

MASSACHUSETTS INSTITUTE OF TECHNOLOGY
Department of Physics

Physics 8.01 TEAL

Fall Term 2004

Exam 3: Equation Summary

Momentum:

$$\vec{p} = m\vec{v}, \quad \vec{F}_{ave} \Delta t = \Delta \vec{p}, \quad \vec{F}_{ext}^{total} = \frac{d\vec{p}^{total}}{dt}$$

$$\text{Impulse: } \vec{I} \equiv \int_{t=0}^{t=t_f} \vec{F}(t) dt = \Delta \vec{p}$$

$$\text{Torque: } \vec{\tau}_S = \vec{r}_{S,P} \times \vec{F}_P \quad |\vec{\tau}_S| = |\vec{r}_{S,P}| |\vec{F}_P| \sin \theta = r_{\perp} F = r F_{\perp}$$

Static Equilibrium:

$$\vec{F}_{total} = \vec{F}_1 + \vec{F}_2 + \dots = \vec{0}; \quad \vec{\tau}_S^{total} = \vec{\tau}_{S,1} + \vec{\tau}_{S,2} + \dots = \vec{0}.$$

$$\text{Rotational dynamics: } \vec{\tau}_S^{total} = \frac{d\vec{L}_S}{dt}$$

$$\text{Angular Velocity: } \vec{\omega} = (d\theta/dt) \hat{k}$$

$$\text{Angular Acceleration: } \vec{\alpha} = (d^2\theta/dt^2) \hat{k}$$

$$\text{Fixed Axis Rotation: } \vec{\tau}_S = I_S \vec{\alpha}$$

$$\tau_S^{total} = I_S \alpha = I_S \frac{d\omega}{dt}$$

$$\text{Moment of Inertia: } I_S = \int_{body} dm (r_{\perp})^2$$

$$\text{Angular Momentum: } \vec{L}_S = \vec{r}_{S,m} \times m\vec{v},$$

$$|\vec{L}_S| = |\vec{r}_{S,m}| |m\vec{v}| \sin \theta = r_{\perp} p = r p_{\perp}$$

Angular Impulse:

$$\vec{J}_S = \int_{t_0}^{t_f} \vec{\tau}_S dt = \Delta \vec{L}_S = \vec{L}_{S,f} - \vec{L}_{S,0}$$

Rotation and Translation:

$$\vec{L}_S^{total} = \vec{L}_S^{orbital} + \vec{L}_{cm}^{spin},$$

$$\vec{L}_S^{orbital} = \vec{r}_{S,cm} \times \vec{p}^{total},$$

$$\vec{L}_{cm}^{spin} = I_{cm} \vec{\omega}_{spin}$$

$$\vec{\tau}_S^{orbit} = \frac{d\vec{L}_S^{orbit}}{dt}, \quad \vec{\tau}_{cm}^{spin} = \frac{d\vec{L}_{cm}^{spin}}{dt}$$

Rotational Energy: $K_{cm} = \frac{1}{2} I_{cm} \omega_{cm}^2$

Rotational Power: $P_{rot} \equiv \frac{dW_{rot}}{dt} = \vec{\tau}_s \cdot \vec{\omega} = \tau_s \omega = \tau_s \frac{d\theta}{dt}$

One Dimensional Kinematics: $\vec{v} = d\vec{r} / dt$, $\vec{a} = d\vec{v} / dt$

$$v_x(t) - v_{x,0} = \int_{t'=0}^{t'=t} a_x(t') dt' \quad x(t) - x_0 = \int_{t'=0}^{t'=t} v_x(t') dt'$$

Constant Acceleration:

$$\begin{aligned} x(t) &= x_0 + v_{x,0}(t-t_0) + \frac{1}{2} a_x(t-t_0)^2 & v_x(t) &= v_{x,0} + a_x(t-t_0) \\ y(t) &= y_0 + v_{y,0}(t-t_0) + \frac{1}{2} a_y(t-t_0)^2 & v_y(t) &= v_{y,0} + a_y(t-t_0) \end{aligned}$$

where $x_0, v_{x,0}, y_0, v_{y,0}$ are the initial position and velocities components at $t = t_0$

Newton's Second Law: Force, Mass, Acceleration

$$\vec{F} \equiv m\vec{a} \quad \vec{F}^{total} = \vec{F}_1 + \vec{F}_2 \quad F_x^{total} = ma_x \quad F_y^{total} = ma_y \quad F_z^{total} = ma_z$$

Newton's Third Law: $\vec{F}_{1,2} = -\vec{F}_{2,1}$

Force Laws:

Universal Law of Gravity: $\vec{F}_{1,2} = -G \frac{m_1 m_2}{r_{1,2}^2} \hat{r}_{1,2}$, attractive

Gravity near surface of earth: $\vec{F}_{grav} = m_{grav} \vec{g}$, towards earth

Contact force: $\vec{F}_{contact} = \vec{N} + \vec{f}$, depends on applied forces

Static Friction: $0 \leq f_s \leq f_{s,max} = \mu_s N$ direction depends on applied forces

Kinetic Friction: $f_k = \mu_k N$ opposes motion

Hooke's Law: $F = k |\Delta x|$, restoring

Kinematics Circular Motion: arc length: $s = R\theta$; angular velocity: $\omega = d\theta/dt$
 tangential velocity: $v = R\omega$; angular acceleration: $\alpha = d\omega/dt = d^2\theta/dt^2$; tangential acceleration $a_\theta = R\alpha$.

Period: $T = \frac{2\pi R}{v} = \frac{2\pi}{\omega}$; **frequency:** $f = \frac{1}{T} = \frac{\omega}{2\pi}$,

Radial Acceleration: $|a_r| = R\omega^2$; $|a_r| = \frac{v^2}{R}$; $|a_r| = 4\pi^2 R f^2$; $|a_r| = \frac{4\pi^2 R}{T^2}$

Center of Mass: $\vec{R}_{cm} = \sum_{i=1}^{i=N} m_i \vec{r}_i / m^{total} \rightarrow \int_{body} dm \vec{r} / m^{total}$;

Velocity of Center of Mass: $\vec{V}_{cm} = \sum_{i=1}^{i=N} m_i \vec{v}_i / m^{total} \rightarrow \int_{body} dm \vec{v} / m^{total}$

Kinetic Energy: $K = \frac{1}{2}mv^2$; $\Delta K = \frac{1}{2}mv_f^2 - \frac{1}{2}mv_0^2$

Work: $W = \int_{r_0}^{r_f} \vec{F} \cdot d\vec{r}$; **Work- Kinetic Energy:** $W^{total} = \Delta K$

Power: $P = \vec{F} \cdot \vec{v} = dK/dt$

Potential Energy: $\Delta U = -W_{conservative} = -\int_A^B \vec{F}_c \cdot d\vec{r}$

Potential Energy Functions with Zero Points:

Constant Gravity: $U(y) = mgy$ with $U(y_0 = 0) = 0$.

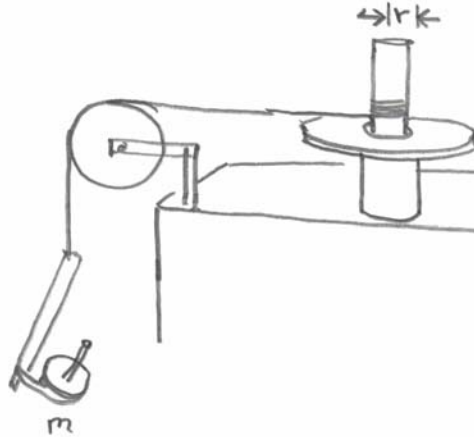
Inverse Square Gravity: $U_{gravity}(r) = -\frac{Gm_1 m_2}{r}$ with $U_{gravity}(r_0 = \infty) = 0$.

Hooke's Law: $U_{spring}(x) = \frac{1}{2}kx^2$ with $U_{spring}(x = 0) = 0$.

Work- Mechanical Energy: $W_{nc} = \Delta K + \Delta U^{total} = \Delta E_{mech} = (K_f + U_f^{total}) - (K_0 + U_0^{total})$

Problem 3: (30 points)

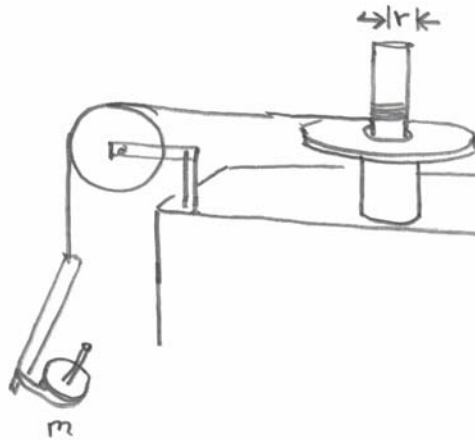
A steel washer, is mounted on a cylindrical rotor of radius r . A massless string, with a mass m attached to the other end, is wrapped around the side of the rotor and passes over a massless pulley. Assume that there is a constant frictional torque about the axis of the rotor. The mass is released and falls. As the mass falls, the rotor undergoes an angular acceleration of magnitude α_1 . After the string detaches from the rotor and the rotor coasts to a stop with an angular acceleration of magnitude α_2 . Let g denote the gravitational constant.



- a) What is the moment of inertia I_R of the rotor assembly (including the washer) about the rotation axis? Show all your work. Express your answer in terms of the given quantities r , α_1 , α_2 , m , and g .
- b) Suppose the rotor has an angular velocity ω_0 , when a second washer of moment of inertia I_W is dropped on top of the first washer. It takes a time Δt for the dropped washer to move at the same angular velocity as the rotor. What is the angular velocity ω_f of the assembly immediately after the collision is finished and the dropped washer and rotor move at the same angular velocity ω_f ? Express your answer in terms of the quantities I_R , I_W , α_2 , Δt , and ω_0 as needed.

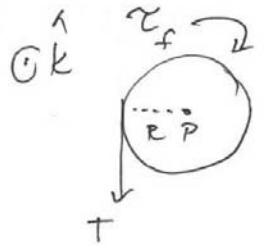
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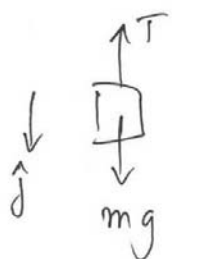
a) The torque diagram for the rotor (seen from overhead) is as follows



$\vec{\tau}_P = I_R \vec{\alpha}_P$
$\hat{k}: RT = \tau_f \quad I_R \alpha_1$

(1)

The force diagram on the mass is



$\vec{F} = m \vec{a}$
$\hat{j}: mg - T = m a$

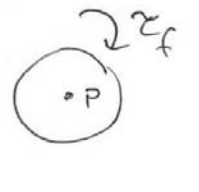
(2)

(2a)

The linear acceleration and angular acceleration

$$a = R \alpha_1 \quad (3)$$

After the string is detached, the only torque left is the frictional torque



$\vec{\tau}_P = I_R \vec{\alpha}$
$-\tau_f = I_R \alpha_2$

(4)

eq (1) becomes (with eq (4))

$$RT + I_R \alpha_2 = I_R \alpha_1 \quad (5)$$

Use eq (2a) for the tension in eq (5)

$$R(mg - ma) = I_R(\alpha_1 - \alpha_2)$$

Now use eq (3)

$$R(mg - mR\alpha_1) = I_R(\alpha_1 - \alpha_2) \quad (6)$$

Solve for I_R :

$$I_R = \frac{mR(g - R\alpha_1)}{(\alpha_1 - \alpha_2)} \quad (7)$$

b) During the collision there is a frictional torque so the angular momentum is not constant

$$-\tau_f \Delta t = L_f - L_0 \quad (8)$$

The final angular momentum is

$$L_f = (I_R + I_w) \omega_f$$

The initial angular momentum is

$$L_0 = I_R \omega_0$$

so eq (8) becomes

$$-\tau_f \Delta t = (I_R + I_w) \omega_f - I_R \omega_0 \quad (9)$$

Use $-\tau_f = I_R \alpha_2$ then eq (9) becomes

$$I_R \alpha_2 \Delta t = (I_R + I_w) \omega_f - I_R \omega_0$$

solve for ω_f :

$$\omega_f = \frac{I_R (\omega_0 + \alpha_2 \Delta t)}{I_R + I_w}$$

note that $\alpha_2 < 0$.