor of 1 is itself, 1.
- **2** has one proper factor. The one and only proper factor of 2 is 1. A natural number that has exactly one proper factor is a **prime number**.
- **3** has one proper factor. The one and only proper factor of 3 is 1. Yes, 3 is a prime number.
- **4** has two proper factors. The proper factors of 4 are 1 and 2. A natural number that has two or more proper factors is a **composite number**.
- **6** has three proper factors. The proper factors of 6 are 1, 2, and 3. **6** is a **composite number**.

whole number, *n*. The whole numbers are 0 and the **natural numbers** (**positive integers**). The whole numbers are 0, 1, 2, 3, and so on.

zillion, *n*. The number of seconds it will take us to finish construction of this stuff and fixx all of the mistrakes.