Physics - 8.02

# Assignment #3

February 22, 2002.

We strongly recommend that you read about a topic before it is covered in lectures.

Lecture Date	Topics Covered	Reading from Giancoli
#9 Mon 2/25	Currents - Resistivity - Ohm's Law	Chapter 25 through Sect. 25-4
#10 Wed 2/27	Batteries - EMF - Energy Conservation - Power Kirchhoff's Rules - Circuits Kelvin Water Dropper	Sect. 25-5 through 25-8 Chapter 26 through Sect. 26-3 (take notes in lecture)
#11 Fri 3/1	Magnetic field - Lorentz force - Torques Electric Motors (DC) Cathode Ray Tube, Oscilloscope	Chapter 27 through Sect. 27-7 Sect. 23-9

## Due before 4 PM Friday, March 1 in 4-339B.

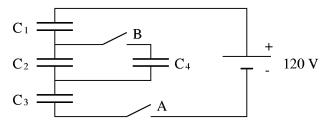
## Problem 3.1

Capacitors in series and parallel. Giancoli 24-23.

## Problem 3.2

Switching Capacitors.

In the diagram below, the four capacitors have the same capacitance; the battery provides 120 V.



Consider two cases, starting in both cases with uncharged capacitors.

#### Case I.

- (a) While switch B is kept open, switch A is closed and then opened after  $C_1$ ,  $C_2$ , and  $C_3$  are fully charged. What is now the electric potential difference across each capacitor?
- (b) Subsequently switch B is closed. What is now the electric potential difference across each capacitor?

## Case II.

- (c) Switch A is open. Switch B is first closed. What is now the electric potential difference across each capacitor?
- (d) Subsequently switch A is closed. What now is the potential difference across each capacitor?

# Problem 3.3.

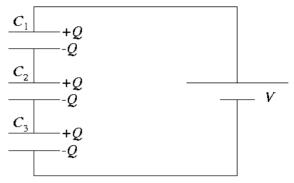
The effect of a dielectric medium on the capacitance. Giancoli 24-60.

# Problem 3.2

Switching capacitors.

(a) With switch B open and switch A closed,  $C_4$  is out of the picture, and the circuit is effectively as shown in the diagram at right. If  $V=120~\mathrm{V}$  is the battery voltage, we must have

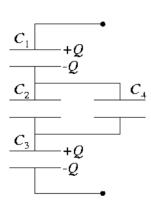
$$V_1 + V_2 + V_3 = V$$
.



By the same sort of charge-conservation argument as in problem 3.1, all three capacitors must necessarily have the same charge  $\pm Q$  on their plates. Since all three have the same capacitance as well (call it "C"), we have

$$Q = CV_1 = CV_2 = CV_3$$
  
 $\Rightarrow V_1 = V_2 = V_3 = V/3 = 40 \text{ V}$ .

Once the capacitors are charged in this manner, the potential across each will remain the same even after the opening of switch A.

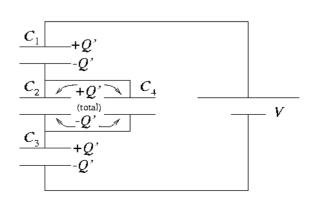


(b) Opening switch A has taken the circuit branch containing the battery out of the game, but not before charging capacitors  $C_1$ ,  $C_2$ , and  $C_3$  as described in part (a). Now, closing switch B brings  $C_4$  back into play, and the circuit is effectively as shown at left.  $C_1$  and  $C_3$  will still each carry the same charge Q as before (opening switch A leaves them isolated from any flow of charge). However, the charge Q that originally resided on  $C_2$  alone will now be distributed between  $C_2$  and  $C_4$ . As  $C_2 = C_4$ , by symmetry we will have a charge Q/2 on each, and the voltage across  $C_2$  and  $C_4$  will thus be half the original voltage across  $C_2$ . So then,

$$V_1 = V_3 = 40 \text{ V} , \quad V_2 = V_4 = 20 \text{ V} .$$

- (c) With the capacitors initially uncharged and switch A open, closing switch B does not provide a potential across any of the capacitors, for the circuit branch containing the battery is effectively disconnected.
- (d) Now we close switch A, with switch B previously closed. The circuit is as shown at right. The effective capacitance of the  $C_2$ - $C_4$  parallel combination is  $C_{24} = C + C = 2C$ . The entire four-capacitor system has an equivalent capacitance

$$\begin{split} C_{\rm eq} &= \left(\frac{1}{C_1} + \frac{1}{C_{24}} + \frac{1}{C_3}\right)^{-1} \\ &= \left(\frac{1}{C} + \frac{1}{2C} + \frac{1}{C}\right)^{-1} = \frac{2}{5}C \ , \end{split}$$



and thus the charge drawn from the battery and onto each of  $C_1$ ,  $C_{24}$ , and  $C_3$  is

$$Q' = C_{
m eq} V = rac{2}{5} C V$$
 .

Now we can determine the potential differences across the various capacitors:

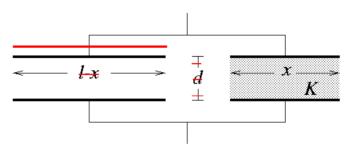
$$V_1 = V_3 = rac{Q'}{C} = rac{2}{5}V = 48 \; ext{V}$$

and

$$V_2 = V_4 = rac{Q'}{2C} = rac{1}{5}V = 24 \; {
m V} \;\; .$$

# Problem 3.3

The effect of a dielectric medium on the capacitance. (Giancoli 24-60.)



(a) To get the total capacitance of this arrangement, we consider it to be the equivalent of two capacitors in parallel, as shown above. The "capacitor" on the left (no dielectric) has plate area l(l-x) and (by Giancoli equation 24-2, p. 615) capacitance  $\epsilon_0 l(l-x)/d$ . The "capacitor" on the right (with dielectric) has area lx and capacitance  $K\epsilon_0 lx/d$ . Thus the total capacitance is the sum of these:

$$C = \frac{\epsilon_0 l}{d} [(K-1)x + l] .$$

When x = 0 this reduces to  $\epsilon_0 l^2/d$ , and when x = l we get  $K \epsilon_0 l^2/d$ ; these are the proper limits.

(b) With a potential difference  $V_0$ , the energy stored in this capacitor will be given by Giancoli equation 24-5 (p. 620):

$$U = \frac{1}{2}CV_0^2 = \frac{\epsilon_0 l}{2d}[(K-1)x + l]V_0^2$$

(c) Suppose we move the slab a little bit further in between the plates, increasing x by an amount  $\Delta x$ . The potential difference (provided by a battery, perhaps) remains constant while the capacitance increases a bit, so the change in stored energy will be

$$\Delta U = \frac{1}{2}C_f V_0^2 - \frac{1}{2}C_i V_0^2$$
$$= \frac{\epsilon_0 L}{2d}(K - 1)\Delta x V_0^2 ,$$