

Do yourself a favor and prepare for lectures!

We recommend strongly that you read about the topics before they are covered in lectures. Reading through the new concepts (for half an hour or so) before the lectures would be a great help to you! Thorough reading and studying the material, of course, is needed before you start the problems.

Lecture Date	Topics Covered	Reading from Giancoli
#1 Wed 2/6	What holds our world together? Electric charges (historical) - Polarization Electric Force - Coulomb's Law	Chapter 21 through Sect. 21-5
#2 Fri 2/8	Electric Field - Field Lines - Superposition Inductive Charging - Dipoles - Induced Dipoles	Sect. 21-6 through 21-11
#3 Mon 2/11	Electric Flux - Gauss's Law - Examples	All of Chapter 22
#4 Wed 2/13	Electrostatic Potential - Electric Energy - eV Conservative Field - Equipotential Surfaces	Chapter 23 through Sect. 23-8
#5 Fri 2/15	$\vec{E} = -\text{grad } V$ More on Equipotential Surfaces - Conductors Electrostatic shielding (Faraday cage)	Sect. 23-5 & 23-7

Due before 4 PM, Friday, February 15 in 4-339B.

Problem 1.1

Relative strengths of Gravitational and Electrostatic forces.

The gravitational force between two concentrated ("point-like") masses is very similar in its mathematical structure to the electrostatic force between two concentrated charges. The "strength" of these two forces is, however, vastly different. To illustrate this, consider the following example. Somewhere in outer space are two identical spherical dust grains, $50\text{ }\mu\text{m}$ in diameter, with mass density 2.5 g/cm^3 . They are at a distance d meters apart. If the grains were electrically neutral, free of other external forces, and have negligible relative velocity initially, they would eventually collide gravitationally.

Now suppose that both grains are electrically charged, each having n "extra" electrons. Find the minimum value of n that would prevent the gravitational collision. Compare this with the approximate total number of electrons contained in one grain.

$1\text{ }\mu\text{m} = 10^{-6}\text{ m}$; for other necessary information, see the inside of the cover of your book. Recall that the total mass of matter is almost entirely due to the neutrons and protons (roughly of equal mass). Electrically neutral material contains an equal number of protons and electrons. Assume that in the grains the number of protons equals the number of neutrons.

Problem 1.2

Electric field along the line passing through two point charges.

A point charge $Q_1 = +3\text{ }\mu\text{C}$ is placed at the origin, and a point charge $Q_2 = -7\text{ }\mu\text{C}$ is placed at $x = 0.4\text{ m}$ on the x -axis of a cartesian coordinate system.

- (a) Determine the electric field, $\vec{E}(x) = E(x)\hat{x}$, at all points along the x -axis.

- (b) Plot $E(x)$ vs. x for $x < 0$.
- (c) At what points, if any, (apart from $|x| = \infty$), is $E(x) = 0$?

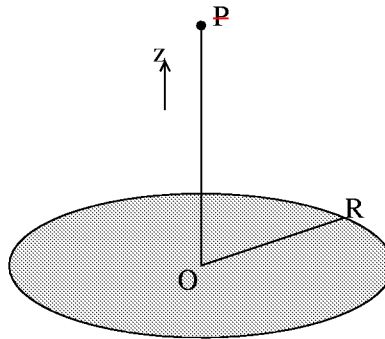
Problem 1.3

~~Continuous charge distribution.~~
Giancoli 21-49.

Problem 1.4

~~E -field of a uniformly charged disk.~~

~~A disk of radius R carries a uniform surface charge density σ (see sketch). The z axis passes through the center O . The total charge, Q , on the disk is thus $Q = \pi R^2 \sigma$.~~



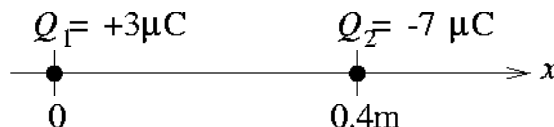
- (a) ~~What is the electric field \vec{E} (magnitude and direction) at a point P a distance z above the center of the disk? The procedure to follow is to sum (integrate) the contributions of all infinitesimal charges $dQ = \sigma dA$ of the surface elements dA on the disk. The integration is relatively easy if you introduce rings with radii r (centered on O) and width dr , and then introduce a new variable $s = \sqrt{r^2 + z^2}$, and observe that $rdr = sds$. Express your answer for $E(z)$ in terms of Q , R , ϵ_0 and z .~~
- (b) ~~Plot $E(z)$ as a function of z for all positive z 's. Use R as your unit on the abscissa; use $Q/(4\pi\epsilon_0 R^2)$ as your unit for $E(z)$.~~
- (c) ~~Using the binomial expansion (see Appendix A in Giancoli) for $\sqrt{z^2 + R^2}$, find simplified expressions for $E(z)$ in two limiting cases:~~
- ~~(i) $z^2 \ll R^2$~~
 - ~~(ii) $z^2 \gg R^2$.~~
- ~~In your calculations retain only the first non-vanishing term of your expansion.~~
- (d) ~~Compare your result in case (ii) with the result you can obtain “effortlessly” by making use of Coulomb’s Law for a point-like charge.~~
- (e) ~~Using Gauss’s law (choose a proper “pillbox”), calculate $E(z)$ near point O for case (i); compare your answer with (c).~~

~~**Gauss’s Law** is a powerful tool in qualitative studies of the electric field for some surmised charge distributions (for example, for surface charge distributions on metals). We will see such an example in the next assignment. Gauss’s Law leads also very quickly to quantitative results if the charge distributions exhibit simple symmetry (spheres, cylinders & large planes) problems 1.4(e), 1.6, 1.7 & 1.8 are classic examples of this.~~

Problem 1.2

Electric field along the line passing through two point charges.

(Note: In these solutions, as in the text, we will always denote vector quantities such as the electric field by boldface font: \mathbf{E} . In lectures, and probably in your own handwritten work, vectors are more commonly written with an arrow over the top: \vec{E} .)



(a) Let us first determine the electric field due to Q_1 alone. From the expression for the electric field due to a point charge,

$$\mathbf{E}_1 = \frac{1}{4\pi\epsilon_0} \frac{Q_1}{r^2} \hat{\mathbf{r}} \quad .$$

If we are at position x on the x -axis, $\mathbf{r} = x\hat{x}$, $r = \sqrt{x^2} = |x|$, and $\hat{\mathbf{r}} = \mathbf{r}/|\mathbf{r}| = (x/|x|)\hat{x}$, and

$$\begin{aligned} \mathbf{E}_1 &= \frac{Q_1}{4\pi\epsilon_0} \frac{1}{|x|^2} \frac{x}{|x|} \hat{x} \\ &= \frac{Q_1}{4\pi\epsilon_0} \frac{x}{|x|^3} \hat{x} \quad . \end{aligned}$$

Note that \mathbf{E}_1 is in the \hat{x} direction for $x > 0$ and in the $-\hat{x}$ direction for $x < 0$, as we would expect, since the charge is located at $x = 0$. Similarly,

$$\mathbf{E}_2 = \frac{Q_2}{4\pi\epsilon_0} \frac{x - 0.4}{|x - 0.4|^3} \hat{x}$$

(variables always measured in SI units unless otherwise noted). Note that \mathbf{E}_2 changes direction depending on whether we are to the left or right of the charge at $x = 0.4$ m. The total \mathbf{E} is the sum of \mathbf{E}_1 and \mathbf{E}_2 :

$$\mathbf{E} = \frac{10^{-6}}{4\pi\epsilon_0} \left[3 \frac{x}{|x|^3} - 7 \frac{x - 0.4}{|x - 0.4|^3} \right] \hat{x} \quad .$$

This expression is good for $-\infty < x < \infty$, but it is worthwhile writing it out as follows:

$$\mathbf{E} = \frac{10^{-6}}{4\pi\epsilon_0} \begin{cases} (-3/x^2 + 7/(x - 0.4)^2) \hat{x} & -\infty < x < 0 \\ (+3/x^2 + 7/(x - 0.4)^2) \hat{x} & 0 < x < 0.4 \\ (+3/x^2 - 7/(x - 0.4)^2) \hat{x} & 0.4 < x < \infty \end{cases} \quad .$$

Note the sign changes as one passes over the positions of the two charges and the field due to that charge reverses.

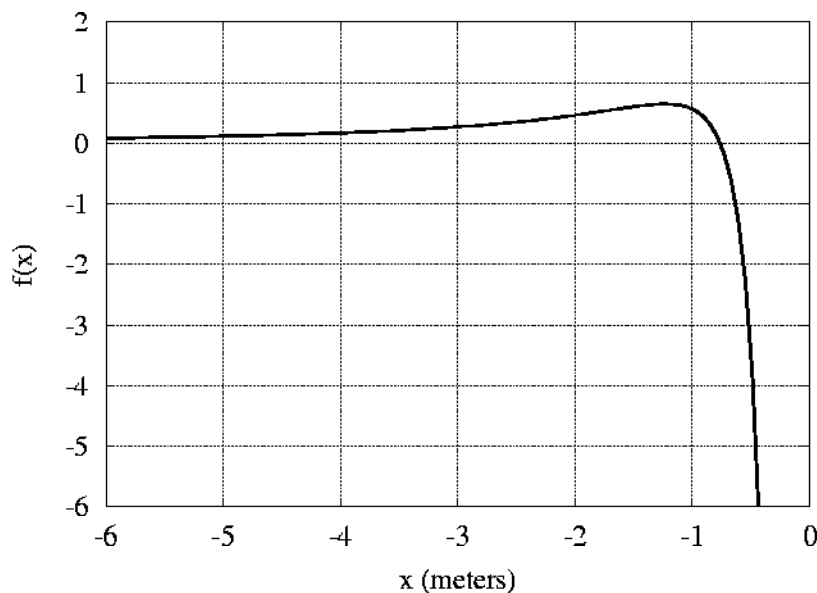
(b) For $x < 0$,

$$\mathbf{E}(x) = \hat{x} \frac{10^{-6}}{4\pi\epsilon_0} f(x) \quad ,$$

where

$$f(x) = -\frac{3}{x^2} + \frac{7}{(x - 0.4)^2} \quad .$$

(See graph at right.)



This function has a zero at $x = -0.758$ m, and a maximum at $x = -1.225$ m, with a value there of 0.652 m^{-2} . As we move toward $x = 0$, $f(x)$ blows up to $-\infty$, because of the positive charge at $x = 0$. As we move toward $x = -\infty$, $f(x)$ goes asymptotically to $+4/x^2$, which simply means that far enough away ($|x| \gg 0.4$) the field looks like that due to a point charge of charge $Q_1 + Q_2 = -4 \mu\text{C}$.

(c) There are no other zeros of $\mathbf{E}(x)$ except at $x = -0.758$ m. A glance at our expression for $\mathbf{E}(x)$ above clearly shows that there can be no zeros in the range $0 < x < 0.4$ (sum of two positive terms can never be zero). You might think that we could get a zero in the range $0.4 < x < \infty$, but the negative charge (which is of greater magnitude than the positive charge) is closer throughout this range, and the electric field is therefore always in the $-\hat{x}$ direction. Solving for the zeros of the expression for $\mathbf{E}(x)$ in the $0.4 < x < \infty$ range will give zeros that are outside of that range and therefore unphysical.
