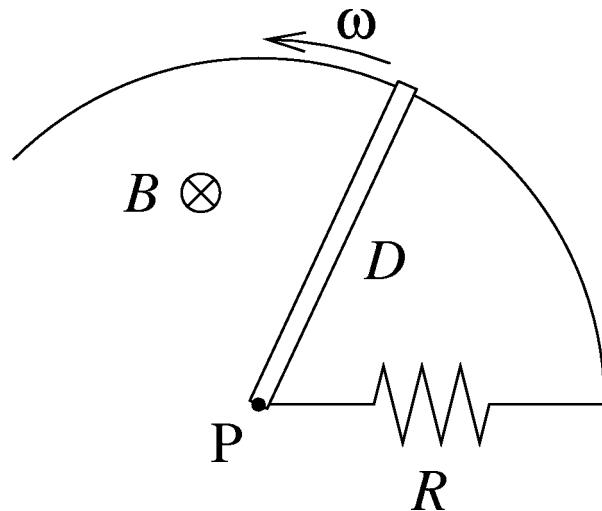


### Problem 7 (16 points)

A conducting bar of length  $D$  rotates with angular frequency  $\omega$  about a pivot P at one end of the bar (see the figure). The other end of the bar is in slipping contact with a stationary conducting wire in the shape of a circle (we only show a small part of that circle to keep the drawing simple). Between point P and the circular wire there is a resistor  $R$  as shown. Thus the bar, the resistor and the arc form a closed conducting loop. The resistance of the bar and the circular wire are negligibly small. There is a **uniform** magnetic field  $B$  **everywhere**, it is perpendicular to the plane of the paper as indicated.

What is the induced current in the loop? Express your answer in terms of  $D, \omega, R$ , and  $B$ .



For tiny elements  $dl$  along the CD wire segment,  $I dl \times \hat{r}$  is purely in the “out of the paper” direction, so the contribution of the CD wire segment to the  $B$  field at P is directed purely out of the paper. By the principle of linear superposition, this contribution will be exactly one half the field at P due to a *complete* infinite straight wire which extends beyond C. The familiar application of Ampère’s law for an “infinitely” long wire gives  $B = \mu_0 I / 2\pi d$ , so our answer is half that value

$$B_P = \frac{\mu_0 I}{4\pi d} \text{ out of the paper.}$$

**Problem 6** (See the lecture supplement of March 15.)

A voltmeter will read a negative value if the (small) current that flows through it enters through the negative terminal, so the current in our circuit must be counterclockwise.

(a) Both voltmeters have equal internal resistances and they are wired in series, so both will have the same 0.1 Volt drop across themselves. The induced EMF driving the current must therefore be

$$\mathcal{E} = 0.2 \text{ Volts (counterclockwise).}$$

(b) The counterclockwise current enters the *positive* terminal of the left hand voltmeter, so it will read

$$V_{\text{left}} = +0.1 \text{ Volts.}$$

**Problem 7**

If  $\theta$  is the angle between the resistor wire and the moving bar, the area of the plane surface bounded by the closed circuit is

$$(\pi D^2) \frac{\theta}{2\pi} = \frac{1}{2} D^2 \theta .$$

The magnetic flux through this surface is then  $\Phi_B = \frac{1}{2} BD^2 \theta$ , and the magnitude of the induced EMF is

$$|\mathcal{E}| = \frac{d\Phi_B}{dt} = \frac{1}{2} BD^2 \frac{d\theta}{dt} = \frac{1}{2} BD^2 \omega .$$

The induced current will be

$$I = \frac{|\mathcal{E}|}{R} = \frac{BD^2 \omega}{2R} .$$

(By Lenz’s law, this current will flow in a counterclockwise sense, so as to counter the changing flux due to the motion of the bar.)