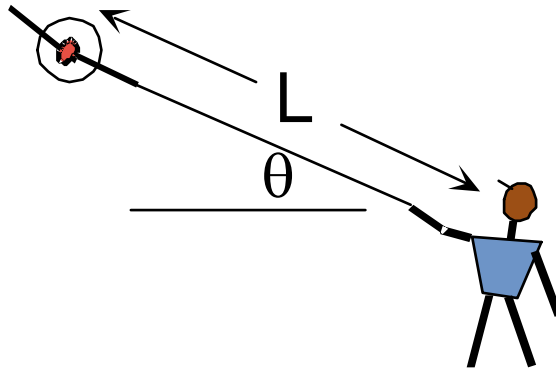


### In-Class-Problem 14: Whirling Objects, U-Control Model Airplane

A U-control airplane of mass  $M$  is attached by wires of length  $L$  (and negligible mass) to the “pilot” who controls the lift provided by the wing. (The wires control the plane’s elevator.) The plane’s engine keeps it moving at constant speed  $v$ .

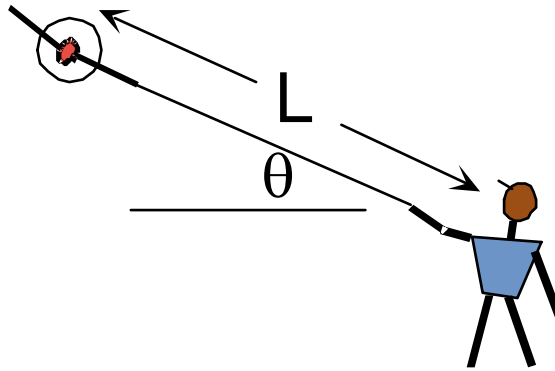
- Briefly describe how you intend to model the motion of the object? What directions are you choosing for analyzing the components of your forces and state why?
- Find the total tension  $T$  in the wires when the plane is flown overhead in a circle so that the wires make an angle  $\theta$  with the ground. Remember that the wings can provide lift only in the direction perpendicular to their area, i.e. in a direction perpendicular to the wires. Think carefully before selecting the angle of your coordinate system.
- The plane will go out of control and crash if the tension is not maintained. Given a particular speed of the plane,  $v$ , is there some angle  $\theta_{\text{crit}}$  which you would advise the pilot not to exceed? If possible, exhibit a speed  $v_{\text{safe}}$ , at which the plane would be safe at any angle?



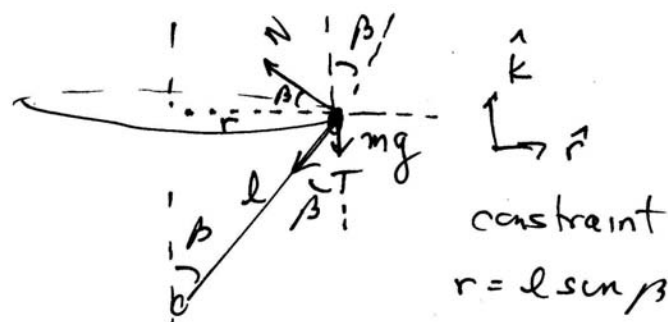
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Solution:



$$\hat{r}: \quad \frac{\vec{F}}{r} = \frac{m \vec{a}}{r} \quad \left| \quad -\frac{mv^2}{r} \right.$$

$$\hat{k}: \quad N \sin \beta - T \cos \beta - mg = 0$$

$$T \sin \beta + N \cos \beta = m r \omega^2 \quad (1)$$

$$N \sin \beta - T \cos \beta - mg = 0 \quad (2)$$

multiply eq (1) by  $\sin \beta$ , and eq (2) by  $\cos \beta$

$$\sin \beta (T \sin \beta + N \cos \beta) = \frac{mv^2}{l \sin \beta} (\sin \beta)$$

$$-\cos \beta (N \sin \beta - T \cos \beta - mg) = mg (-\cos \beta)$$

$$\text{add} \quad T = \frac{mv^2}{l} - mg \cos \beta$$

$$T = 0 \text{ when } \frac{mv^2}{l} = mg \cos \beta, \text{ string goes slack}$$

$$\text{so } v^2 > lg \cos \beta$$

Notice that  $\beta = \pi/2$  the rope lies in the plane of the airplane's circular orbit.

$\cos(\pi/2) = 0$ , and the tension  $T = mv^2/l > 0$  for all velocities. The other extreme value occurs when  $\beta \rightarrow 0$ . This corresponds to the radius of the orbit  $r \rightarrow 0$ . Then  $\cos(0) \rightarrow 1$  and the tension is  $T \rightarrow (mv^2/l) - mg$ . In order for the tension to stay positive  $v > \sqrt{gl}$ .