

$$F = C_1rv + C_2r^2v^2 \quad F = \frac{mMG}{r^2} \quad \mathbf{F} = \frac{d\mathbf{p}}{dt} = m\mathbf{a} \quad dW = \mathbf{F} \cdot d\mathbf{r} \quad a_{cent} = \frac{v^2}{r} = \omega^2r$$

$$U = \frac{-mMG}{r} \quad U = mgh \quad U = \frac{1}{2}kx^2 \quad K = \frac{1}{2}mv^2 \quad K = \frac{1}{2}I\omega^2$$

$$E_{\text{tot}} = K + U = \frac{1}{2}mv^2 - \frac{mMG}{r} = \frac{-mMG}{2a} \quad \mathbf{L} = \mathbf{r} \times \mathbf{p} \quad I = \sum_i m_i r_i^2$$

$$m_1r_1 = m_2r_2 \quad v = \omega r \quad T^2 = \frac{4\pi^2(r_1 + r_2)^3}{G(m_1 + m_2)} \quad \omega = \frac{d\theta}{dt} \quad \omega = \sqrt{k/m}$$

$$\boldsymbol{\tau} = \mathbf{r} \times \mathbf{F} = I\boldsymbol{\alpha} = \frac{d\mathbf{L}}{dt} \quad \alpha = \frac{d\omega}{dt} \quad T = \frac{2\pi}{\omega} \quad L = I\omega \quad \mathbf{I} = \int_0^{\Delta t} \mathbf{F} dt = \mathbf{p}_f - \mathbf{p}_i$$

$$\omega = \sqrt{g/l} \quad \omega_{\text{pr}} = \frac{\tau}{L_s} \quad T^2 = \frac{4\pi^2a^3}{GM}$$

$$\text{Solid disk of mass } M \text{ and radius } R \text{ rotating about its cylindrical axis: } I = \frac{1}{2}MR^2$$

$$\text{Solid sphere of mass } M \text{ and radius } R \text{ rotating about an axis through its center: } I = \frac{2}{5}MR^2$$

$$v_f - v_i = -u \ln\left(\frac{m_f}{m_i}\right) - gt \quad I = I_{\text{cm}} + Md^2 \quad I_z = I_x + I_y$$

$$f' = f \left(1 + \frac{v}{c} \cos \theta\right) \quad \lambda' = \lambda \left(1 - \frac{v}{c} \cos \theta\right) \quad \frac{dP}{dy} = -\rho g$$

$$PV = nRT \quad R = 8.31\text{J/K} \quad PV = NkT \quad k = 1.38 \times 10^{-23}\text{J/K} \quad N_A = 6.02 \times 10^{23}$$

$$\lambda = \frac{h}{p} \quad E = hf = \frac{hc}{\lambda} \quad \Delta L = \alpha L \Delta T \quad \Delta V = \beta V \Delta T$$

$$h = 6.6 \times 10^{-34} \text{ J} \cdot \text{sec} \quad \hbar = \frac{h}{2\pi} = 1.05 \times 10^{-34} \text{ J} \cdot \text{sec}$$

$$\frac{1}{2}\rho v_1^2 + \rho g y_1 + P_1 = \frac{1}{2}\rho v_2^2 + \rho g y_2 + P_2$$

### Problem 7 (25 points)

Imagine a spherical, non-rotating planet of mass  $M$  and radius  $R$ . The planet has no atmosphere. A spacecraft of mass  $m$  ( $m \ll M$ ), is launched from the surface of the planet with speed  $v_0$  at an angle of  $30^\circ$  to the local vertical. The rocket burn is very short. Thus you may assume that when the spacecraft has a speed  $v_0$ , it has not yet moved any appreciable distance.

- a. (4) The speed  $v_0$  is so high that the orbit is not bound. What is the minimum speed for which this is the case?

~~Now imagine that the orbit is bound and that in its subsequent orbit the spacecraft reaches a maximum distance of  $15R$  from the center of the planet. At this distance the speed is  $V$ .~~

- ~~b. (4) What is the ratio of  $v_0/V$ ?~~
- ~~c. (4) What is the total energy of the spacecraft immediately after launch?~~
- ~~d. (5) What is the total energy of the spacecraft when it is farthest away from the planet?~~
- ~~e. (8) Write down one equation which would allow you to solve for  $v_0$  in terms of  $M$ ,  $G$ , and  $R$  (we are not asking you to solve this equation).~~

**Problem 7** 25 points

4 pts a)  $\frac{1}{2}mv^2 = \frac{mMG}{R} \rightarrow v = \sqrt{\frac{2MG}{R}}$

~~4 pts b)  $mv_0 R \sin 30^\circ = mV(15R) \rightarrow \frac{v_0}{V} = \frac{15}{\sin 30^\circ} = 30$~~

~~4 pts c)  $\frac{1}{2}mv_0^2 = \frac{mMG}{R}$~~

~~5 pts d) same as in c. or  $\frac{1}{2}mV^2 = \frac{mMG}{15R} = \frac{1}{2}m\frac{v_0^2}{900} = \frac{mMG}{15R}$~~

~~8 pts e)  $\frac{1}{2}mv_0^2 = \frac{mMG}{R} = \frac{1}{2}m\frac{v_0^2}{900} = \frac{mMG}{15R}$~~   
 ~~$\frac{1}{2}v_0^2 = \frac{MG}{R} = \frac{1}{2}\frac{v_0^2}{900} = \frac{MG}{15R}$~~