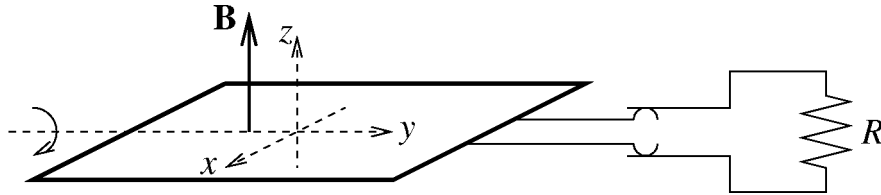


### Problem 6 (7 points)

A simple electric generator (as shown below) is rotating about the y-axis with a frequency of  $f$  Hz. There is a uniform magnetic field  $B = 0.5$  Tesla in the  $+z$ -direction. The rotor consists of a coil of  $n$  windings each with an area of  $S$  m<sup>2</sup>. The generator, through slipping contacts, is powering a light bulb whose resistance is  $R$   $\Omega$  (see the figure). The ohmic resistance of the coil is negligibly small compared to that of the light bulb. You may also assume here, for simplicity, that the self-inductance of the coil is negligibly small.



- a. (3) What is the maximum value ( $I_{max}$ ) of the induced alternating current? Also indicate in the figure one of the two positions of the coil when this maximum current occurs.
- b. (4) What is the time-averaged mechanical power (in Watts) that must be supplied to maintain the rotation (neglect friction in the bearings)?

(c) When we disconnect the battery before inserting the dielectric, each capacitor carries a charge  $Q = CV$ , giving a total charge of  $2CV$  on the parallel combination. After inserting the dielectric, charge will flow between the left and right capacitors so as to equalize the voltages across them at some new value  $V'$ . The *total* charge on the two will remain constant, though. The capacitance of the right hand capacitor is unchanged, so its new charge is  $Q_R = CV'$ . The capacitance of the left hand capacitor has increased by a factor of  $\kappa$ , thus its charge is now  $Q_L = \kappa CV'$ . Enforcing charge conservation as required, we have

$$Q_R + Q_L = (1 + \kappa)CV' = 2CV \Rightarrow V' = \frac{2V}{(1 + \kappa)}.$$

The electric field between the plates of each capacitor is then

$$E = V'/d = \frac{2V}{(1 + \kappa)d} = 2V/3d \quad (\text{directed downward}).$$

## Problem 6

(a) If the coil lies in the  $x$ - $y$  plane at time  $t = 0$ , then the angle between the coil and the  $x$ - $y$  plane at later times will be  $2\pi ft$ . The magnetic flux through the plane surface bounded by the coil will be  $\Phi_B = nSB \cos(2\pi ft)$ . The induced EMF is  $\mathcal{E} = -d\Phi_B/dt = 2\pi fnSB \sin(2\pi ft)$ , and the induced current is  $I = \mathcal{E}/R = (2\pi fnSB/R) \sin(2\pi ft)$ . The maximum value of this induced current is thus  $I_{\max} = 2\pi fnSB/R$ , or using the given numerical value for  $B$ ,  $I_{\max} = \pi fnS/R$  Amperes. This maximum occurs when the plane of the coil is at a right angle to the  $x$ - $y$  plane.

(b) The time-averaged mechanical power supplied to maintain rotation must equal the time-averaged power  $\bar{P}$  dissipated in the resistor:

$$\bar{P} = \bar{I}^2 R = I_{\max}^2 R \overline{\sin^2(2\pi ft)} = \frac{1}{2} I_{\max}^2 R = \frac{(\pi fnS)^2}{2R} \text{ Watts.}$$

## Problem 7

(a) The phase is of the form  $(ky + \omega t)$ , so propagation is in the  $\hat{y}$  direction.

(b) In order for  $\vec{E}_0 \times \vec{B}_0$  to be in the  $\hat{y}$  direction given  $\vec{E}_0$  in the  $+\hat{z}$  direction,  $\vec{B}_0$  must be in the  $-\hat{x}$  direction.  $|\vec{E}_0|/|\vec{B}_0| = v = c/n$  is the wavespeed in the medium, so

$$|\vec{B}_0| = \frac{n|\vec{E}_0|}{c} = \frac{(1.5)(2)}{3 \times 10^8} = 10^{-8} \Rightarrow \vec{B}_0 = -10^{-8} \hat{x}.$$

(c)

$$\bar{v} = \frac{c}{n} = \lambda f = \frac{\lambda \omega}{2\pi} \Rightarrow \lambda = \frac{2\pi c}{n\omega} = \frac{2\pi(3 \times 10^8)}{(1.5)(4\pi \times 10^{15})} = 10^{-7} \text{ m}.$$