

**MASSACHUSETTS INSTITUTE OF TECHNOLOGY**  
**Department of Physics**

**Physics 8.01 TEAL**

**Fall Term 2004**

**Exam 3: Equation Summary**

**Momentum:**

$$\vec{p} = m\vec{v}, \quad \vec{F}_{ave} \Delta t = \Delta \vec{p}, \quad \vec{F}_{ext}^{total} = \frac{d\vec{p}^{total}}{dt}$$

$$\text{Impulse: } \vec{I} \equiv \int_{t=0}^{t=t_f} \vec{F}(t) dt = \Delta \vec{p}$$

$$\text{Torque: } \vec{\tau}_S = \vec{r}_{S,P} \times \vec{F}_P \quad |\vec{\tau}_S| = |\vec{r}_{S,P}| |\vec{F}_P| \sin \theta = r_{\perp} F = r F_{\perp}$$

**Static Equilibrium:**

$$\vec{F}_{total} = \vec{F}_1 + \vec{F}_2 + \dots = \vec{0}; \quad \vec{\tau}_S^{total} = \vec{\tau}_{S,1} + \vec{\tau}_{S,2} + \dots = \vec{0}.$$

$$\text{Rotational dynamics: } \vec{\tau}_S^{total} = \frac{d\vec{L}_S}{dt}$$

$$\text{Angular Velocity: } \vec{\omega} = (d\theta/dt) \hat{k}$$

$$\text{Angular Acceleration: } \vec{\alpha} = (d^2\theta/dt^2) \hat{k}$$

$$\text{Fixed Axis Rotation: } \vec{\tau}_S = I_S \vec{\alpha}$$

$$\tau_S^{total} = I_S \alpha = I_S \frac{d\omega}{dt}$$

$$\text{Moment of Inertia: } I_S = \int_{body} dm (r_{\perp})^2$$

$$\text{Angular Momentum: } \vec{L}_S = \vec{r}_{S,m} \times m\vec{v},$$

$$|\vec{L}_S| = |\vec{r}_{S,m}| |m\vec{v}| \sin \theta = r_{\perp} p = r p_{\perp}$$

**Angular Impulse:**

$$\vec{J}_S = \int_{t_0}^{t_f} \vec{\tau}_S dt = \Delta \vec{L}_S = \vec{L}_{S,f} - \vec{L}_{S,0}$$

**Rotation and Translation:**

$$\vec{L}_S^{total} = \vec{L}_S^{orbital} + \vec{L}_{cm}^{spin},$$

$$\vec{L}_S^{orbital} = \vec{r}_{S,cm} \times \vec{p}^{total},$$

$$\vec{L}_{cm}^{spin} = I_{cm} \vec{\omega}_{spin}$$

$$\vec{\tau}_S^{orbit} = \frac{d\vec{L}_S^{orbit}}{dt}, \quad \vec{\tau}_{cm}^{spin} = \frac{d\vec{L}_{cm}^{spin}}{dt}$$

**Rotational Energy:**  $K_{cm} = \frac{1}{2} I_{cm} \omega_{cm}^2$

**Rotational Power:**  $P_{rot} \equiv \frac{dW_{rot}}{dt} = \vec{\tau}_s \cdot \vec{\omega} = \tau_s \omega = \tau_s \frac{d\theta}{dt}$

**One Dimensional Kinematics:**  $\vec{v} = d\vec{r} / dt$  ,  $\vec{a} = d\vec{v} / dt$

$$v_x(t) - v_{x,0} = \int_{t'=0}^{t'=t} a_x(t') dt' \quad x(t) - x_0 = \int_{t'=0}^{t'=t} v_x(t') dt'$$

**Constant Acceleration:**

$$\begin{aligned} x(t) &= x_0 + v_{x,0}(t-t_0) + \frac{1}{2} a_x(t-t_0)^2 & v_x(t) &= v_{x,0} + a_x(t-t_0) \\ y(t) &= y_0 + v_{y,0}(t-t_0) + \frac{1}{2} a_y(t-t_0)^2 & v_y(t) &= v_{y,0} + a_y(t-t_0) \end{aligned}$$

where  $x_0, v_{x,0}, y_0, v_{y,0}$  are the initial position and velocities components at  $t = t_0$

**Newton's Second Law: Force, Mass, Acceleration**

$$\vec{F} \equiv m\vec{a} \quad \vec{F}^{total} = \vec{F}_1 + \vec{F}_2 \quad F_x^{total} = ma_x \quad F_y^{total} = ma_y \quad F_z^{total} = ma_z$$

**Newton's Third Law:**  $\vec{F}_{1,2} = -\vec{F}_{2,1}$

**Force Laws:**

Universal Law of Gravity:  $\vec{F}_{1,2} = -G \frac{m_1 m_2}{r_{1,2}^2} \hat{r}_{1,2}$  , attractive

Gravity near surface of earth:  $\vec{F}_{grav} = m_{grav} \vec{g}$  , towards earth

Contact force:  $\vec{F}_{contact} = \vec{N} + \vec{f}$  , depends on applied forces

Static Friction:  $0 \leq f_s \leq f_{s,max} = \mu_s N$  direction depends on applied forces

Kinetic Friction:  $f_k = \mu_k N$  opposes motion

Hooke's Law:  $F = k|\Delta x|$  , restoring

**Kinematics Circular Motion:** arc length:  $s = R\theta$  ; angular velocity:  $\omega = d\theta/dt$   
 tangential velocity:  $v = R\omega$  ; angular acceleration:  $\alpha = d\omega/dt = d^2\theta/dt^2$  ; tangential acceleration  $a_\theta = R\alpha$  .

**Period:**  $T = \frac{2\pi R}{v} = \frac{2\pi}{\omega}$ ; **frequency:**  $f = \frac{1}{T} = \frac{\omega}{2\pi}$ ,

**Radial Acceleration:**  $|a_r| = R\omega^2$ ;  $|a_r| = \frac{v^2}{R}$ ;  $|a_r| = 4\pi^2 R f^2$ ;  $|a_r| = \frac{4\pi^2 R}{T^2}$

**Center of Mass:**  $\vec{R}_{cm} = \sum_{i=1}^{i=N} m_i \vec{r}_i / m^{total} \rightarrow \int_{body} dm \vec{r} / m^{total}$ ;

**Velocity of Center of Mass:**  $\vec{V}_{cm} = \sum_{i=1}^{i=N} m_i \vec{v}_i / m^{total} \rightarrow \int_{body} dm \vec{v} / m^{total}$

**Torque:**  $\vec{\tau}_S = \vec{r}_{S,P} \times \vec{F}_P$   $|\vec{\tau}_S| = |\vec{r}_{S,P}| |\vec{F}_P| \sin \theta = r_{\perp} F = r F_{\perp}$

**Static Equilibrium:**  $\vec{F}_{total} = \vec{F}_1 + \vec{F}_2 + \dots = \vec{0}$ ;  $\vec{\tau}_S^{total} = \vec{\tau}_{S,1} + \vec{\tau}_{S,2} + \dots = \vec{0}$ .

**Kinetic Energy:**  $K = \frac{1}{2}mv^2$ ;  $\Delta K = \frac{1}{2}mv_f^2 - \frac{1}{2}mv_0^2$

**Work:**  $W = \int_{r_0}^{r_f} \vec{F} \cdot d\vec{r}$ ; **Work- Kinetic Energy:**  $W^{total} = \Delta K$

**Power:**  $P = \vec{F} \cdot \vec{v} = dK/dt$

**Potential Energy:**  $\Delta U = -W_{conservative} = -\int_A^B \vec{F}_c \cdot d\vec{r}$

**Potential Energy Functions with Zero Points:**

Constant Gravity:  $U(y) = mgy$  with  $U(y_0 = 0) = 0$ .

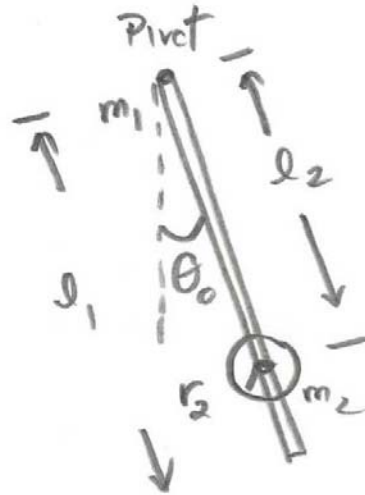
Inverse Square Gravity:  $U_{gravity}(r) = -\frac{Gm_1m_2}{r}$  with  $U_{gravity}(r_0 = \infty) = 0$ .

Hooke's Law:  $U_{spring}(x) = \frac{1}{2}kx^2$  with  $U_{spring}(x = 0) = 0$ .

**Work- Mechanical Energy:**  $W_{nc} = \Delta K + \Delta U^{total} = \Delta E_{mech} = (K_f + U_f^{total}) - (K_0 + U_0^{total})$

### Problem 5: Experiment 7: Physical Pendulum

A physical pendulum consists of a rod of mass  $m_1$  pivoted at one end. The rod has length  $l_1$  and moment of inertia  $I_1$  about the pivot point. A disc of mass  $m_2$  and radius  $r_2$  with moment of inertia  $I_{cm}$  about its center of mass, is rigidly attached a distance  $l_2$  from the pivot point. The pendulum is initially displaced to an angle  $\theta_0$  and then released from rest.



- What is the moment of inertia of the physical pendulum about the pivot point?
- What is the angular acceleration of the pendulum about the pivot point?
- What is the angular velocity of the pendulum when the pendulum is at the bottom of its swing?
- If the disc is moved further from the pivot, will the period of the pendulum increase or decrease or stay the same?

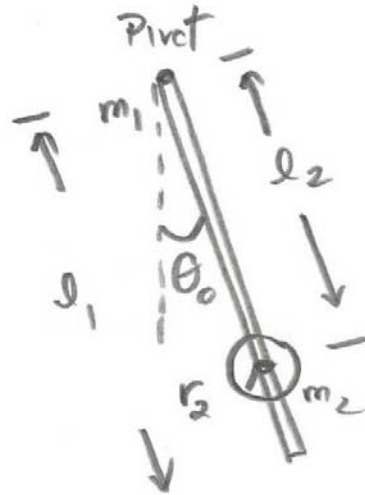
### ~~Problem 6: Experiment 11 Angular Momentum~~

~~A steel washer, is mounted on the shaft of a small motor. The moment of inertia of the motor and washer is  $I_0$ . Assume that the frictional torque on the axle remains the same throughout the slowing down. The washer is set into motion. When it reaches an initial angular velocity  $\omega_0$ , at  $t = 0$ , the power to the motor is shut off, and the washer slows down until it reaches an angular velocity of  $\omega_b$  at time  $t_b$ . At that instant, a second steel washer with a moment of inertia  $I_w$  is dropped on top of the first washer. Assume that the second washer is only in contact with the first washer. The collision takes place over a time  $\Delta t = t_a - t_b$ . Assume the frictional torque on the axle remains the same. The two washers continue to slow down until at they stop at  $t_f$ .~~

- ~~Describe how you measured the moment of inertia in the experiment.~~
- ~~What is the angular deceleration while the first washer and motor is slowing down?~~

### Problem 5: Experiment 7: Physical Pendulum

A physical pendulum consists of a rod of mass  $m_1$  pivoted at one end. The rod has length  $l_1$  and moment of inertia  $I_1$  about the pivot point. A disc of mass  $m_2$  and radius  $r_2$  with moment of inertia  $I_{cm}$  about its center of mass, is rigidly attached a distance  $l_2$  from the pivot point. The pendulum is initially displaced to an angle  $\theta_0$  and then released from rest.

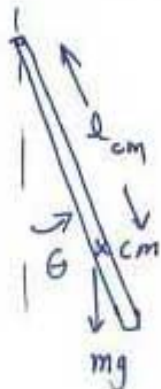


- What is the moment of inertia of the physical pendulum about the pivot point?
- What is the angular acceleration of the pendulum about the pivot point?
- What is the angular velocity of the pendulum when the pendulum is at the bottom of its swing?
- If the disc is moved further from the pivot, will the period of the pendulum increase or decrease or stay the same?

Solution:  $I_P = \underbrace{I_1}_{\text{rod}} + \underbrace{m_2 l_2^2}_{\text{disc}} + I_{cm}$

a)

$\hat{k} \odot$



$$l_{cm} = \frac{m_1 \frac{l_1}{2} + m_2 l_2}{m_1 + m_2}$$

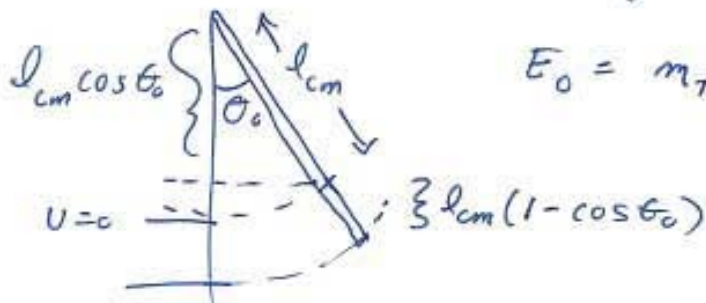
$$\vec{\tau}_P = I_P \vec{\alpha}$$

$$-mgl_{cm} \sin \theta = I_P \alpha$$

$$b) \alpha = \frac{-mgl_{cm} \sin \theta}{I_P} = - \frac{(m_1 + m_2) g \left( \frac{m_1 l_1}{2} + m_2 l_2 \right) \sin \theta}{I_1 + m_2 l_2^2 + I_{cm}}$$

$$\alpha = \frac{-g \left( \frac{m_1 l_1}{2} + m_2 l_2 \right) \sin \theta}{I_1 + m_2 l_2^2 + I_{cm}}$$

c) Use conservation of energy about pivot



$$E_0 = m_T g l_{cm} (1 - \cos \theta_0)$$

$$E_f = \frac{1}{2} I_{pivot} \omega_f^2$$

$$E_0 = E_f \Rightarrow$$

$$m_T g l_{cm} (1 - \cos \theta_0) = \frac{1}{2} I_p \omega_f^2$$

$$\omega_f = \left( \frac{m_T g l_{cm} (1 - \cos \theta_0)}{I_p} \right)^{1/2}$$

$$\omega_f = \left( \frac{g \left( \frac{m_1 l_1}{2} + m_2 l_2 \right) (1 - \cos \theta_0)}{I_1 + m_2 l_2^2 + I_{cm}} \right)^{1/2}$$

d) For the small angle approximation

$$T = \frac{2\pi}{\sqrt{\frac{m_T g l_{cm}}{I_p}}} = 2\pi \sqrt{\frac{I_p}{m_T g l_{cm}}}$$

If the disc is slid to a new position  $l_2'$

$$I_p' = I_1 + I_{cm} + m_2 l_2'^2 = I_p + m_2 (l_2' - l_c)$$

so this has changed

The center of mass has also changed

$$l_{cm}' = \frac{m_1 l + m_2 l_2'}{m_1 + m_2} = l_{cm} + \frac{m_2 (l_2' - l_c)}{m_1 + m_2}$$

so

$$T' = 2\pi \sqrt{\frac{I_{cm}'}{m_T g l_{cm}'}} = 2\pi \sqrt{\frac{I_p + m_2 (l_2'^2 - l_c^2)}{m_T g (l_{cm} + \frac{m_2 (l_2' - l_c)}{m_1 + m_2})}}$$

$$= 2\pi \sqrt{\frac{I_p (1 + \frac{m_2 (l_2'^2 - l_c^2)}{I_p})}{m_T g l_{cm} (1 + \frac{m_2 (l_2' - l_c)}{(m_1 + m_2) l_{cm}})}}$$

$$\text{If } 1 + \frac{m_2 (l_2'^2 - l_c^2)}{I_p} > 1 + \frac{m_2 (l_2' - l_c)}{(m_1 + m_2) l_{cm}}$$

period increases!