

c) Work done by the Inverse Square Gravitational Force: Consider an object of mass m moving directly towards the sun (mass m_s). Initially the object is at a distance r_0 from the center of the sun. The object moves to a final distance r_f from the center of the sun. How much work does the gravitational force between the sun and the object do on the object during this motion?

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Solution: The gravitational force between the sun and the body is given by

$$\vec{\mathbf{F}}_g = F_g \hat{\mathbf{r}} = -\frac{GMm}{r^2} \hat{\mathbf{r}}.$$

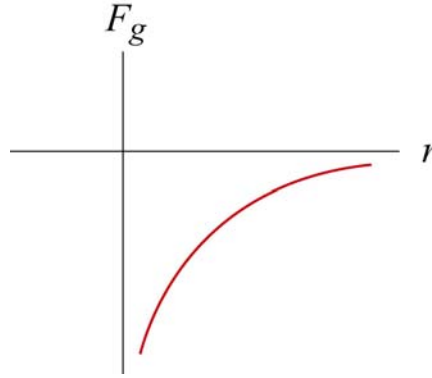


Figure 3 Graph of force between the sun and the body as a function of r

The work done by this gravitational force on the body is the area under the curve in Figure 8.5.4 and is given by the integral

$$W = \int_{r_0}^{r_f} F_g dr = \int_{r_0}^{r_f} \left(-\frac{GMm}{r^2} \right) dr.$$

Upon evaluation of this integral, we have for the work

$$W = \int_{r_0}^{r_f} \left(-\frac{GMm}{r^2} \right) dr = \frac{GMm}{r} \Big|_{r_0}^{r_f} = GMm \left(\frac{1}{r_f} - \frac{1}{r_0} \right).$$

Since the body has moved closer to the sun, $r_f < r_0$, hence $1/r_f > 1/r_0$. Thus the work done by gravitational force between the sun and the body on the body is positive,

$$W = Gmm_s \left(\frac{1}{r_f} - \frac{1}{r_0} \right) > 0.$$

We expect this result because the gravitational force points along the direction of motion, so the work done by the force is positive. Also we expect that the sign of the work is the same for a body moving closer to the sun as a body falling towards the earth in a constant gravitational field. We shall see that the positive work done by the gravitational force will decrease the potential energy stored in the gravitational field.