

~~8.02X Electricity and Magnetism~~

~~Problem Set 4~~

~~Issued: Thu, Feb 24~~

~~Due: Fri, Mar 4 4PM < note Date + Time!~~

~~Reading suggestions (from Young & Freedman)~~

~~Fri, 2/25 : Electric Potential, Capacitance:23-4,24-1~~

~~Mon, 2/28: Energy Storage in Capacitors, Dielectrics:24-3, 24-4~~

~~Wed, 3/2: Capacitors in Circuits:24-2~~

~~Fri, 3/4: Conductors and Insulators, EF Experiment 25-1~~

**Homework Problems (30 points total)**

**Problem 1 (8 points)** Two point-like charges  $Q_1 = 1\text{C}$  and  $Q_2 = -1\text{C}$  are separated by a distance of 1m. Suppose in an x-y coordinate system  $Q_1$  sits at  $(-0.5\text{m}, 0)$  and  $Q_2$  sits at  $(+0.5\text{m}, 0)$ .

- What is the force on charge  $Q_1$  due to  $Q_2$ ?
- Find the minimum of the x-component of the field between  $-0.5\text{ m} < x < 0.5\text{m}$ . What is the magnitude of the field in units of  $[\text{V/m}]$ ?
- Draw graphs of the x-component and y-component of the total electric field  $E_x(x,y)$  and  $E_y(x,y)$  vs  $x$  between  $-0.5\text{m} < x < 0.5\text{m}$  for  $y=0$ ,  $y=-10\text{cm}$ ,  $y=+10\text{cm}$  (the three curves for each component can be combined into one graph, if properly labeled).
- Sketch the electric field of this charge configuration using fieldlines. Does this sketch correspond to the graphs from (c)?

~~**Problem 2 (8 points)** Two point-like charges  $Q_1 = 1\text{C}$  and  $Q_2 = -2\text{C}$  sit at  $x_1 = -0.5\text{m}$  and  $x_2 = +0.5\text{m}$  along the x-axis of some coordinate system.~~

- ~~Draw a graph the electric potential due to  $Q_1$ ,  $Q_2$  separately and the total electric potential from  $x = -2\text{m}$  to  $x = +2\text{m}$ .~~
- ~~How could one approximate the total potential of  $Q_1+Q_2$  for distances  $x \gg 1\text{m}$ ?~~
- ~~Draw a graph of the potential energy for a charge  $Q_3$  of  $-0.1\text{C}$  in the potential created by  $Q_1$  and  $Q_2$  between  $-2\text{m} < x < 2\text{m}$~~

~~**Problem 3 (6 points)** Young&Freedman, Problem 24-60~~

~~**Problem 4 (8 points)** Young&Freedman, Problem 24-71~~

~~**Note that check-off and experiment write-up (FROM BOTH PARTNERS) for experiment 'HVPS' are due on Fri, 3/4. HVPS questions have been provided in a separate document. There will be 2 bonus points for HVPS on 2/28 and 1 bonus point on 3/1.**~~

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## 8.02x – Problem Set 4 Solutions

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### Problem 1 (8 points)

a) The force on charge  $Q_1$  due to charge  $Q_2$  is attractive, therefore pointing in the  $x+$  direction. The distance between the charges is 1m. The magnitude of the force is

$$F = k \frac{Q_1 Q_2}{d^2} \approx 9.0 \times 10^9 \text{N}. \quad (1)$$

b) The  $x$ -component of the electric field will be the smallest exactly in the middle between the charges. If we moved away from  $x = 0$ , the field from one of the charges would increase more than the amount the field from the other charge would decrease. Therefore it will be the smallest in the middle. It will point in the  $x+$  direction, with magnitude

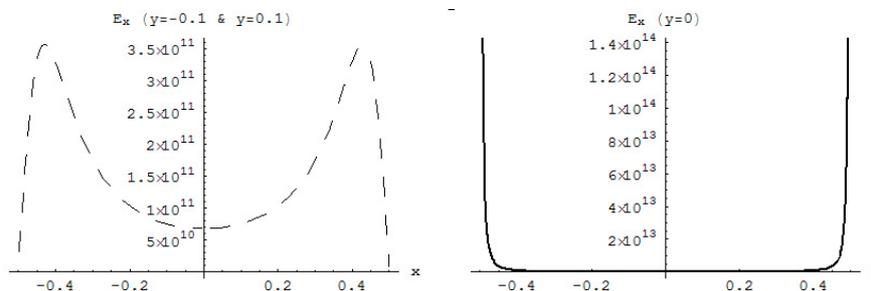
$$E = k \frac{|Q_1|}{(d/2)^2} + k \frac{|Q_2|}{(d/2)^2} = 8k \frac{Q_1}{d^2} \approx 72.0 \times 10^9 \text{V/m}. \quad (2)$$

c) We will plot the components of the electric field,  $E_x$  and  $E_y$  for  $y = -0.1$ ,  $y = 0$  and  $y = 0.1$ . The electric field has components

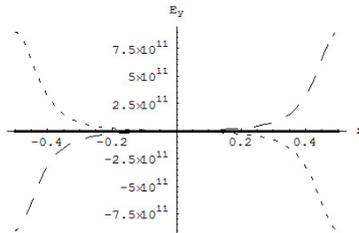
$$\begin{aligned} E_x &= E_x^{(1)} + E_x^{(2)} = & (3) \\ &= k \frac{Q_1}{(x-x_1)^2 + y^2} \frac{x-x_1}{\sqrt{(x-x_1)^2 + y^2}} + k \frac{|Q_2|}{(x-x_2)^2 + y^2} \frac{x-x_2}{\sqrt{(x-x_2)^2 + y^2}}, \end{aligned}$$

$$\begin{aligned} E_y &= E_y^{(1)} + E_y^{(2)} = & (4) \\ &= k \frac{Q_1}{(x-x_1)^2 + y^2} \frac{y}{\sqrt{(x-x_1)^2 + y^2}} - k \frac{|Q_2|}{(x-x_2)^2 + y^2} \frac{y}{\sqrt{(x-x_2)^2 + y^2}}, \end{aligned}$$

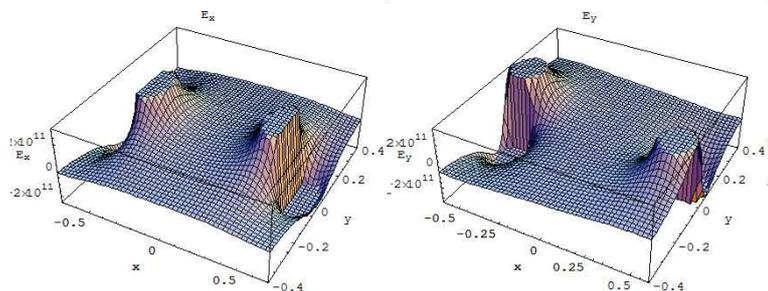
First the  $E_x$  component plots (dashed line:  $y = -0.1$  and  $y = 0.1$ , thick line:  $y = 0$ ):



And the  $E_y$  plots (dashed line:  $y = -0.1$ , thick line:  $y = 0$ , dotted line:  $y = 0.1$ ):



Just as an extra, we also add a 3D plot of  $E_x$  and  $E_y$ ...



d) For the plot of the field lines see the book, picture 21.26(b). From that we can see that the  $x$ -component at  $y = 0$  goes to infinity at the point charges, and is smaller between them. For  $y = \pm 0.1$ , the  $x$ -component is almost zero at  $x = \pm 0.5$ , because the field lines are almost vertical. When moving away from the charges at small  $y$ ,  $E_x$  quickly finds its maximum (dense field lines turning towards  $x$ ) and then decreases, because the field lines become much less dense.

On the other hand, the  $E_y$ -component is zero for  $y = 0$ , nonzero for  $y = \pm 0.1$ , and changes sign at  $x = 0$ , which agrees with our plots.

**Problem 2** (8 points)

~~a) We have two charges,  $Q_1 = 1\text{ C}$  and  $Q_2 = -2\text{ C}$ , positioned at  $x_1$  and  $x_2$ . The potentials are~~

~~$$V_1 = k \frac{Q_1}{|x - x_1|}, \quad V_2 = k \frac{Q_2}{|x - x_2|}. \quad (5)$$~~

~~The plot of the potentials  $V_1$ ,  $V_2$  and both of them combined:~~

