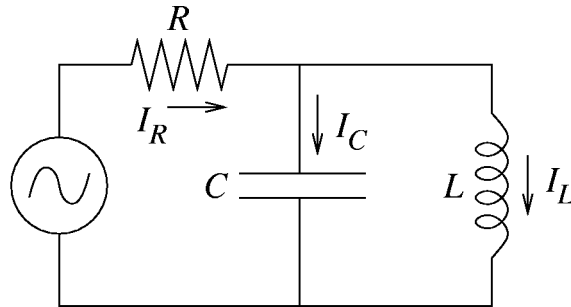


### Problem 11 (8 points)

The LRC circuit as shown is driven by a power supply whose EMF =  $V_o \cos(\omega t)$ . In steady state, the current through the ideal self-inductor is  $I_L$ , the current through the ideal capacitor is  $I_C$ , and the current through the resistor is  $I_R$ . **Steady state means that you wait a long time so that all transient phenomena have died out. Don't even THINK of writing down a differential equation. This problem is designed to see whether you have an appreciation for how a capacitor and a self inductor behave in extreme situations. No fancy math is needed. Express all your answers in terms of  $L$ ,  $R$ ,  $C$  and  $V_o$ .**



- a. (2) What are the maximum values of  $I_L$ ,  $I_C$ , and  $I_R$  in case  $\omega = 0$  (zero frequency means that the power supply is now a simple battery with zero internal resistance). **We are asking you for steady state solutions, NOT transient solutions.**
- b. (3) Answer the same question as under “a”, for the other extreme when  $\omega$  approaches a value which is infinitely high.
- c. (2) Do you expect the maximum value of the current  $I_R$  to be higher or lower than the value you found under “a” in the case that the frequency is somewhere in between the above two extremes. **Give your reasons.**
- d. (1) There is one frequency (**in steady state**) for which  $I_R$  is zero. This is not so intuitive, but given the fact that this is so, what do you think that frequency is? **Please, do not try to calculate this frequency.**

~~(c) 100% Reflection  $\Rightarrow$  no energy is absorbed.~~

### Problem 11

(a) In the  $\omega = 0$  steady state, all currents are constant in time. The self-inductor acts as a wire with zero resistance, shorting out the capacitor and ensuring  $I_C = 0$  (there would be no current in this branch even if we replaced the capacitor with a resistor). Thus we effectively have a resistor  $R$  in series with a zero-resistance wire, and  $I_L = I_R = V_0/R$ . (Notice, even though it is irrelevant here, that the reactance of the capacitor ( $1/\omega C$ ) is infinitely high.)

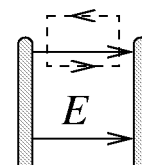
(b) With  $\omega$  infinitely high, the reactance of the self-inductor ( $\omega L$ ) becomes infinitely large and thus  $I_L = 0$ . The reactance of the capacitor ( $1/\omega C$ ) goes to zero. We effectively have a resistor  $R$  in series with a zero-reactance capacitor, and our peak current values are simply  $I_C = I_R = V_0/R$ .

(c) We would expect the peak value of  $I_R$  to **decrease** from the value  $V_0/R$  found in part (a) as we increase  $\omega$  to non-zero values. The effective impedance of the parallel combination of the capacitor and the self-inductor was zero with  $\omega = 0$ . For  $\omega \neq 0$  it can only increase, as the self-inductor will no longer act as zero-Ohm wire and the reactance of the capacitor ( $1/\omega C$ ) will also be non-zero. (This expectation can be confirmed by an exact calculation, if you are feeling ambitious.)

(d) The frequency is  $\omega = 1/\sqrt{LC}$ . (What else could it be?) This is the natural oscillation frequency of the  $LC$  sub-circuit alone. We will have an oscillating current in the  $LC$  sub-circuit that dissipates no energy. The voltage across the capacitor will be exactly offset by the power-supply voltage at all times, so that there will never be any voltage across the resistor. Hence,  $I_R = 0$ . (None of this is very intuitive.)

### ~~Problem 12 (See homework problem 5.7 and Exam 2, problem 4)~~

~~Assuming an abrupt drop to zero, integrating around the dashed path shown gives  $\oint \mathbf{E} \cdot d\mathbf{l} \neq 0$ , for there is an  $E$ -field between the plates. Since this is a static situation, we have  $d\Phi_B/dt = 0$  for the rate of change of magnetic flux through the open surface bounded by our loop, and thus Faraday's law asserts  $\oint \mathbf{E} \cdot d\mathbf{l} = 0$ . Therefore the  $E$ -field cannot drop abruptly to zero outside the capacitor.~~



### ~~Problem 13~~

~~(a)  $V_{\text{right}}$  will read 0.1 Volt, since it has the same internal resistance as  $V_{\text{left}}$  but has its terminals oppositely oriented with respect to the current in the circuit.~~

~~(b) If  $V_{\text{left}}$  reads +0.1 Volt, then the current must enter through the positive terminal. Thus the current flows counterclockwise. From Ohm's law,~~

$$~~I = V_{\text{left}}/R_{\text{left}} = (0.1)/(10^4) = 10^{-5} \text{ Amperes.}~~$$

~~The induced EMF is counterclockwise as well, and is given by~~

$$~~\mathcal{E} = I(R + R_{\text{left}} + R_{\text{right}}) = (10^{-5})(2.5 \times 10^4) = 0.25 \text{ Volts.}~~$$