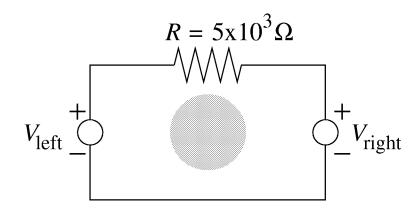
## Problem 13 (6 points)

Two voltmeters,  $V_{\text{right}}$  and  $V_{\text{left}}$ , each with an internal resistance of  $10^4~\Omega$  are in series with a resistor R of  $5\times10^3~\Omega$ . They are connected through wires of negligible resistance (see the circuit below). The "+" side of both voltmeters is up as shown. A changing magnetic field is present in the shaded area. At a particular moment in time  $V_{\text{left}}$  reads +0.1 Volt.

- a. (3) At that moment in time, what is the reading of  $V_{\text{right}}$ ?
- b. (3) What, at that moment, is the current (in A) in the circuit (magnitude and direction), and what is the induced EMF?



(c) 100% Reflection  $\Longrightarrow$  no energy is absorbed.

## Problem 11

- (a) In the  $\omega = 0$  steady state, all currents are constant in time. The self-inductor acts as a wire with zero resistance, shorting out the capacitor and ensuring  $I_C = 0$  (there would be no current in this branch even if we replaced the capacitor with a resistor). Thus we effectively have a resistor R in series with a zero-resistance wire, and  $I_L = I_R = V_0/R$ . (Notice, even though it is irrelevant here, that the reactance of the capacitor  $(1/\omega C)$  is infinitely high.)
- (b) With  $\omega$  infinitely high, the reactance of the self-inductor ( $\omega L$ ) becomes infinitely large and thus  $I_L = 0$ . The reactance of the capacitor ( $1/\omega C$ ) goes to zero. We effectively have a resistor R in series with a zero-reactance capacitor, and our peak current values are simply  $I_C = I_R = V_0/R$ .
- (c) We would expect the peak value of  $I_R$  to decrease from the value  $V_0/R$  found in part (a) as we increase  $\omega$  to non-zero values. The effective impedance of the parallel combination of the capacitor and the self-inductor was zero with  $\omega = 0$ . For  $\omega \neq 0$  it can only increase, as the self-inductor will no longer act as zero-Ohm wire and the reactance of the capacitor  $(1/\omega C)$  will also be non-zero. (This expectation can be confirmed by an exact calculation, if you are feeling ambitious.)
- (d) The frequency is  $\omega = 1/\sqrt{LC}$ . (What else could it be?) This is the natural oscillation frequency of the LC sub-circuit alone. We will have an oscillating current in the LC sub-circuit that dissipates no energy. The voltage across the capacitor will be exactly offset by the power-supply voltage at all times, so that there will never be any voltage across the resistor. Hence,  $I_R = 0$ . (None of this is very intuitive.)

**Problem 12** (See homework problem 5.7 and Exam 2, problem 4) Assuming an abrupt drop to zero, integrating around the dashed path shown gives  $\oint \mathbf{E} \cdot d\mathbf{l} \neq 0$ , for there is an E-field between the plates. Since this is a static situation, we have  $d\Phi_B/dt = 0$  for the rate of change of magnetic flux through the open surface bounded by our loop, and thus Faraday's law asserts  $\oint \mathbf{E} \cdot d\mathbf{l} = 0$ . Therefore the E-field cannot drop abruptly to zero outside the capacitor.



## Problem 13

- (a)  $V_{\text{right}}$  will read -0.1 Volt, since it has the same internal resistance as  $V_{\text{left}}$  but has its terminals oppositely oriented with respect to the current in the circuit.
- (b) If  $V_{\text{left}}$  reads +0.1 Volt, then the current must enter through the positive terminal. Thus the current flows counterclockwise. From Ohm's law,

$$I = V_{\text{left}}/R_{\text{left}} = (0.1)/(10^4) = 10^{-5} \,\text{Amperes}.$$

The induced EMF is counterclockwise as well, and is given by

$$\mathcal{E} = I(R + R_{\text{left}} + R_{\text{right}}) = (10^{-5})(2.5 \times 10^4) = 0.25 \text{ Volts}.$$