Massachusetts Institute of Technology *Spring Term 2005*

8.02X Electricity and Magnetism

Problem Set 4

Issued: Thu, Feb 24
Due: Fri, Mar 4 4PM <- note Date + Time!

Reading suggestions (from Young & Freedman)

Fri, 2/25 : Electric Potential, Capacitance:23-4,24-1
Mon, 2/28: Energy Storage in Capacitors, Dielectrics:24-3, 24-4
Wed, 3/2: Capacitors in Circuits:24-2
Fri, 3/4: Conductors and Insulators, EF Experiment 25-1

Homework Problems (30 points total)

Problem 1 (8 points) Two point-like charges $Q_1 = 1C$ and $Q_2 = -1C$ are separated by a distance of 1m. Suppose in an x-y coordinate system Q_1 sits at (-0.5m, 0) and Q_2 sits at (+0.5m,0).

(a) What is the force on charge Q_1 due to Q_2?
(b) Find the minimum of the x-component of the field between $-0.5\ m < x < 0.5m$. What is the magnitude of the field in units of [V/m]?.
(c) Draw graphs of the x-component and y-component of the total electric field $E_x(x,y)$ and $E_y(x,y)$ vs x between $-0.5m < x < 0.5m$ for y=0, y=-10cm, y=+10cm (the three curves for each component can be combined into one graph, if properly labeled).
(d) Sketch the electric field of this charge configuration using fieldlines. Does this sketch correspond to the graphs from (c)?

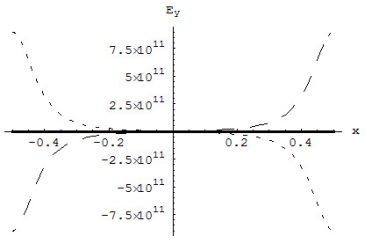
Problem 2 (8 points) Two point-like charges $Q_1 = 1C$ and $Q_2 = -2C$ sit at $x_1 = -0.5m$ and $x_2 = +0.5m$ along the x-axis of some coordinate system.

a. Draw a graph the electric potential due to Q_1, Q_2 separately and the total electric potential from x = -2m to x = +2m.
b. How could one approximate the total potential of Q_1+Q_2 for distances $x >> 1m$?
c. Draw a graph of the potential energy for a charge Q_3 of –0.1C in the potential created by Q_1 andQ_2 between $-2m < x < 2m$

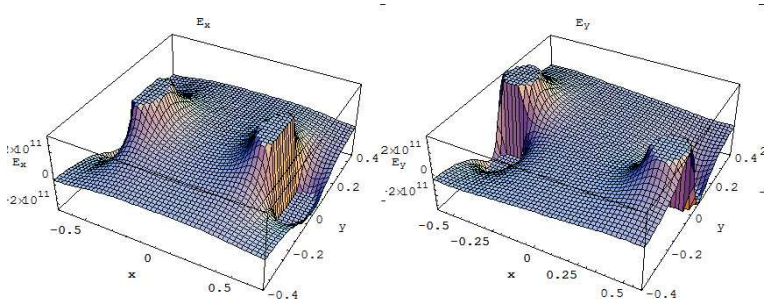
Problem 3 (6 points) Young&Freedman, Problem 24-60
Problem 4 (8 points) Young&Freedman, Problem 24-71

Note that check-off and experiment write-up (FROM BOTH PARTNERS) for experiment 'HVPS' are due on Fri, 3/4. HVPS questions have been provided in a separate document. There will be 2 bonus points for HVPS on 2/28 and 1 bonus point on 3/1.

And the E_y plots (dashed line: $y = -0.1$, thick line: $y = 0$, dotted line: $y = 0.1$):



Just as an extra, we also add a 3D plot of E_x and E_y...



d) For the plot of the field lines see the book, picture 21.26(b). From that we can see that the x-component at $y = 0$ goes to infinity at the point charges, and is smaller between them. For $y = \pm 0.1$, the x-component is almost zero at $x = \pm 0.5$, because the field lines are almost vertical. When moving away from the charges at small y, E_x quickly finds its maximum (dense field lines turning towards x) and then decreases, because the field lines become much less dense.

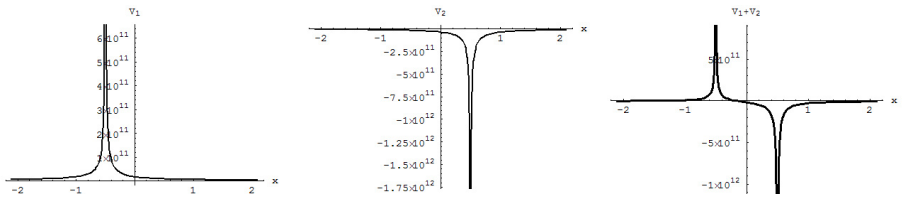
On the other hand, the E_y-component is zero for $y = 0$, nonzero for $y = \pm 10$, and changes sign at $x = 0$, which agrees with our plots.

Problem 2 (8 points)

a) We have two charges, $Q_1 = 1$ C and $Q_2 = -2$ C, positioned at x_1 and x_2. The potentials are

$$V_1 = k\frac{Q_1}{|x - x_1|}, \qquad V_2 = k\frac{Q_2}{|x - x_2|}. \tag{5}$$

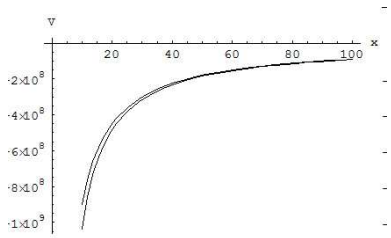
The plot of the potentials V_1, V_2 and both of them combined:



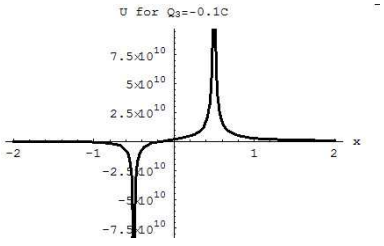
b) For distances $x \gg 1$m, the sum of the potentials can be roughly approximated by a potential from a single charge $Q_1 + Q_2 = -1$ C, located at $x = 0$. We can support our claim by a simple computation:

$$\begin{aligned} V &= V_1 + V_2 = k\frac{Q_1}{|x - x_1|} + k\frac{Q_2}{|x - x_2|} \approx \\ &\approx k\frac{Q_1 + Q_2}{|x|} = V_{\approx}. \end{aligned} \tag{6}$$

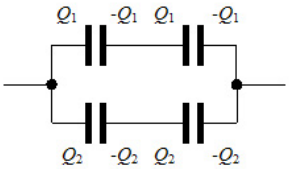
Just to check where this rough approximation makes sense, we plot V and $V_{\approx}$ for large x.



c) The potential energy for $Q_3 = -0.1$C in the potential of the two charges is $U_3 = Q_3(V_1 + V_2)$.



~~Y&F 24.60~~ ~~(6 points)~~



~~a) With the switch open, the charges on the capacitors distribute themselves as on Fig.3. The capacitance of the top line is the same as the capacitance of the bottom line (two capacitors in series) $C_{top} = 2\mu$F. From the symmetry we find that the charge will distribute itself evenly, $Q_1 = Q_2 = Q/2$. The total capacitance of the net of capacitors is $C = 4\mu$F. From~~

$$~~V_{ab} = \frac{Q}{C} = \frac{2Q_1}{C}~~$$