

Problem 2.5

~~Electric potential of flat ring with hole in its center.~~

~~Giancoli 23-78. Assume that the charge Q is uniformly distributed.~~

Problem 2.6

Electric breakdown fields. This problem may help you qualitatively to understand why the observed value of the breakdown field in air (1 atm) is of the order of 10^6 V/m. Recall from the lectures that it takes about 10 eV to ionize an air molecule.

Under standard conditions of temperature and pressure, dry air will break down (i.e., lose its insulating quality), and sparking will result when the electric field exceeds about 3×10^6 V/m.

- (a) Suppose that we wish to maintain an isolated metallic sphere at -4000 V. What is the minimum radius of the sphere for which there is no sparking?
- (b) At this minimum radius, what is the total charge on the sphere?
- (c) Now consider a free electron in the air, very near the charged sphere. If the electron is initially at rest, what radial distance will it travel before reaching a kinetic energy of 10 eV? Assume that no collision with the air molecules takes place during this interval of travel.
- (d) Compare your answer in (c) with the typical electron mean-free path in air which is about 10^{-6} m.
Note: the mean free path is the distance that a particle travels on average between two successive collisions; it is an experimentally measurable quantity.

Problem 2.7

~~5.2 keV protons.~~

~~Giancoli 23-72.~~

Problem 2.8

~~Dropping charged objects in the Earth's electric field.~~

~~Giancoli 23-74.~~

Recitations.

~~There are 28 recitation sections (see the 8.02 Website). If for any reason you want to change section, please see Maria Springer in 4352.~~

Problem 2.6*Electric breakdown fields.*

(a) Let a be the radius of our sphere. We know from our previous dealings with charged conducting spheres that just outside the surface the electric field is given by

$$E(r = a) = \frac{Q}{4\pi\epsilon_0 a^2}$$

and the potential (relative to ∞) at its surface is

$$V(r = a) = \frac{Q}{4\pi\epsilon_0 a} .$$

From these two equations, we easily deduce the relation between E and V at $r = a$ to be

$$E(r = a) = \frac{V(r = a)}{a} .$$

For a fixed voltage of -4000 V, E on the surface becomes larger and larger as a becomes smaller and smaller. For E to have a magnitude less than 3×10^6 V/m, we need

$$\begin{aligned} a &> \frac{4000}{3 \times 10^6} = 1.33 \times 10^{-3} \text{ m} \\ \Rightarrow a &> 1.33 \text{ mm for no breakdown.} \end{aligned}$$

(b) From above, $Q = 4\pi\epsilon_0 aV$, or for our numbers, $Q = -6 \times 10^{-10}$ Coulomb.

(c) An electron in the air just outside the sphere will be accelerated radially away from the sphere by the electric field. If it were moving in a constant electric field of magnitude 3×10^6 V/m, it would pick up an energy of 10 eV in a distance l satisfying

$$e(3 \times 10^6)l = (10)e .$$

This equation is simply force (qE) times distance is equal to energy, where we have used the fact that 1 eV (in Joules) is the charge of an electron (in Coulombs) times 1 Volt. Solving the above equation for l gives

$$l = \frac{10}{3 \times 10^6} = 3.3 \times 10^{-6} \text{ m} .$$

Now in fact the E field is not constant outside the sphere: it falls off as $1/r^2$, of course. But in 3.3×10^{-6} m it falls off only a very tiny amount, so that the assumption of a constant field over the distance l is a very good one.

(d) We may conclude from the result of part (c) that an electron can gain 10 eV within three mean free paths or so, if one mean free path is 10^{-6} m.

We can understand “breakdown” as follows. The energy required to ionize an air molecule is ~ 10 eV. If $E \ll 3 \times 10^6$ V/m, a “stray” electron in the air will pick up energy between collisions, but not enough to ionize an air molecule when it collides. If $E > 3 \times 10^6$ V/m, the “stray” electron will pick up enough energy to ionize an air molecule. This gives us two free electrons which will then become 4, and then 8, etc. This cascade, or chain reaction, will eventually reach macroscopic scales as a spark, and the sphere will partially discharge through the air.

Problem 2.7

5.2 keV protons. (Giancoli 23-72.)

(a) Denote the electric field acting upon the particles by $\mathbf{E}(\mathbf{r})$. The force exerted on the proton at any position \mathbf{r} will be $e\mathbf{E}(\mathbf{r})$ (here, $e > 0$ is the magnitude of the electron or proton charge). The energy acquired by the proton during its trip from P to Q is thus

$$(KE)_p = \int_P^Q e\mathbf{E}(\mathbf{r}) \cdot d\mathbf{l} = 5.2 \text{ keV} \quad .$$

The force on the electron at any given \mathbf{r} will be $-e\mathbf{E}(\mathbf{r})$, and hence the energy it would gain in traveling from Q to P is

$$(KE)_e = \int_Q^P [-e\mathbf{E}(\mathbf{r})] \cdot d\mathbf{l} = \int_P^Q [-e\mathbf{E}(\mathbf{r})] \cdot (-d\mathbf{l}) = \int_P^Q e\mathbf{E}(\mathbf{r}) \cdot d\mathbf{l} \quad .$$

So we see that

$$(KE)_e = (KE)_p = 5.2 \text{ keV} \quad .$$

(b) We can find the ratio of particle speeds from

$$\begin{aligned} 1 &= \frac{(KE)_e}{(KE)_p} = \frac{m_e v_e^2/2}{m_p v_p^2/2} \\ \Rightarrow \frac{v_e}{v_p} &= \sqrt{\frac{m_p}{m_e}} = \sqrt{\frac{1.67 \times 10^{-27}}{9.11 \times 10^{-31}}} \simeq 43 \quad . \end{aligned}$$