

Centripetal Motion (cont'd)
w/ Physics
Astronomy
Optional Lesson

[Note: this lesson will only make sense to students who have already covered Newton's Laws of motion before the start of this lesson. Otherwise, use the "Centripetal Motion Impressionistic" lesson.]

[Start off this lesson with a Hot Wheels demonstration. Set up the Hot Wheels track with a hill and loops (either one or two). Experiment with students changing the speed of the initial car and seeing how far the car makes it through the loop. If you have a large group of students (more than 10) you may want to have more than one track and let small groups experiment. The goal of this activity is to have students notice a general relationship between speed and the ability of the car to stay on the track.]

So, we've already talked about roller coaster rides and what forces are acting. Imagine you're hurtling down the track, and hit a loop. You circle upside down. What force is holding you into your seat or pushing against you as you travel upside down? This is actually a bit of a trick question! If you remember our diagram [pull out the same force diagram from "Centripetal Motion" lesson], there is NO FORCE holding you in your seat!

As you move along the track and the track curves upwards, the cars (and your body) want to keep moving in a straight line, and they try to. But the track curving upwards applies a force that pushes you into a loop.

The yellow lines represent the straight path you and the cars *want* to continue following. The cars are moving very fast, so at some point, they want to head straight up! But the track continues to curve, applying a force and pushing the cars around.

The question is **not** 'What's holding the cars up?' The correct question is 'What's keeping the cars from flying straight up?'. Do you remember which force it is that pulls on the cars toward the center of rotation? [Centripetal Force]

The cars are also accelerating. This is not due to a change in speed. What is the other variable that can change that affects acceleration? [Direction] Yes, the cars are changing direction, which changes their acceleration. Scientists tell us that acceleration due to centripetal force is v^2/r , where r is the radius of the circle. If we remember that Newton's Second Law of Motion is $F = ma = mv^2/r$.

Staying in the Loop

But, how do we ensure that our velocity is great enough to keep us in a roller coaster loop?

Well, that gets us into energy...If I hold a marble five feet above the ground, is there any energy expended, or is there anything resulting from the fact that the force of gravity is pulling down on the marble? [No] But, there is potential, right? If I were to drop the marble, then gravity would pull the marble toward the ground, thereby causing the marble to go from an acceleration of zero to an acceleration greater than zero.

This stored energy is the potential energy of the marble, the energy held by the object as it is held up in the air before the force of gravity acts on it. The higher I hold the marble, the greater the force of gravity when I drop it (remember the sand experiments); the greater the initial height of the object, the greater the *potential* force of gravity. So, the Potential Energy of an object is directly dependent on its height, its mass, and the thing pulling it down or gravity. In other words:

$$PE = m * g * h$$

But, what happens when the marble is actually dropped? That energy released by the movement of the marble is the kinetic energy of the marble. The faster the marble, or any body, moves due to gravity, the greater its kinetic energy. The greater the mass and speed of the object, the more kinetic energy it has. Why does this make sense? ($F = ma$) (If your students have had calculus, you can find the derivative of acceleration, otherwise, I just tell my students to trust me on the $\frac{1}{2}$ and the squared until they get to calculus – usually they think it's cool that they're learning about advanced math!) So, the kinetic energy of an object is directly dependent on its velocity coming down from the hill or being dropped and its mass. In other words:

$$KE = \frac{1}{2} m * v^2$$

How does all of this energy relate to roller coasters? In the simplest type of roller coasters, there are no mechanical forces *pushing* the cars. Rather, the cars get pulled to the top of the first hill and then dropped over the other side and the work of gravity and the inertia of the cars move them along the track.

If we were to examine a typical roller coaster, where would the most potential energy for the cars lie? Kinetic energy? (top of hill, bottom of hill, respectively). Now, try to create your own beginning hill and subsequent hill and find out how much potential energy you need at the beginning to have enough velocity to make it over the second hill. [For this activity, each student needs a piece of foam tubing sliced in half length-wise and a metal marble and tape to secure the coaster, or any other home-made roller coaster. I have students experiment

for a while in small groups until they start to realize that the first hill must be higher than the second hill, otherwise the marble will never make it to the end. If groups finish early, have them experiment with loops in their coaster.]

Summarize

The total energy available to the cars is the potential energy given to them by lifting them to the initial height h of the first hill, and we already know the potential energy is:

$$PE = m \cdot g \cdot h$$

Because the track is not completely frictionless, meaning the cars experience some resistance, they lose energy, so the cars can never reach a height h again in the ride.

You can also calculate how fast the cars will be moving at the bottom of a hill by using what you know about kinetic energy:

$$KE = \frac{1}{2}m \cdot v^2$$

Here's where things get pretty neat: At the bottom of the hill, all of the potential energy will have been changed to kinetic energy—ignore friction for a moment. So at the bottom, the kinetic energy is the same as the potential energy was at the top, $KE = PE$.

$$\begin{aligned} KE &= PE \\ \frac{1}{2} m \cdot v^2 &= m \cdot g \cdot h \\ m \cdot v^2 &= 2 \cdot m \cdot g \cdot h \\ v^2 &= 2 \cdot g \cdot h \\ v &= \sqrt{2gh} \end{aligned}$$

This tells us what the velocity of the cars are at the bottom of the hill starting from a height of h . Notice, that there isn't any mass involved in this equation, which means that the velocity of everything from a 2 ton gorilla to a marble in the car will be the same!

[After this lesson, my students take a trip to an amusement park for a Physics Day. Their job is find out the height of the various first hills on all of the roller coasters, calculate their velocities at the bottom of the hill, document through pictures what it “looks like” to travel at those varying velocities, and find three examples of centrifugal force (not all roller coaster loops) and document those while also writing a description of what it feels like to experience centripetal force in their own words. You can find all of this in the assessment file.]