

~~4.7 SHO - Tunnel through the Earth~~ page 410, problem 43. I did this problem on **PIVoT**.

4.8 *Pendulum*. Watch **PIVoT**.

At the end of a string of length 2 m hangs a bob with a mass of 3 kg. The bob is released from rest with the string at 30° to the vertical [$g = 10 \text{ m/sec}^2$].

- a) How fast is the bob moving at the bottom of the swing?
- b) What is its kinetic energy at the instant when the string makes an angle of 10° to the vertical?
- c) How would your answers under a) and b) change if the mass of the bob were 6 kg?

~~4.9 Work~~ page 178, problem 8.

~~4.10 Pushing a Box~~ page 178, problem 10.

~~4.11 Roller Coaster~~ page 182, problem 52. Watch **PIVoT** under *roller coaster*.

~~4.12 Horizontal spring with friction. Watch **PIVoT** under friction (Spring on Table with Friction).~~

~~A 3.0 kg block rests on a leveled table. The coefficients of friction between the block and the table are $\mu_s = 0.30$ and $\mu_k = 0.20$. The block is attached to a wall by means of a horizontal spring of spring constant $k = 80 \text{ N/m}$. We pull on the block and stretch the spring and then let go with the block initially at rest. Use $g = 10 \text{ m/sec}^2$.~~

- ~~a) What is the maximum extension of the spring for which the block will remain stationary when released?~~
- ~~b) The block is placed in this position and then given a very gentle push towards the wall. At what position will the block reach its maximum speed?~~

~~4.13 Pendulum in action. I did this problem on **PIVoT** under *pendulum*.~~

~~A pendulum consists of a "massless" string of length l and a bob of mass m . We release the bob (without speed) when the string is at an angle θ_0 with respect to the vertical. A pin is located a distance L below the top of the string. When the pendulum swings down, the string hits the pin.~~

- ~~a) What is the maximum angle α that the string below the pin makes with the vertical after hitting the pin?~~
- ~~b) If the bob had been released with an initial tangential velocity v_0 , what would then be the maximum angle α . Does the direction of this tangential velocity matter?~~

~~4.14 Drop suitcase on conveyor belt.~~

- ~~a) A stationary suitcase is placed on a moving conveyor belt. Is the friction static or kinetic?~~
- ~~b) The belt is moving to the right. Thus, relative to the belt, the suitcase is moving to the left, thus there is a force on the suitcase to the right. The suitcase accelerates to the right until it reaches the speed of the belt. Is the work that friction does on the suitcase positive, negative, or zero?~~
- ~~c) After the suitcase reaches the speed of the belt, what is the horizontal component of the force acting on the suitcase?~~

Reminder.

~~There are 25 recitation sections. If you want to change, for whatever reason, please go to the physics education office (4-352).~~

$$x = A \cos(\omega t + \delta)$$

with

$$\omega = \sqrt{\frac{GM}{R^3}}$$

The period is then given by

$$\tau = \frac{2\pi}{\omega} = 2\pi \sqrt{\frac{R^3}{GM}}$$

(c) Since the train starts at rest from San Francisco, the amplitude of the motion is half the tunnel length. Thus the time to travel from San Francisco to Washington, D.C. would be

$$\Delta T = \frac{\tau}{2} = \pi \sqrt{\frac{R^3}{GM}} \approx 2550 \text{ s}$$

The tunnel length, l , is worked out in example 5 on page 18. Thus the amplitude of motion is

$$A = \frac{l}{2} = \frac{3.8 \times 10^3 \text{ km}}{2} = 1.9 \times 10^3 \text{ km}$$

The motion is then given by

$$x = A \cos(\omega t) \quad \Rightarrow \quad v = -\omega A \sin(\omega t)$$

It takes the train the time $t = \frac{\tau}{4}$ to reach the midpoint of the tunnel, thus the maximum speed is given by

$$|v| = \omega A \sin\left(\frac{2\pi}{\tau} \cdot \frac{\tau}{4}\right) = \omega A \sin\left(\frac{\pi}{2}\right) = \omega A = \frac{2\pi A}{\tau} = \frac{2\pi \times 1.9 \times 10^3 \text{ km}}{5100 \text{ s}} \approx 2.3 \text{ km/s}$$

Problem 4.8

Let $l = 2 \text{ m}$ and $m = 3 \text{ kg}$ and $g = 10 \text{ m/s}^2$. We are going to measure the vertical height from the bottom of the swing, and let θ be the angle of the string relative to the vertical. If you work out the geometry, the height of the bob, relative to the bottom, as a function of angle, θ , is given by

$$h = l(1 - \cos \theta)$$

(Please see Figure 15.16 on page 391 for a clear depiction of the geometry.) Therefore the potential energy as a function of θ is

$$U = mgh = mgl(1 - \cos \theta)$$

The total energy is conserved for such a pendulum. (The tension always acts perpendicular to the motion, thus it does no work; hence energy is conserved. This is also shown on page 392.) The bob is released from rest at an angle $\theta = 30^\circ$ relative to the vertical. Thus the total energy initially, which is solely potential, is

$$E = mgl(1 - \cos 30^\circ) \approx 8.038 \text{ J}$$

(a) At the bottom of the swing the total energy, which is solely kinetic, is

$$E = \frac{1}{2}mv^2 \quad \implies \quad |v| = \sqrt{\frac{2E}{m}} = \sqrt{2gl(1 - \cos 30^\circ)} \approx 2.3 \text{ m/s}$$

(b) The consider when the string makes an angle $\theta = 10^\circ$ relative to the vertical. The energy is given by

$$E = mgl(1 - \cos 10^\circ) + K \quad \implies \quad K = mgl(\cos 10^\circ - \cos 30^\circ) \approx 7.13 \text{ J}$$

(c) The answer to part (a) was independent of m ; thus the speed at the bottom wouldn't change if the mass doubled. The answer to part (b) was linear in m ; thus the kinetic energy at 10° would double if the mass doubled.

Problem 4.9 ~~(Ohanian, page 178, problem 8)~~

~~The work done by a force on a particle as it travels from position P_1 to position P_2 is given by~~

$$~~W = \int_{P_1}^{P_2} F dx~~$$

~~For the current problem, we can evaluate this integral by summing areas of four regions. The regions are defined as: (1) $0 \leq x \leq 2$, (2) $2 \leq x \leq 4$, (3) $4 \leq x \leq 6$, and (4) $6 \leq x \leq 8$. In region (1) the force as a function of position is~~

$$~~F_x = \left(\frac{2 \text{ N} - 0 \text{ N}}{2 \text{ m} - 0 \text{ m}} \right) x = (1 \text{ N/m})x~~$$