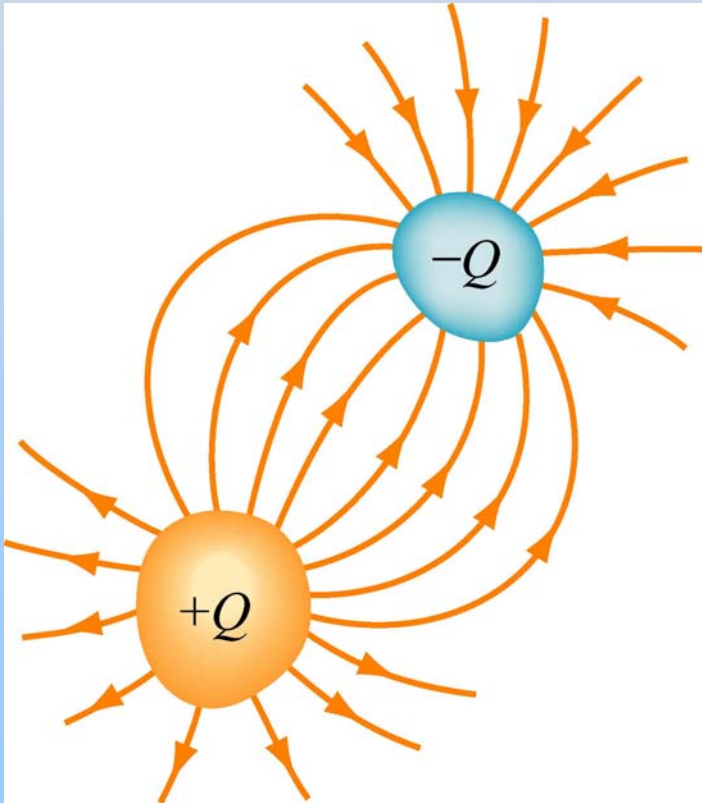


Capacitors and Capacitance

Capacitors: Store Electric Energy

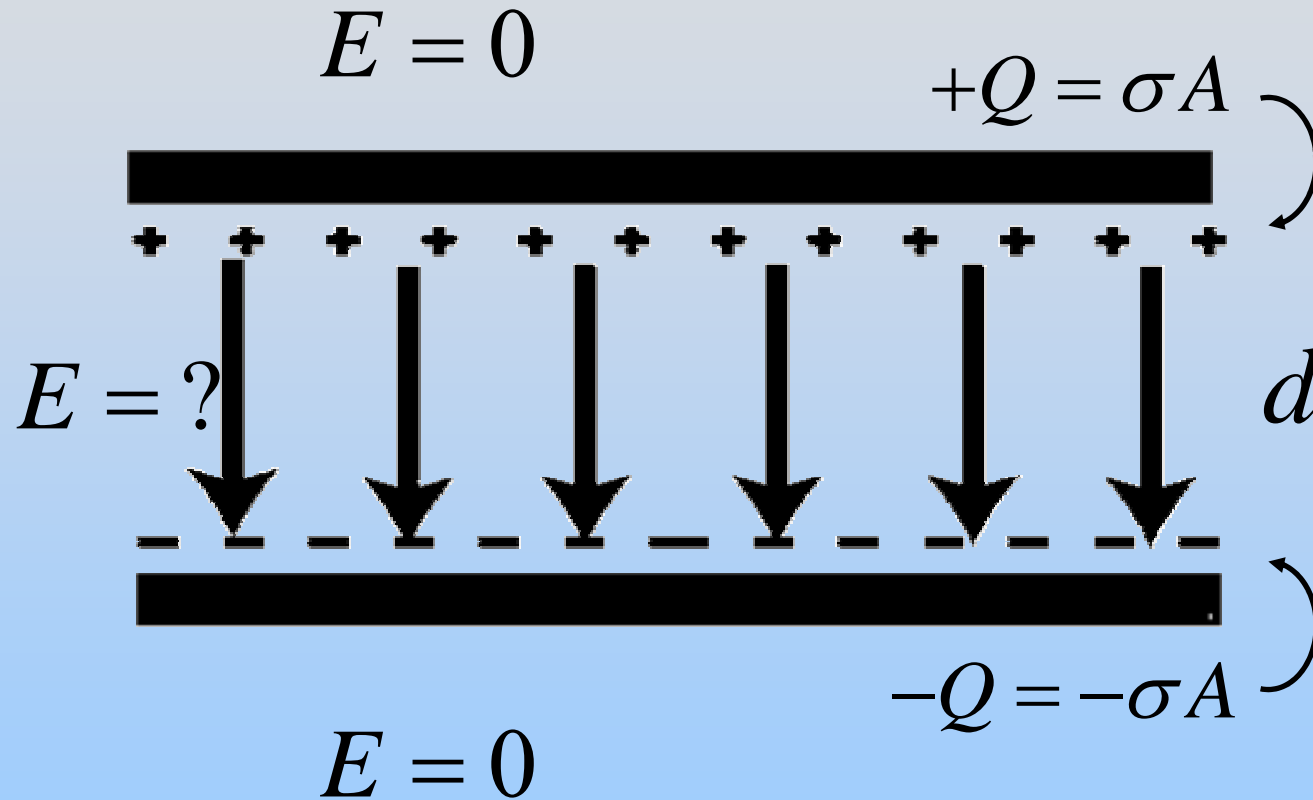
Capacitor: two isolated conductors with equal and opposite charges Q and potential difference ΔV between them.



$$C = \frac{Q}{|\Delta V|}$$

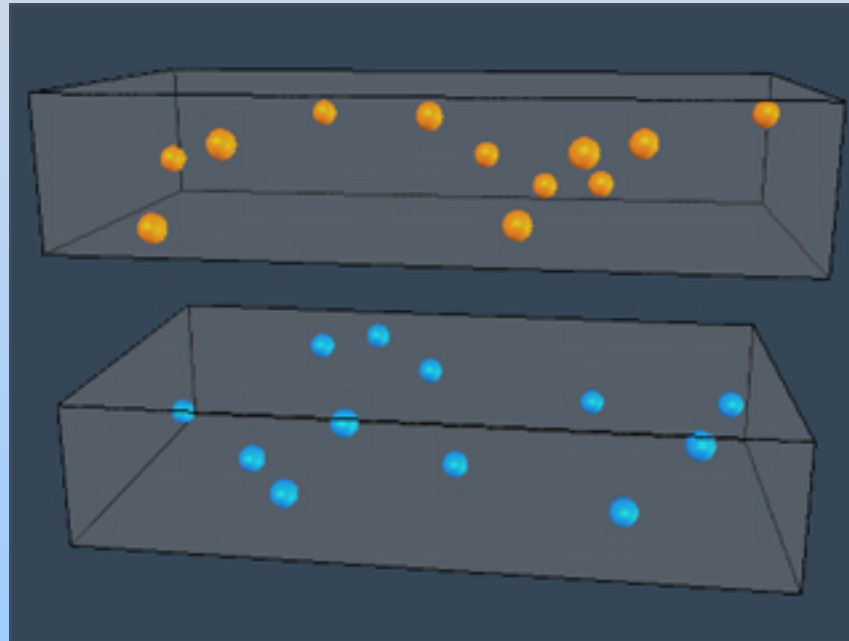
**Units: Coulombs/Volt or
Farads**

Parallel Plate Capacitor



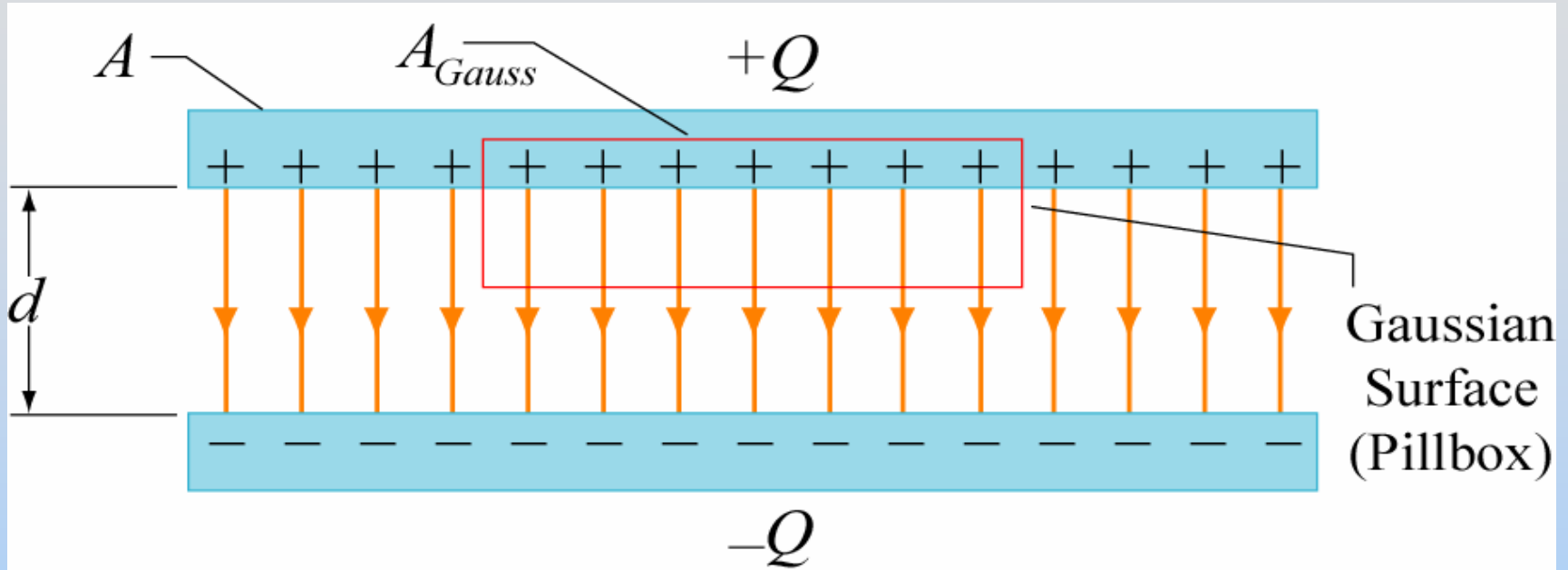
Parallel Plate Capacitor

When you put opposite charges on plates, charges move to the inner surfaces of the plates to get as close as possible to charges of the opposite sign



<http://ocw.mit.edu/ans7870/8/8.02T/f04/visualizations/electrostatics/35-capacitor/35-capacitor320.html>

Calculating E (Gauss's Law)

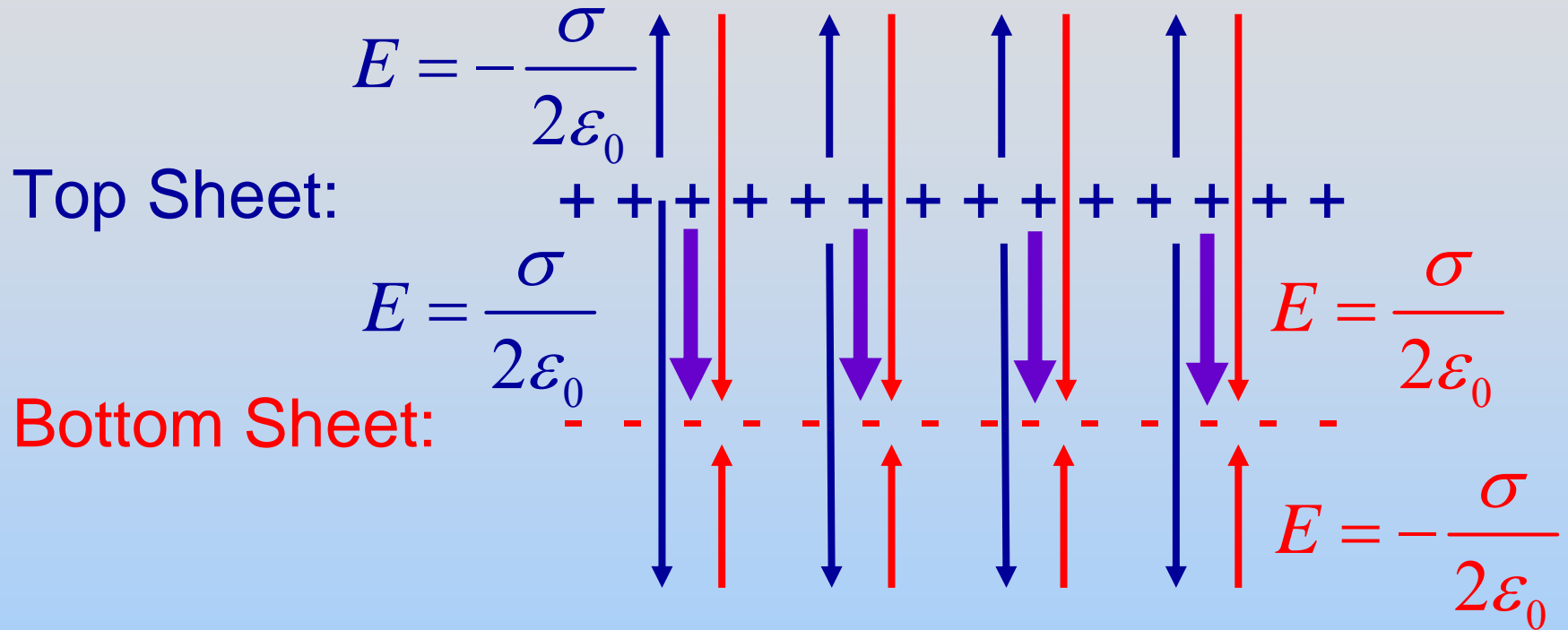


$$\oiint_S \vec{E} \cdot d\vec{A} = \frac{q_{in}}{\epsilon_0} \quad E(A_{Gauss}) = \frac{\sigma A_{Gauss}}{\epsilon_0}$$

$$E = \frac{\sigma}{\epsilon_0} = \frac{Q}{A\epsilon_0}$$

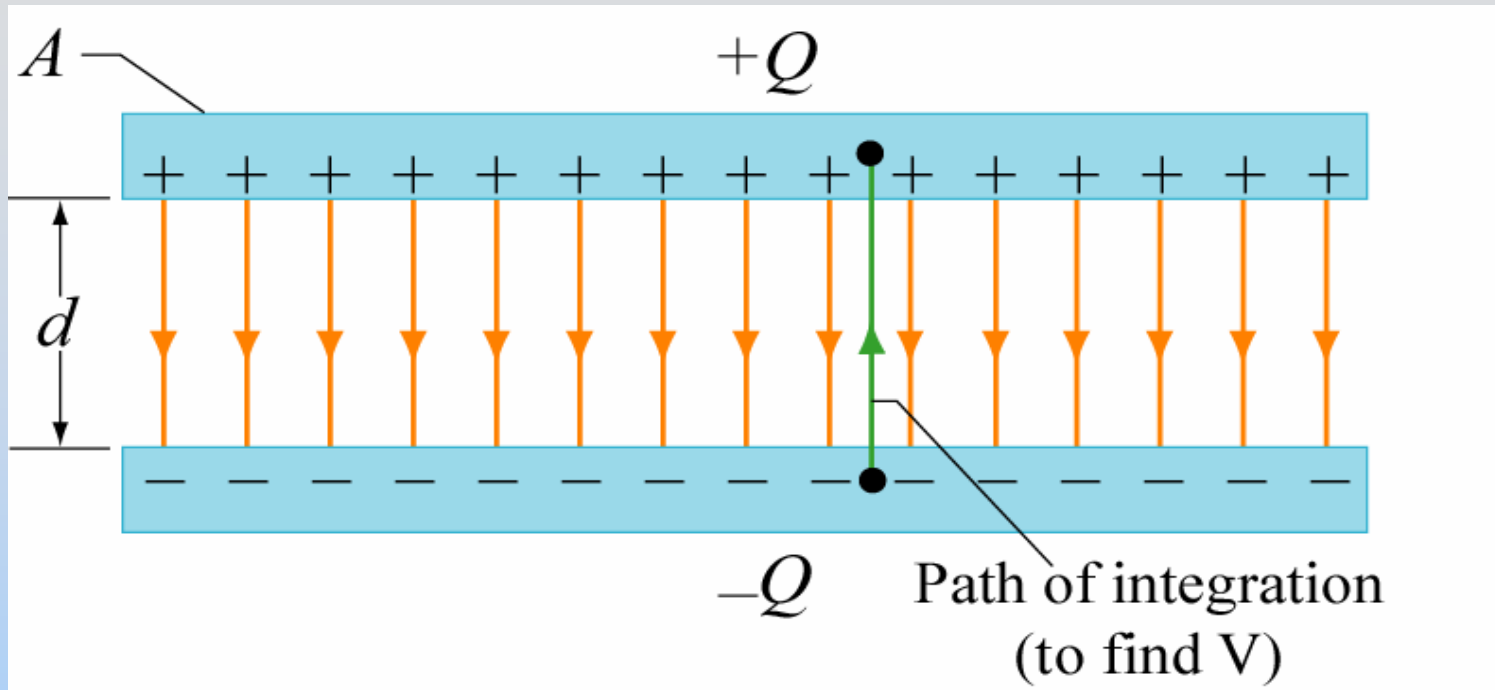
Note: We only “consider” a single sheet! Doesn't the other sheet matter?

Alternate Calculation Method



$$E = \frac{\sigma}{2\epsilon_0} + \frac{\sigma}{2\epsilon_0} = \frac{\sigma}{\epsilon_0} = \frac{Q}{A\epsilon_0}$$

Parallel Plate Capacitor



$$\Delta V = - \int_{bottom}^{top} \vec{\mathbf{E}} \cdot d\vec{\mathbf{S}} = Ed = \frac{Q}{A\epsilon_0} d$$

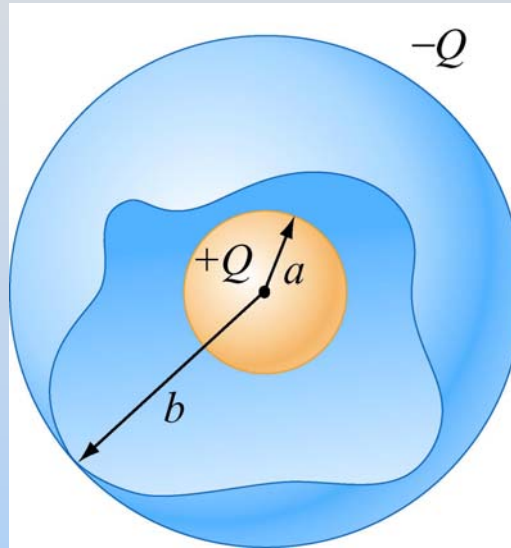
$$C = \frac{Q}{|\Delta V|} = \frac{\epsilon_0 A}{d}$$

C depends only on geometric factors A and d

Demonstration: Big Capacitor

Spherical Capacitor

Two concentric spherical shells of radii a and b



What is E ?

Gauss's Law $\rightarrow E \neq 0$ only for $a < r < b$,
where it looks like a point charge:

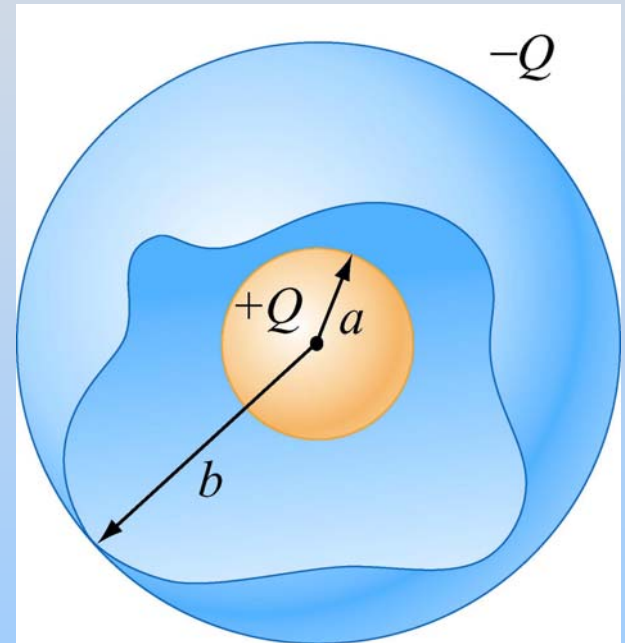
$$\vec{E} = \frac{Q}{4\pi\epsilon_0 r^2} \hat{r}$$

Spherical Capacitor

$$\Delta V = - \int_{\text{inside}}^{\text{outside}} \vec{\mathbf{E}} \cdot d\vec{\mathbf{S}} = - \int_a^b \frac{Q\hat{\mathbf{r}}}{4\pi\epsilon_0 r^2} \cdot dr \hat{\mathbf{r}} = \frac{Q}{4\pi\epsilon_0} \left(\frac{1}{b} - \frac{1}{a} \right)$$

Is this positive or negative? Why?

$$C = \frac{Q}{|\Delta V|} = \frac{4\pi\epsilon_0}{\left(a^{-1} - b^{-1}\right)}$$



For an isolated spherical conductor of radius a :

$$C = 4\pi\epsilon_0 a$$

Capacitance of Earth

For an isolated spherical conductor of radius a :

$$C = 4\pi\epsilon_0 a$$

$$\epsilon_0 = 8.85 \times 10^{-12} \text{ F/m} \quad a = 6.4 \times 10^6 \text{ m}$$

$$C = 7 \times 10^{-4} \text{ F} = 0.7 \text{ mF}$$

A Farad is REALLY BIG! We usually use pF (10^{-12}) or nF (10^{-9})

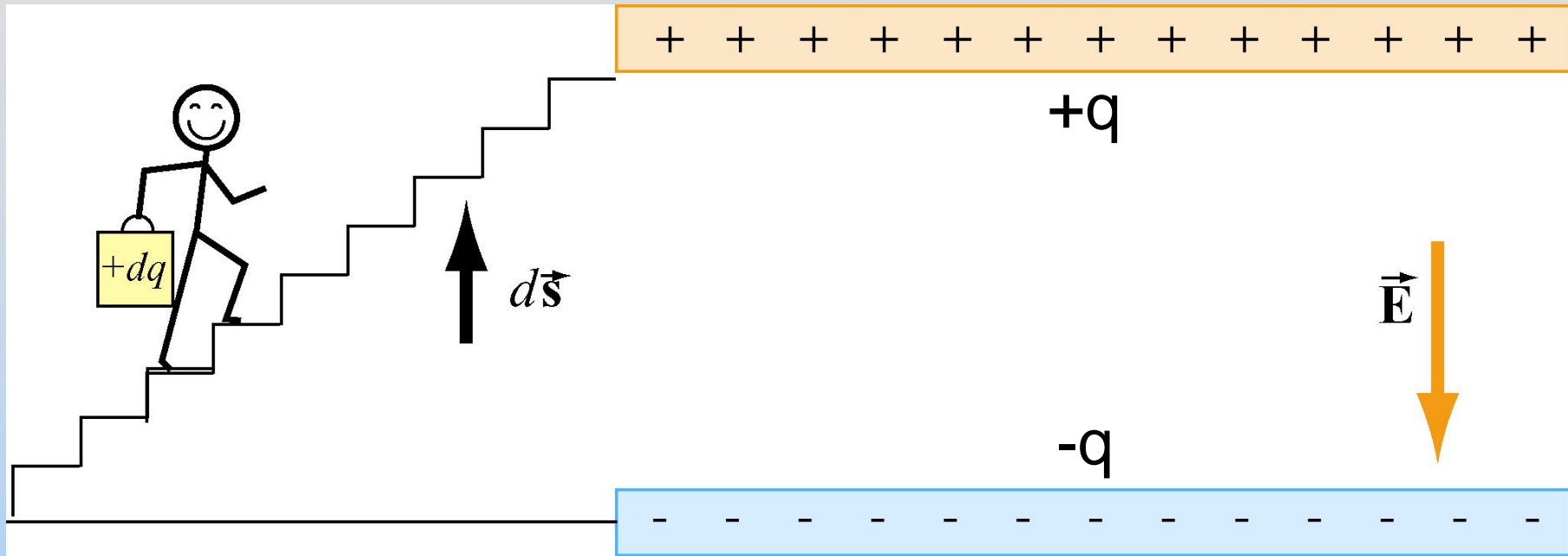
1 Farad Capacitor

How much charge?

$$\begin{aligned} Q &= C |\Delta V| \\ &= (1 \text{ F})(12 \text{ V}) \\ &= 12 \text{ C} \end{aligned}$$

Energy Stored in Capacitor

Energy To Charge Capacitor



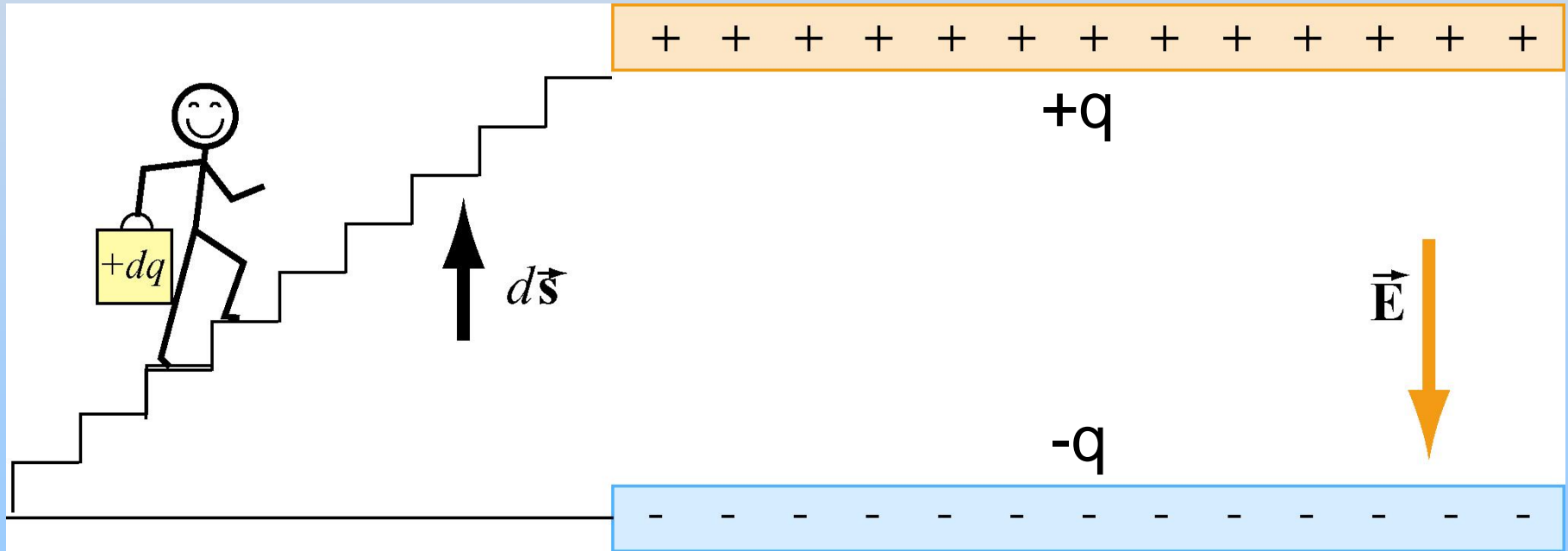
1. Capacitor starts uncharged.
2. Carry $+dq$ from bottom to top.
Now top has charge $q = +dq$, bottom $-dq$
3. Repeat
4. Finish when top has charge $q = +Q$, bottom $-Q$

Work Done Charging Capacitor

At some point top plate has $+q$, bottom has $-q$

Potential difference is $\Delta V = q / C$

Work done lifting another dq is $dW = dq \Delta V$



Work Done Charging Capacitor

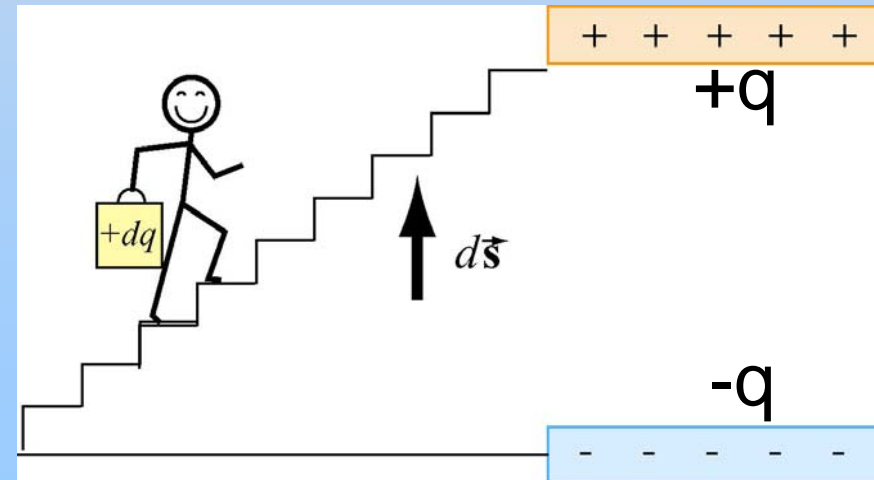
So work done to move dq is:

$$dW = dq \Delta V = dq \frac{q}{C} = \frac{1}{C} q dq$$

Total energy to charge to $q = Q$:

$$W = \int dW = \frac{1}{C} \int_0^Q q dq$$

$$= \frac{1}{C} \frac{Q^2}{2}$$



Energy Stored in Capacitor

$$\text{Since } C = \frac{Q}{|\Delta V|}$$

$$U = \frac{Q^2}{2C} = \frac{1}{2} Q |\Delta V| = \frac{1}{2} C |\Delta V|^2$$

Where is the energy stored???

Energy Stored in Capacitor

Energy stored in the E field!

Parallel-plate capacitor: $C = \frac{\epsilon_o A}{d}$ and $V = Ed$

$$U = \frac{1}{2} C V^2 = \frac{1}{2} \frac{\epsilon_o A}{d} (Ed)^2 = \frac{\epsilon_o E^2}{2} \times (Ad) = u_E \times (volume)$$

$$u_E = E \text{ field energy density} = \frac{\epsilon_o E^2}{2}$$

1 Farad Capacitor - Energy

How much energy?

$$\begin{aligned}U &= \frac{1}{2} C |\Delta V|^2 \\&= \frac{1}{2} (1 \text{ F}) (12 \text{ V})^2 \\&= 72 \text{ J}\end{aligned}$$

Compare to capacitor charged to 3kV:

$$\begin{aligned}U &= \frac{1}{2} C |\Delta V|^2 = \frac{1}{2} (100 \mu\text{F}) (3 \text{ kV})^2 \\&= \frac{1}{2} (1 \times 10^{-4} \text{ F}) (3 \times 10^3 \text{ V})^2 = 450 \text{ J}\end{aligned}$$