

**Problem 4 (12 points)**

Apply Faraday's law to show that a static electric field between the plates of a parallel-plate capacitor cannot drop abruptly to zero at the edges of the capacitor.

## MIT 8.02 Spring 2002 Exam #2 Solutions

### **Problem 1** (*Homework problem 6.6; simplified version*)

**(a)** Before the switch is closed, all currents are zero. Immediately after the switch is closed,  $I_3$  will still be zero, for the inductor prevents instantaneous changes in  $I_3$ . That leaves us with  $V = R_1 I_1 + R_2 I_2$ , and  $I_1 = I_2$  because  $I_3 = 0$ . So  $I_1 = I_2 = V/(R_1 + R_2)$ .

**(b)** The ohmic resistance of the self-inductor is zero. So if we wait a long time, until the self-inductor is no longer opposing changes in  $I_3$ , we have a wire with zero resistance in parallel with  $R_2$ . Thus  $I_2$  will be zero, and  $I_1 = I_3$ . Applying Ohm's law, we find  $I_1 = V/R_1 (= I_3)$ .

### **Problem 2**

**One way:** The Lorentz force on the left segment is  $IbB$  to the left (with uniform force per unit length), the force on the right segment is  $IbB$  to the right, the force on the top segment is  $IaB$  upward, and the force on the bottom segment is  $IaB$  downward. This combination of forces produces no torque on the loop.

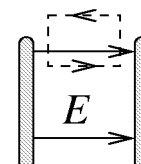
**Another way:** The loop's magnetic moment  $\mu$  has direction perpendicular to the plane surface bounded by the loop.  $B$  is also perpendicular to that surface. So the torque is  $\tau = \mu \times B = 0$ .

### **Problem 3** (*Homework problem 4.6.*)

The kinetic energy acquired during acceleration is  $Mv^2/2 = (2e)V$ , where  $v$  is the ion's final speed. In the  $B$ -field region, (Lorentz force) = (centripetal force) gives  $(2e)vB = Mv^2/d$ . Eliminating  $v$  and solving for  $M$  gives  $M = eB^2 d^2/V$ .

### **Problem 4** (*Homework problem 5.7.*)

Assuming an abrupt drop to zero, integrating around the dashed path shown gives  $\oint \mathbf{E} \cdot d\mathbf{l} \neq 0$ , for there is an  $E$ -field between the plates. Since this is a static situation, we have  $d\Phi_B/dt = 0$  for the rate of change of magnetic flux through the open surface bounded by our loop, and thus Faraday's law asserts  $\oint \mathbf{E} \cdot d\mathbf{l} = 0$ . Therefore the  $E$ -field *cannot* drop abruptly to zero outside the capacitor.



### **Problem 5**

Tiny elements  $d\mathbf{l}$  along the AC wire segment are parallel to the unit vector  $\hat{\mathbf{r}}$  directed from the element to point P, giving  $I d\mathbf{l} \times \hat{\mathbf{r}} = 0$ . So, by the Biot-Savart law, the AC wire segment makes no contribution to the magnetic field at P.