

## 8.02x Electricity and Magnetism

### Problem Set 10

Issued: Sun, April 17

~~Due: Fri, April 22, 4PM < note Date & Time!~~

~~Reading suggestions from Young & Freedman~~

~~Mon, 4/18 Patriots day vacation~~

~~Wed, 4/20: AC Circuits, Inductors, RL Circuits 31.2, 30.2, 30.4~~

~~Fri, 4/22: RLC circuits, Oscillations: 30.5, 30.6~~

~~Problem 1 (6 points):~~

~~Compare the oscillations of an LRC circuit to the vibration of a mass  $m$  on a spring. What do  $L$  and  $C$  correspond to in the mechanical system? What is the mechanical analog to  $R$ ?~~

**Problem 2(6 points)**

You have probably noticed that when a circuit carrying a large current is interrupted, a spark occurs between the poles of the switch (or the poles of a plug that is pulled).

- (a) Explain this phenomenon. Where does the energy for the spark come from?
- (b) Assume an inductor  $L=1\text{mH}$  and a resistor  $R=10\Omega$  are connected in series to a battery providing  $V=100\text{V}$ . How much energy is stored in the inductor a long time after the circuit is closed?.

~~Problem 3 (6 points) Young&Freedman, Problem 30.10~~

~~Problem 4 (6 points) Young&Freedman, Problem 30.14~~

~~Problem 5 (6 points) Young&Freedman, Problem 30.26~~

## ~~PROBLEM SET 10; 802x; SPRING 2005~~

1) ~~The differential equation governing an RLC circuit is:~~

$$\begin{aligned} &\text{L di/dt} + Ri + q/C = 0. \text{ Using } i = dq/dt, \text{ we have,} \\ &\text{L d}^2q/\text{dt}^2 + R dq/dt + q/C = 0. \end{aligned}$$

~~The differential equation governing a mass on a spring is (with velocity proportional viscous damping):~~

$$\text{m d}^2x/\text{dt}^2 + b dx/dt + Kx = 0. \text{ Here the mass is m, K is the spring constant and b is the coefficient of proportionality between velocity and the viscous retarding force.}$$

~~Thus: M and L play the same roles; b and R play the same roles; and, K plays the same role as 1/C.~~

~~Mv is the momentum that will persist unless changed by a force, and Li is the flux in an inductor that will persist unless changed by an external agent. The kinetic energy stored in motion is (1/2) mv<sup>2</sup>, while energy is stored in the inductor as (1/2) Li<sup>2</sup>. The resistor is an agent for energy loss at the rate i<sup>2</sup>R. Energy is lost to viscosity at the rate bv<sup>2</sup>. Energy is stored in a capacitor as (1/2) q<sup>2</sup>/C and energy is stored in the spring as (1/2) Kx<sup>2</sup>.~~

2) The self-inductance of the circuit causes the current to persist until the voltage developed across the gap acting as a capacitor causes it to stop. Now this gap usually has a very small capacitance and the current, which we have assumed to be large, can charge the gap to a very large voltage. Thus the spark develops when the air brakes down. The energy for the spark comes from the energy stored in the self-inductance of the circuit, (1/2) Li<sup>2</sup>.

The equilibrium current is  $i = V/R = 100/10 = 10$  amps. The energy stored in the inductor is  $(1/2) Li^2 = (1/2)(1/1000)(100) = 1/20$  joule.

3) a. ~~Compare figure 30.18 and fig 30.6b. Note that points a and b are reversed. Thus, according to equation 30.8,  $dI/dt = (V_b - V_a)/L = -1.04V/0.260H = -4$  A/s. Thus, the current is decreasing.~~

b. ~~From a. we know that  $di = (-4 \text{ A/s}) dt$ . After integrating both sides of the expression with respect to t, we obtain  $\Delta I = (-4 \text{ A/s}) \Delta t$  and so  $I = (12.0 \text{ A}) - 4 \text{ A/s} * 2 \text{ s} = 4 \text{ A}$ .~~

4) a.  ~~$U = P * t = (200 \text{ W})(24 \text{ h/day} * 3600 \text{ s/h}) = 1.73 * 10^7 \text{ J}$ .~~

b.  ~~$U = \frac{1}{2} L I^2$  and therefore  $L = 2U/I^2 = 2 (1.73 * 10^7 \text{ J}) / (80 \text{ A}^2) = 5406 \text{ H}$ .~~

5) ~~When switch 1 is closed and switch 2 is open, the loop rule gives  $L di/dt + IR = 0$  and therefore  $dI/dt = -I R/L$ . Integrating from  $I_0$  to I on the LHS and 0 to t on the RHS gives  $\ln(I/I_0) = -R/L t$  and therefore  $I(t) = I_0 \exp(-t/(L/R))$~~