

**Problem 8: (Archimedes's Principle)**

A cylindrical beaker of mass  $m_b = 1.3 \text{ kg}$  contains  $1.5 \times 10^3 \text{ ml}$  of water. The beaker is placed on a scale and then a rock of mass  $m_r = 2.2 \text{ kg}$ , suspended by a massless string, is totally immersed in the water. The water level rises by  $1.5 \text{ cm}$ . The diameter of the beaker is  $0.2 \text{ m}$ .

- a) What mass does the scale measure before the rock is lowered into the water?
- b) What mass does the scale measure after the rock is lowered into the water?
- c) What is the density of the rock?

A plastic hollow sphere of the same volume as the rock has density  $\rho_s = 0.6 \times 10^3 \text{ kg/m}^3$ . The sphere is held just below the surface of the water by a rope that is attached to the bottom of the beaker.

- d) What mass does the scale now read?

**Problem 9: (Planetary Orbits)**

A satellite of mass  $m_1$  is in an elliptical orbit around a planet of mass  $m_2$  whose center is at F, one of the two foci of the ellipse ( $m_2 \gg m_1$ ). P is the point of closest approach of the two masses and is a distance  $r_P$  from the foci F. The satellite's speed at the point P is  $v_P$ . The two masses are furthest apart at the point A which is a distance  $r_A = 5 r_P$ .

- a) What is the speed of the satellite at the point A?
- b) What is the energy of the satellite?

**Problem 10: (Energy, Force, and Kinematics)**

A child's playground slide is  $5.0 \text{ m}$  in length and is at an angle of  $2.0 \times 10^{-1}$  degrees with respect to the ground. A child of mass  $2.0 \times 10^1 \text{ kg}$  starts from rest at the top of the slide. The coefficient of sliding friction for the slide is  $\mu_k = 0.2$ .

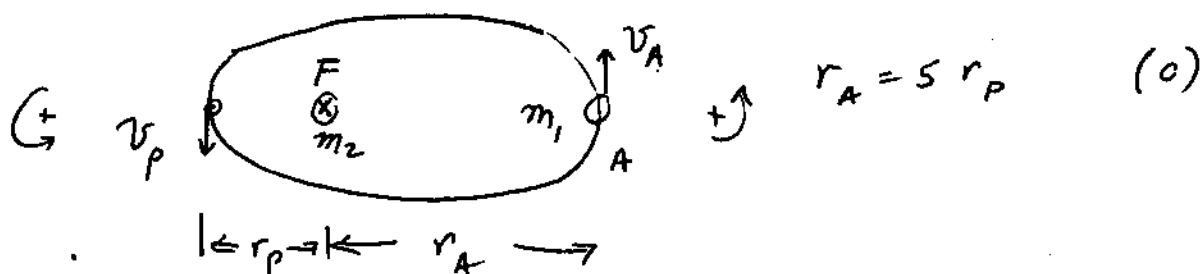
- a) What is the total work done by the friction force on the child?
- b) What is the speed of the child at the bottom of the slide?
- c) How long does the child take to slide down the ramp?

**Problem 9 (Planetary Orbits)**

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a) What is the speed of the satellite at the point A?

b) What is the energy of the satellite?



Angular Momentum is conserved about the point F: So  $L_{FA} = L_{FP}$

$$r_A m_1 v_A = r_P m_1 v_P \Rightarrow r_A v_A = r_P v_P$$

$$\text{Since } r_A = 5 r_P \Rightarrow 5 r_P v_A = r_P v_P$$

$$v_A = v_P / 5 \quad (1)$$

Conservation of Energy:

$$E = \frac{1}{2} m_1 v_A^2 - \frac{G m_1 m_2}{r_A} = \frac{1}{2} m_1 v_P^2 - \frac{G m_1 m_2}{r_P}$$

$$\frac{1}{2} m_1 v_A^2 - \frac{G m_1 m_2}{5 r_P} = \frac{1}{2} m_1 (5 v_A)^2 - \frac{G m_1 m_2}{r_P}$$

$$G m_1 m_2 \left( \frac{1}{r_P} - \frac{1}{5 r_P} \right) = \frac{1}{2} m_1 (25 v_A^2 - v_A^2) \quad (2)$$

We can solve eq (2) for  $v_A$

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$$v_A = \left( \frac{G m_2}{15 r_p} \right)^{1/2}$$

The energy is then

$$E = \frac{1}{2} m_1 (5v_A)^2 - \frac{G m_1 m_2}{r_p}$$

$$= \frac{1}{2} m_1 \left( \frac{25 G m_2}{15 r_p} \right) - \frac{G m_1 m_2}{r_p}$$

$$= -\frac{1}{6} \frac{G m_1 m_2}{r_p} < 0 \quad \text{as it should be for a bound orbit}$$