

In-Class Problem 15: *Uniform circular motion and the moon's period*)

In this problem assume that the moon is only under the influence of the earth's gravitational force given by a magnitude $\vec{F}_{e,m} = -G \frac{m_e m_m}{r_{e,m}^2} \hat{r}_{e,m}$. Also assume that the moon is moving in a circular orbit around the earth and that the moon travels with a constant speed in that orbit. Let $G = 6.67 \times 10^{-11} \text{ N} \cdot \text{m}^2 / \text{kg}^2$. The mass of the earth is $m_e = 5.98 \times 10^{24} \text{ kg}$. The mass of the moon is $m_m = 7.36 \times 10^{22} \text{ kg}$. The radius of the orbit is $r_{e,m} = 3.8 \times 10^8 \text{ m}$.

- a) Briefly describe how you intend to model the motion of the object? What directions are you choosing for analyzing the components of your forces and state why?
- b) Calculate the period of the moon's orbit around the earth.
- c) Is this the same period as the time between full moons as seen from the earth? Explain your reasoning.

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Solution

Moon's orbit: circular motion, use radial unit vector, acceleration is inward $a_{rad} = -r \frac{4\pi^2}{T^2}$



Equation of motion:

$$\vec{F} = m\vec{a}$$

$$\frac{-Gm_e M_m}{r^2} = -m_r \frac{4\pi^2}{T^2} r$$

$$\Rightarrow T^2 = \frac{4\pi^2 r^3}{Gm_e}$$

Period:

$$\Rightarrow T = 2\pi \left(\frac{r^3}{Gm_e} \right)^{1/2}$$

$$= 2\pi \left(\frac{(3.8 \times 10^8 \text{ m})^3}{(6.67 \times 10^{-11} \text{ N} \cdot \text{m}^2 / \text{kg}^2)(5.98 \times 10^{24} \text{ kg})} \right)^{1/2}$$

$$= 2.3 \times 10^6 \text{ s} = 27.0 \text{ days}$$

2nd full moon moon has moved an additional time equal to

$$\frac{1}{12} T = 2.25 \text{ days}$$

$$\Delta T_{\text{full moon}} = 29.25 \text{ days}$$

