

3.8 Solved Problems

3.8.1 Electric Potential Due to a System of Two Charges

Consider a system of two charges shown in Figure 3.8.1.

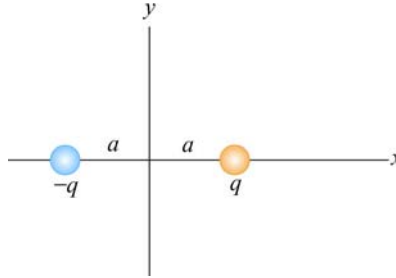


Figure 3.8.1 Electric dipole

Find the electric potential at an arbitrary point on the x axis and make a plot.

Solution:

The electric potential can be found by the superposition principle. At a point on the x axis, we have

$$V(x) = \frac{1}{4\pi\epsilon_0} \frac{q}{|x-a|} + \frac{1}{4\pi\epsilon_0} \frac{(-q)}{|x+a|} = \frac{q}{4\pi\epsilon_0} \left[\frac{1}{|x-a|} - \frac{1}{|x+a|} \right]$$

The above expression may be rewritten as

$$\frac{V(x)}{V_0} = \frac{1}{|x/a - 1|} - \frac{1}{|x/a + 1|}$$

where $V_0 = q/4\pi\epsilon_0 a$. The plot of the dimensionless electric potential as a function of x/a is depicted in Figure 3.8.2.

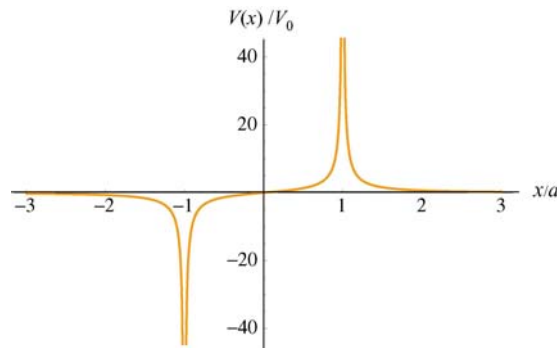


Figure 3.8.2

As can be seen from the graph, $V(x)$ diverges at $x/a = \pm 1$, where the charges are located.

3.8.2 Electric Dipole Potential

Consider an electric dipole along the y-axis, as shown in the Figure 3.8.3. Find the electric potential V at a point P in the x-y plane, and use V to derive the corresponding electric field.

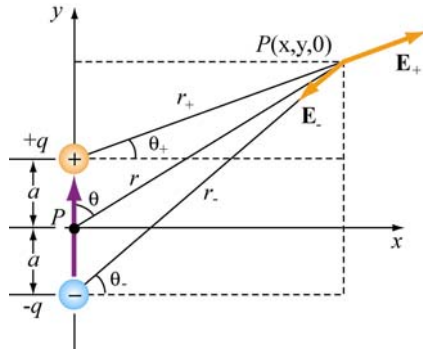


Figure 3.8.3

By superposition principle, the potential at P is given by

$$V = \sum_i V_i = \frac{1}{4\pi\epsilon_0} \left(\frac{q}{r_+} - \frac{q}{r_-} \right)$$

where $r_{\pm}^2 = r^2 + a^2 \mp 2ra \cos \theta$. If we take the limit where $r \gg a$, then

$$\frac{1}{r_{\pm}} = \frac{1}{r} \left[1 + (a/r)^2 \mp 2(a/r) \cos \theta \right]^{-1/2} = \frac{1}{r} \left[1 - \frac{1}{2} (a/r)^2 \pm (a/r) \cos \theta + \dots \right]$$

and the dipole potential can be approximated as

$$\begin{aligned} V &= \frac{q}{4\pi\epsilon_0 r} \left[1 - \frac{1}{2} (a/r)^2 + (a/r) \cos \theta - 1 + \frac{1}{2} (a/r)^2 + (a/r) \cos \theta + \dots \right] \\ &\approx \frac{q}{4\pi\epsilon_0 r} \cdot \frac{2a \cos \theta}{r} = \frac{p \cos \theta}{4\pi\epsilon_0 r^2} = \frac{\vec{p} \cdot \hat{r}}{4\pi\epsilon_0 r^2} \end{aligned}$$

where $\vec{p} = 2aq \hat{j}$ is the electric dipole moment. In spherical polar coordinates, the gradient operator is

$$\vec{\nabla} = \frac{\partial}{\partial r} \hat{r} + \frac{1}{r} \frac{\partial}{\partial \theta} \hat{\theta} + \frac{1}{r \sin \theta} \frac{\partial}{\partial \phi} \hat{\phi}$$

$$V(z = 3d) = \frac{\lambda}{4\pi\epsilon_0} \ln\left(\frac{3d+d}{3d-d}\right) = \frac{\lambda}{4\pi\epsilon_0} \ln 2$$

The electric potential difference between the two points is

$$\Delta V = V(z = 3d) - V(z = 4d) = \frac{\lambda}{4\pi\epsilon_0} \ln\left(\frac{6}{5}\right) > 0$$

Using the fact that the electric potential difference ΔV is equal to the change in potential energy per unit charge, we have

$$\Delta U = q\Delta V = -\frac{|e|\lambda}{4\pi\epsilon_0} \ln\left(\frac{6}{5}\right) < 0$$

where $q = -|e|$ is the charge of the electron.

(c) If the electron starts out at rest at $z = 4d$ then the change in kinetic energy is

$$\Delta K = \frac{1}{2}mv_f^2$$

By conservation of energy, the change in kinetic energy is

$$\Delta K = -\Delta U = \frac{|e|\lambda}{4\pi\epsilon_0} \ln\left(\frac{6}{5}\right) > 0$$

Thus, the magnitude of the velocity at $z = 3d$ is

$$v_f = \sqrt{\frac{2|e|\lambda}{4\pi\epsilon_0 m} \ln\left(\frac{6}{5}\right)}$$

3.9 Conceptual Questions

1. What is the difference between electric potential and electric potential energy?
2. A uniform electric field is parallel to the x -axis. In what direction can a charge be displaced in this field without any external work being done on the charge?
3. Is it safe to stay in an automobile with a metal body during severe thunderstorm? Explain.

4. Why are equipotential surfaces always perpendicular to electric field lines?
5. The electric field inside a hollow, uniformly charged sphere is zero. Does this imply that the potential is zero inside the sphere?

3.10 Additional Problems

3.10.1 Cube

How much work is done to assemble eight identical point charges, each of magnitude q , at the corners of a cube of side a ?

3.10.2 Three Charges

Three charges with $q = 3.00 \times 10^{-18}$ C and $q_1 = 6 \times 10^{-6}$ C are placed on the x -axis, as shown in the figure 3.10.1. The distance between q and q_1 is $a = 0.600$ m.

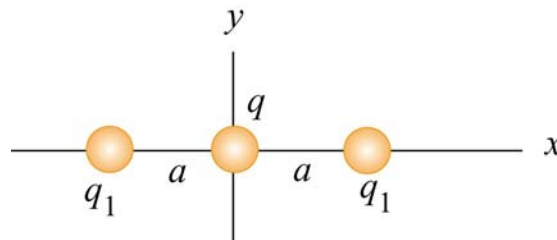


Figure 3.10.1

- (a) What is the net force exerted on q by the other two charges q_1 ?
- (b) What is the electric field at the origin due to the two charges q_1 ?
- (c) What is the electric potential at the origin due to the two charges q_1 ?

3.10.3 Work Done on Charges

Two charges $q_1 = 3.0 \mu\text{C}$ and $q_2 = -4.0 \mu\text{C}$ initially are separated by a distance $r_0 = 2.0$ cm. An external agent moves the charges until they are $r_f = 5.0$ cm apart.

- (a) How much work is done by the *electric field* in moving the charges from r_0 to r_f ? Is the work positive or negative?
- (b) How much work is done by the *external agent* in moving the charges from r_0 to r_f ? Is the work positive or negative?

- (c) What is the potential energy of the initial state where the charges are $r_0 = 2.0\text{ cm}$ apart?
- (d) What is the potential energy of the final state where the charges are $r_f = 5.0\text{ cm}$ apart?
- (e) What is the change in potential energy from the initial state to the final state?

3.10.4 Calculating E from V

Suppose in some region of space the electric potential is given by

$$V(x, y, z) = V_0 - E_0 z + \frac{E_0 a^3 z}{(x^2 + y^2 + z^2)^{3/2}}$$

where a is a constant with dimensions of length. Find the x , y , and the z -components of the associated electric field.

3.10.5 Electric Potential of a Rod

A rod of length L lies along the x -axis with its left end at the origin and has a non-uniform charge density $\lambda = \alpha x$, where α is a positive constant.

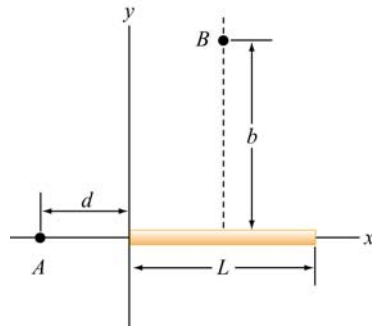


Figure 3.10.2

- (a) What are the dimensions of α ?
- (b) Calculate the electric potential at A .
- (c) Calculate the electric potential at point B that lies along the perpendicular bisector of the rod a distance b above the x -axis.

3.10.6 Electric Potential

Suppose that the electric potential in some region of space is given by

$$V(x, y, z) = V_0 \exp(-k |z|) \cos kx.$$

Find the electric field everywhere. Sketch the electric field lines in the $x - z$ plane.

3.10.7 Calculating Electric Field from the Electric Potential

Suppose that the electric potential varies along the x -axis as shown in Figure 3.10.3 below.

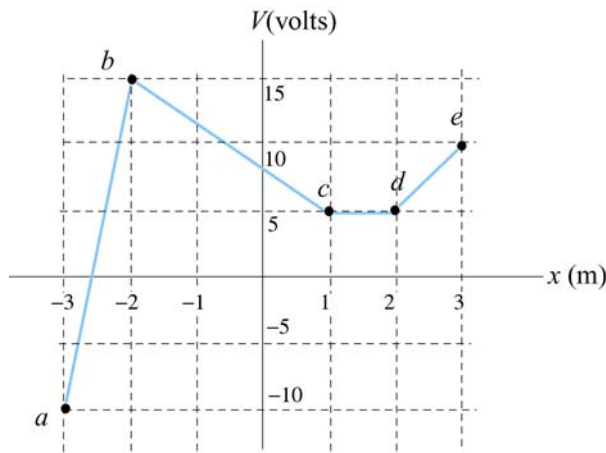


Figure 3.10.3

The potential does not vary in the y - or z -direction. Of the intervals shown (ignore the behavior at the end points of the intervals), determine the intervals in which E_x has

- (a) its greatest absolute value. [Ans: 25 V/m in interval ab .]
- (b) its least. [Ans: (b) 0 V/m in interval cd .]
- (c) Plot E_x as a function of x .
- (d) What sort of charge distributions would produce these kinds of changes in the potential? Where are they located? [Ans: sheets of charge extending in the yz direction located at points b , c , d , etc. along the x -axis. Note again that a sheet of charge with charge per unit area σ will *always* produce a jump in the normal component of the electric field of magnitude σ/ϵ_0 .]

3.10.8 Electric Potential and Electric Potential Energy

A right isosceles triangle of side a has charges q , $+2q$ and $-q$ arranged on its vertices, as shown in Figure 3.10.4.

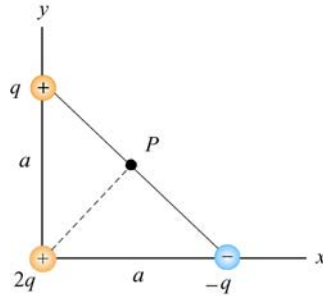


Figure 3.10.4

- What is the electric potential at point P , midway between the line connecting the $+q$ and $-q$ charges, assuming that $V = 0$ at infinity? [Ans: $q/\sqrt{2} \pi \epsilon_0 a$.]
- What is the potential energy U of this configuration of three charges? What is the significance of the sign of your answer? [Ans: $-q^2/4\sqrt{2} \pi \epsilon_0 a$, the negative sign means that work was done on the agent who assembled these charges in moving them in from infinity.]
- A fourth charge with charge $+3q$ is slowly moved in from infinity to point P . How much work must be done in this process? What is the significance of the sign of your answer? [Ans: $+3q^2/\sqrt{2} \pi \epsilon_0 a$, the positive sign means that work was done by the agent who moved this charge in from infinity.]

3.10.9. Electric Field, Potential and Energy

Three charges, $+5Q$, $-5Q$, and $+3Q$ are located on the y -axis at $y = +4a$, $y = 0$, and $y = -4a$, respectively. The point P is on the x -axis at $x = 3a$.

- How much energy did it take to assemble these charges?
- What are the x , y , and z components of the electric field \vec{E} at P ?
- What is the electric potential V at point P , taking $V = 0$ at infinity?
- A fourth charge of $+Q$ is brought to P from infinity. What are the x , y , and z components of the force \vec{F} that is exerted on it by the other three charges?
- How much work was done (by the external agent) in moving the fourth charge $+Q$ from infinity to P ?