Massachusetts Institute of Technology - Physics Department

Physics - 8.02

Assignment #8

April 17, 2002.

We strongly recommend that you read about a topic before it is covered in lectures.

Lecture Date	Topics Covered	Reading from Giancoli
#26 Fri 4/19	Traveling Waves - Standing Waves Musical Instruments	Giancoli (Vol. I) Chapter 15 through 15-9
#27 Mon 4/22	Resonance - Destructive resonance Electromagnetic Waves - Speed of Light Radio - TV Distance determinations using radar and lasers	Sect. 31-6 Chapter 32 through Sect. 32-9
#28 Wed 4/24	Index of Refraction - Poynting Vector Oscillating charges - Radiation Pressure Comet Tails Polarization (linear, elliptical, and circular)	Sect. 32-7 & 32-8 Chapter 33 through Sect. 33-2 Sect. 36-11

Due before 4 PM, Wednesday, April 24 in 4-339B.

Problem 8.1

An LRC circuit.

A circuit contains a self-inductance L in series with a capacitor C and a resistor R. This circuit is driven by an alternating voltage $V = V_0 \sin(\omega t)$. We have $L = 15 \,\mathrm{mH}$, $R = 80 \,\Omega$, $C = 5 \,\mu\mathrm{F}$, and $V_0 = 40 \,\mathrm{volts}$.

- (a) What is the value of the resonance frequency, ω_0 ?
- (b) Consider three separate cases for which $\omega = 0.25\,\omega_0$, $\omega = \omega_0$, and $\omega = 4\omega_0$ respectively. For each case calculate the the peak current I_0 .
- (c) Find the energy $U_C(t)$ and the energy $U_L(t)$ stored, respectively, in the capacitor and in the inductor as a function of time for $\omega = \omega_0$.

Problem 8.2

Average power dissipated in an LRC circuit. Giancoli 31-20.

Problem 8.3

Width of resonance peak. Giancoli 31-30.

Problem 8.4

Traveling waves on a string.

The equation of a transverse wave traveling along a string is given by $y = 0.4 \sin[\pi(0.5x - 200t)]$ where y and x are measured in cm and t in seconds.

(a) Find the amplitude, wavelength, wave number, frequency, period, and speed of the wave.

- (b) Carefully draw the wave (y versus x) at t = 0, and at t = 1/400 sec.
- (c) Find the maximum transverse speed of any mass element of the string.
- (d) Suppose that you clamp the string at two points L cm apart and you observe a standing wave of the same wavelength as in part (a). For what values of L less than 10 cm is this possible?

Problem 8.5

Standing waves on a string.

The equation of a transverse standing wave on a string is given by $y = 0.3 (\sin 3x) (\cos 1200t)$ where y and x are in cm, and t in seconds.

- (a) What is the wavelength, wave number, frequency, and period, of this wave?
- (b) Carefully draw the wave (y versus x) at t = 0, at $t = 1.31 \times 10^{-3}$ sec, and at $t = 2.62 \times 10^{-3}$ sec.
- (c) What is the maximum transverse speed?
- (d) What is the speed of propagation (of a transverse disturbance) along the string?

Problem 8.6

Distance sensing with sound.

A bat can sense its distance from the wall of a cave (or whatever) by emitting a sharp ultrasonic pulse that reflects off the wall. The bat can tell the distance from the time the echo takes to return.

- (a) If a bat is to determine the distance to a wall 8 m away with an error of less than ± 0.2 m, how accurately must it sense the time interval between emission and return of the pulse?
- (b) Suppose that a bat flies into a cave filled with methane (swamp gas). By what factor will this gas distort the bat's perception of distances? At 20°C, the speed of sound in methane is 432 m/s.

Problem 8.7

Design a flute.

A flute can be regarded as a tube open at both ends. It will emit a musical note if the flutist excites a standing wave in the air column inside the tube.

- (a) The lowest musical note that can be played on a flute is C (261.7 Hz). What must be the length of the tube? Assume that the air column is vibrating in its fundamental mode.
- (b) In order to produce higher musical notes, the flutist opens valves along the tube. Since the holes in these valves are large, an open valve has the same effect as shortening the tube. The flutist opens one valve to play $C^{\#}$, two valves to play D, etc. Calculate the successive spacing between the valves of a flute for one complete octave. (The actual spacings used on flutes differ slightly from the results of this simple theoretical evaluation because the mouth cavity of the flutist affects the frequency.)

Problem 8.4

Traveling waves on a string.

(a) For any traveling wave of the form

$$y = a\sin(kx - \omega t) \quad ,$$

a is the amplitude, k is the wavenumber, $\lambda=2\pi/k$ is the wavelength, ω is the frequency in radians per second $(f=\omega/2\pi)$ is the frequency in Hertz), $T=2\pi/\omega$ is the period, and $v=\omega/k$ is the speed of the wave. For our wave,

$$y = 0.4 \sin[\pi(0.5x - 200t)]$$
 (x, y in cm, t in sec)

and we have

$$a = 0.4 \, \text{cm}$$

 $k = 0.5\pi = 1.57 \, \text{cm}^{-1}$
 $\lambda = 2\pi/k = 4 \, \text{cm}$
 $\omega = 200\pi = 628 \, \text{rad/s}$
 $(f = \omega/2\pi = 100 \, \text{Hz})$
 $T = 2\pi/\omega = 0.01 \, \text{sec}$
 $v = \omega/k = 400 \, \text{cm/s}$.

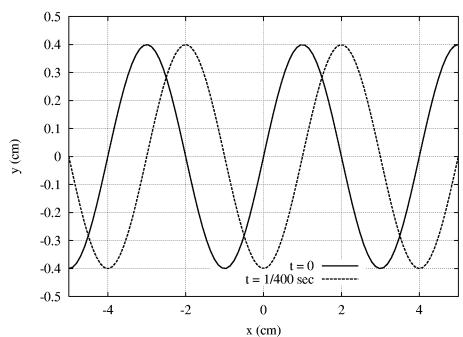
(b) At t = 0, the equation of the wave is

$$y(x,t)|_{t=0} = 0.5\sin(1.57x)$$
.

At $t = 1/400 \sec (= T/4)$, the equation is

$$y(x,t)|_{t=1/400} = 0.5\sin(1.57x - \pi/2) = -0.5\cos(1.57x)$$
.

Following is a plot of y vs. x for these two times:



(c) The transverse (y-direction) speed of a point on our string is

$$\frac{dy}{dt} = a\omega\cos(kx - \omega t) \quad ,$$

with maximum value

$$\frac{dy}{dt}\Big|_{\text{max}} = a\omega = 251 \,\text{cm/s}$$
 .

(d) If we clamp the string at two points a distance L cm apart and observe a standing wave of the same wavelength as above, then L must be an integer number of half-wavelengths:

$$L = n\lambda/2$$
 $(n = 1, 2, 3, 4, ...)$.

For our value of $\lambda = 4$ cm, the first three possibilities are:

$$n = 1$$

$$L = 2 \text{ cm}$$

$$n = 2$$

$$L = 4 \text{ cm}$$

$$L = 6 \text{ cm}$$

If L is to be strictly less than 10 cm, there are 4 possible values for L: 2 cm, 4 cm, 6 cm, and 8 cm.

Problem 8.5

Standing waves on a string.

(a) A standing wave of the form

$$y = a\sin(kx)\cos(\omega t)$$

has the same wavelength, etc. as given in 8.3(a) above. Thus, for

$$y = 0.3\sin(3x)\cos(1200t)$$

with x, y in centimeters and t in seconds, we have

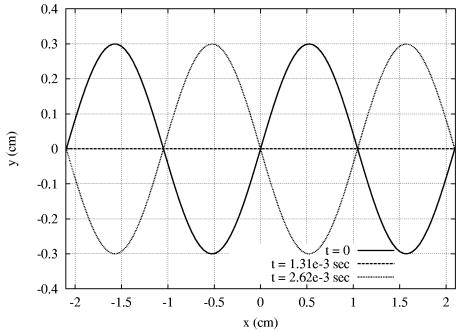
$$k = 3 \, \mathrm{cm}^{-1}$$

 $\lambda = 2\pi/k = 2.1 \, \mathrm{cm}$
 $\omega = 1200 \, \mathrm{rad/s}$
 $(f = \omega/2\pi = 191 \, \mathrm{Hz})$
 $T = 2\pi/\omega = 5.24 \times 10^{-3} \, \mathrm{s}$.

(b) The times of interest are t = 0, $t = 1.31 \times 10^{-3}$ s = T/4, and $t = 2.62 \times 10^{-3}$ s = T/2:

$$\begin{aligned} y(x,t)|_{t=0} &= 0.3\sin(3x) \\ y(x,t)|_{t=1.31\times10^{-3}} &= 0.3\sin(3x)\cos(\pi/2) \equiv 0 \\ y(x,t)|_{t=2.62\times10^{-3}} &= 0.3\sin(3x)\cos(\pi) = -0.3\sin(3x) \end{aligned}.$$

Following is a plot of y vs. x for these three times:



Compare the evolution of this standing wave with the evolution of the traveling wave of problem 8.3.

(c) As in 8.3(c), the maximum transverse speed is

$$\left. \frac{dy}{dt} \right|_{\text{max}} = a\omega = 360 \,\text{cm/s}$$
.

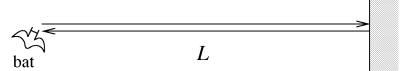
(d) One could interpret this question in one of two ways. If by "speed of propagation" we mean the so-called phase speed, then it is

$$v = \omega/k = 400 \,\mathrm{cm/s}$$
 .

If on the other hand we mean the speed with which this *particular* waveform (standing wave) travels along the string, then it is zero. (See plot under (b) above.)

Problem 8.6

Distance sensing with sound.



(a) The time T between the emission and return of the bat's pulse to a wall L meters away is $T=2L/v_{\rm a}$, where $v_{\rm a}$ is the speed of sound in air. Thus a distance uncertainty ΔL corresponds to a time uncertainty $\Delta T=2\Delta L/v_{\rm a}$. For $\Delta L=\pm 0.2\,{\rm m}$ and $v_{\rm a}=344\,{\rm m/s}$ (1 atm at 20°C), we have

$$\Delta T = \pm 1.2 \times 10^{-3} \,\mathrm{sec} \ .$$

(b) Suppose the bat in our methane-filled cave sends out a pulse which covers a distance L_{actual} . The bat will receive the reflected pulse after a time $T = 2L_{\text{actual}}/v_{\text{m}}$, where v_{m} is the speed of sound in methane. If the bat perceives that he is in air, he will interpret this time delay as being due to an apparent distance

$$L_{\text{apparent}} = \frac{v_{\text{a}}T}{2} = \frac{v_{\text{a}}}{v_{\text{m}}}L_{\text{actual}}$$
.

For $v_a = 344 \,\mathrm{m/s}$ and $v_m = 432 \,\mathrm{m/s}$, this gives

$$L_{\rm apparent} = 0.8 L_{\rm actual}$$
.

So the bat will perceive things as being closer than they actually are by a factor of 0.8, because the signals return faster than he expects.