

10.4 *Archimede's Principle and Oscillations*

A cylindrical wooden rod is weighted at one end so that it floats upright in water. The length of the submerged portion of the rod is 3 m. The rod is displaced vertically from its equilibrium position and released. What is the period of oscillation? If the atmospheric pressure increases by 5%, by what fraction will the period change?

~~10.5 *Ultimate Tensile Strength* page 378, problem 63~~

~~10.6 *Beam Dump - Heat Power* page 537, problem 13~~

~~10.7 *Density of Atoms* page 512, problem 17~~

~~10.8 *Diving Bell - Ideal gas Law* page 513, problem 24~~

~~10.9 *Scuba Diving* page 512, problem 16~~

~~10.10 *High Altitude Balloons* page 512, problem 21~~

~~10.11 *The Earth Atmosphere* page 513, problem 28~~

~~10.12 *Oil Pipeline* page 535, problem 5~~

~~$x = -0.07$ m at which point the rope will start to go slack and there will be no restoring force, only the force of gravity. The motion will no longer be completely simple harmonic motion.~~

~~(d) The ultimate tensile strength is given by Table 14.1 on page 366 as $\left(\frac{F}{A}\right)_{\max} = 3.2 \times 10^8$ N/m². The maximum mass will then be~~

$$\del{M_{\max} = \frac{A}{g} \left(\frac{F}{A}\right)_{\max} \approx 2.6 \times 10^3 \text{ kg}}$$

~~You can only calculate a conservative lower limit to the amount by which the rope will stretch. As we have seen in lectures, Hooke's law (thus $\frac{\Delta L}{L} = \frac{F}{YA}$) no longer holds near the breaking point. The lower limit would be~~

$$\del{\Delta L = \frac{L}{Y} \left(\frac{F}{A}\right)_{\max} \approx 0.44 \text{ m}}$$

~~Thus a conservative lower limit for the length of the rope is 5.44 m.~~

Problem 10.3

~~Let $\rho = 1$ gm/cm³ be the density of water, ρ_w be the density of the wood, ρ_o be the density of the oil, and V be the volume of wood. For the water,~~

$$\del{\rho \cdot \frac{2}{3}V = \rho_w \cdot V \quad \implies \quad \rho_w = \frac{2}{3} \cdot \rho \approx 0.67 \text{ gm/cm}^3}$$

~~For the oil,~~

$$\del{\rho_o \cdot 0.9 \cdot V = \rho_w \cdot V \quad \implies \quad \rho_o = \frac{1}{0.9} \cdot \rho_w \approx 0.74 \text{ gm/cm}^3}$$

Problem 10.4

Let p_0 be atmospheric pressure, $\rho = 10^3$ kg/m³ the density of water, M the mass of the rod, A its cross-section area, $d = 3$ m its equilibrium depth, and x its displacement down from its equilibrium depth. The equilibrium depth is given by

$$Mg = \rho g d A$$

The force equation is

$$p_0 A + Mg - (p_0 + \rho g(d + x))A = M\ddot{x}$$

where $p_0 + \rho g(d + x)$ is the pressure at the bottom of the rod. If we use the expression $Mg = \rho g d A$ in the above equation we have

$$- \rho g A x = M \ddot{x}$$

If we use the equivalent expression $\rho g A = \frac{Mg}{d}$ in the above equation and cancel a factor of M we have

$$-\frac{g}{d} \cdot x = \ddot{x} \quad \implies \quad T = 2\pi \sqrt{\frac{d}{g}} \approx 3.5 \text{ s}$$

Notice that it was unnecessary to specify the value of $\text{kg} \cdot \text{m/s}_0$ to evaluate the expression for T ; hence a change in atmospheric pressure of 5% will not change the period of oscillation at all.

Problem 10.5 (Ohanian, page 378, problem 63)

Let $\rho = 7.8 \times 10^3 \text{ kg}$ be the density of this steel. Now focus on one half of the meter stick, and let x denote the distance from the axis of rotation to a particular bit of length dx . If the cross-section area is A then the corresponding mass is $dm = \rho A dx$. The centripetal force required to accelerate this bit of steel is $dF = dm \cdot \omega^2 x = \rho A \omega^2 x dx$ where ω is the angular velocity of the meter stick. The total tension at the center of the meter stick is then

$$T = \int dF = \int_0^{\frac{1}{2}} \rho A \omega^2 x dx = \rho A \omega^2 \left. \frac{1}{2} x^2 \right|_0^{\frac{1}{2}} = \frac{\omega^2 \rho A}{8} \implies$$

$$\omega = \sqrt{\frac{8T}{\rho A}}$$

where $\frac{1}{2}$ is the length of half the meter stick. The maximum angular velocity is

$$\omega_{\max} = \sqrt{\frac{8}{\rho} \left(\frac{T}{A} \right)_{\max}} \approx 624 \text{ radian/s}$$

where the ultimate tensile strength for steel, $\left(\frac{T}{A} \right)_{\max} = 3.8 \times 10^8 \text{ N/m}^2$, is given in Table 14.1 on page 366.

Problem 10.6 (Ohanian, page 537, problem 13)

(a) The rate of increase of heat is the amount of energy deposited by each electron ($3.2 \times 10^{-9} \text{ J}$) times the rate for electrons ($3.0 \times 10^{14} \text{ s}^{-1}$).

$$\frac{\Delta Q}{\Delta t} = 3.2 \times 10^{-9} \cdot 3.0 \times 10^{14} = 9.6 \times 10^5 \text{ J/s} \approx 230 \text{ kcal/s}$$

where the conversion used is $1 \text{ cal} = 4.186 \text{ J}$.