

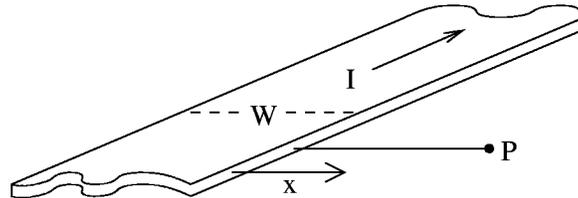
**Problem 5.2**

~~Ampere's law in action~~ Giancoli 28-27.

**Problem 5.3**

*Magnetic Field of a Current Carrying Ribbon.*

A thin, flat, "infinitely" long ribbon of width  $W$  carries a uniform current  $I$ . Determine the magnetic field at a point  $P$  that is in the plane of the ribbon at a distance  $x$  from one edge, as shown in the diagram. Test your result in the limit for  $W \rightarrow 0$ .



**Problem 5.4**

~~Force on loop~~ Giancoli 29-14.

**Problem 5.5**

~~Betatron~~ Giancoli 29-49.

**Problem 5.6**

~~Intuition Breaks Down~~ Do the problems marked "Test 1" and "Test 2" in the Lecture Supplement of March 15.

**Problem 5.7**

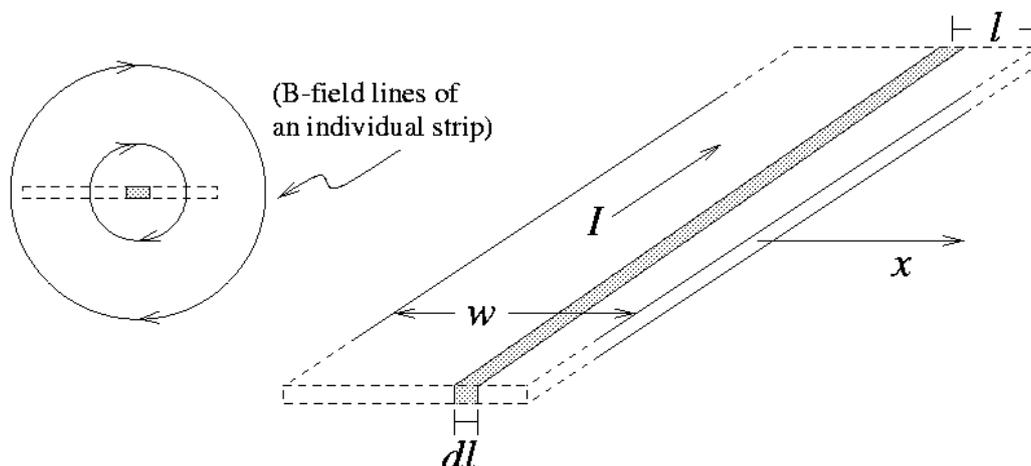
~~Fringe Fields~~ Giancoli 29-69.

***Recitations.***

~~There are 13 recitation instructors (see the 8.02 Website). If for any reason you are unhappy with your present recitation, see Maria Springer in 4-352 and CHANGE!!~~

**Problem 5.3**

Magnetic field of a current-carrying ribbon.



To solve for the magnetic field of a current-carrying ribbon, we break it up conceptually (and mathematically) into many infinitesimal strips which may be regarded as wires. The principle of linear superposition tells us that the magnetic field of the ribbon will be the vector sum of the magnetic field contributions of the individual strips.

Consider a little strip of width  $dl$ , located a distance  $l$  from the right edge of the ribbon as shown in the diagram above. The strip carries a current  $I dl/w$  and is located a distance  $l+x$  from the point in the plane of the ribbon at which we wish to determine the field. The differential contribution to the field at the point of interest due to this strip is just the field due to a long straight wire:

$$dB = \frac{\mu_0 I dl/w}{2\pi(x+l)} ,$$

directed downward by the right-hand rule. To find the total field we simply integrate over the ribbon from  $l = 0$  to  $l = w$  (making the substitution  $u = x+l$ ,  $du = dl$ ):

$$B = \int dB = \frac{\mu_0 I}{2\pi w} \int_0^w \frac{dl}{(x+l)} = \frac{\mu_0 I}{2\pi w} \int_x^{x+w} \frac{du}{u} = \frac{\mu_0 I}{2\pi w} \ln \left( 1 + \frac{w}{x} \right) .$$

(Again, the field is directed downward, perpendicular to both the plane of the ribbon and the direction of current flow.)

To examine the limit  $w \rightarrow 0$ , we make use of the first-order Taylor expansion of the natural logarithm:  $\ln(1+\delta) \simeq \delta$  for  $\delta \ll 1$  (check out Giancoli Appendix A-3). As  $w \rightarrow 0$ ,  $w/x \ll 1$ , so

$$B \rightarrow \frac{\mu_0 I}{2\pi w} \left( \frac{w}{x} \right) = \frac{\mu_0 I}{2\pi x} .$$

This is just what we would expect a distance  $x$  away from a wire carrying current  $I$ .

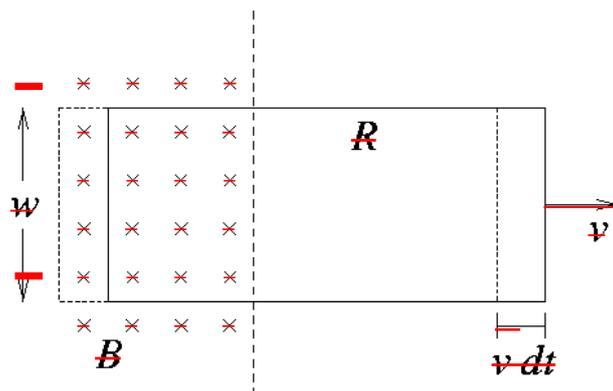
We have taken the limit  $w \rightarrow 0$  at fixed  $x$  (the “skinny ribbon limit”), but we could equally well view this as the limit  $x \rightarrow \infty$  at fixed  $w$  (the “far away limit”): far enough away, *any* ribbon will look like a wire. What is important is that  $w/x \ll 1$ .

### ~~Problem 5.4~~

~~Force on loop. (Ciancoli 29-14.)~~

~~For convenience, we'll define the following variables to work with:~~

$$\begin{aligned} w &= 0.350 \text{ m} \\ B &= 0.450 \text{ T} \\ R &= 0.230 \Omega \\ v &= 3.40 \text{ m/s} \end{aligned}$$



~~In a small time interval  $dt$ , the loop moves a distance  $v dt$ , and thus the area of the loop within the B-field region decreases by an amount  $dA = wv dt$ . The field magnitude remains constant, so the magnetic flux through the plane surface bounded by the loop decreases by an amount  $d\Phi_B = B dA = Bwv dt$ . By Faraday's law (Ciancoli Equation (29-2a), p. 736), the magnitude of the induced emf will be~~

$$|\mathcal{E}| = \frac{d\Phi_B}{dt} = Bwv .$$

~~This will induce a clockwise current in the loop (clockwise to oppose the change in flux: that's Lenz's law) of magnitude~~

$$I = |\mathcal{E}|/R = Bwv/R .$$

~~As this current flows upwards through the left edge of the loop in the B-field region, that edge will experience a force directed to the left as given by the Lorentz force law (in the form of Ciancoli Equation (27-3), p. 691):~~

$$F = IwB = B^2 w^2 v / R = (0.450)^2 (0.350)^2 (3.40) / (0.230) = 0.367 \text{ N} .$$

~~So to keep the loop moving with constant speed, someone or something must pull the loop to the right with an equal 0.367 N of force.~~