

MASSACHUSETTS INSTITUTE OF TECHNOLOGY
Department of Physics

Physics 8.01 TEAL

Fall Term 2004

Exam 3: Equation Summary

Momentum:

$$\vec{p} = m\vec{v}, \quad \vec{F}_{ave} \Delta t = \Delta \vec{p}, \quad \vec{F}_{ext}^{total} = \frac{d\vec{p}^{total}}{dt}$$

$$\text{Impulse: } \vec{I} \equiv \int_{t=0}^{t=t_f} \vec{F}(t) dt = \Delta \vec{p}$$

$$\text{Torque: } \vec{\tau}_S = \vec{r}_{S,P} \times \vec{F}_P \quad |\vec{\tau}_S| = |\vec{r}_{S,P}| |\vec{F}_P| \sin \theta = r_{\perp} F = r F_{\perp}$$

Static Equilibrium:

$$\vec{F}_{total} = \vec{F}_1 + \vec{F}_2 + \dots = \vec{0}; \quad \vec{\tau}_S^{total} = \vec{\tau}_{S,1} + \vec{\tau}_{S,2} + \dots = \vec{0}.$$

$$\text{Rotational dynamics: } \vec{\tau}_S^{total} = \frac{d\vec{L}_S}{dt}$$

$$\text{Angular Velocity: } \vec{\omega} = (d\theta/dt) \hat{k}$$

$$\text{Angular Acceleration: } \vec{\alpha} = (d^2\theta/dt^2) \hat{k}$$

$$\text{Fixed Axis Rotation: } \vec{\tau}_S = I_S \vec{\alpha}$$

$$\tau_S^{total} = I_S \alpha = I_S \frac{d\omega}{dt}$$

$$\text{Moment of Inertia: } I_S = \int_{body} dm (r_{\perp})^2$$

$$\text{Angular Momentum: } \vec{L}_S = \vec{r}_{S,m} \times m\vec{v},$$

$$|\vec{L}_S| = |\vec{r}_{S,m}| |m\vec{v}| \sin \theta = r_{\perp} p = r p_{\perp}$$

Angular Impulse:

$$\vec{J}_S = \int_{t_0}^{t_f} \vec{\tau}_S dt = \Delta \vec{L}_S = \vec{L}_{S,f} - \vec{L}_{S,0}$$

Rotation and Translation:

$$\vec{L}_S^{total} = \vec{L}_S^{orbital} + \vec{L}_{cm}^{spin},$$

$$\vec{L}_S^{orbital} = \vec{r}_{S,cm} \times \vec{p}^{total},$$

$$\vec{L}_{cm}^{spin} = I_{cm} \vec{\omega}_{spin}$$

$$\vec{\tau}_S^{orbit} = \frac{d\vec{L}_S^{orbit}}{dt}, \quad \vec{\tau}_{cm}^{spin} = \frac{d\vec{L}_{cm}^{spin}}{dt}$$

Rotational Energy: $K_{cm} = \frac{1}{2} I_{cm} \omega_{cm}^2$

Rotational Power: $P_{rot} \equiv \frac{dW_{rot}}{dt} = \vec{\tau}_s \cdot \vec{\omega} = \tau_s \omega = \tau_s \frac{d\theta}{dt}$

One Dimensional Kinematics: $\vec{v} = d\vec{r} / dt$, $\vec{a} = d\vec{v} / dt$

$$v_x(t) - v_{x,0} = \int_{t'=0}^{t'=t} a_x(t') dt' \quad x(t) - x_0 = \int_{t'=0}^{t'=t} v_x(t') dt'$$

Constant Acceleration:

$$\begin{aligned} x(t) &= x_0 + v_{x,0}(t-t_0) + \frac{1}{2} a_x(t-t_0)^2 & v_x(t) &= v_{x,0} + a_x(t-t_0) \\ y(t) &= y_0 + v_{y,0}(t-t_0) + \frac{1}{2} a_y(t-t_0)^2 & v_y(t) &= v_{y,0} + a_y(t-t_0) \end{aligned}$$

where $x_0, v_{x,0}, y_0, v_{y,0}$ are the initial position and velocities components at $t = t_0$

Newton's Second Law: Force, Mass, Acceleration

$$\vec{F} \equiv m\vec{a} \quad \vec{F}^{total} = \vec{F}_1 + \vec{F}_2 \quad F_x^{total} = ma_x \quad F_y^{total} = ma_y \quad F_z^{total} = ma_z$$

Newton's Third Law: $\vec{F}_{1,2} = -\vec{F}_{2,1}$

Force Laws:

Universal Law of Gravity: $\vec{F}_{1,2} = -G \frac{m_1 m_2}{r_{1,2}^2} \hat{r}_{1,2}$, attractive

Gravity near surface of earth: $\vec{F}_{grav} = m_{grav} \vec{g}$, towards earth

Contact force: $\vec{F}_{contact} = \vec{N} + \vec{f}$, depends on applied forces

Static Friction: $0 \leq f_s \leq f_{s,max} = \mu_s N$ direction depends on applied forces

Kinetic Friction: $f_k = \mu_k N$ opposes motion

Hooke's Law: $F = k |\Delta x|$, restoring

Kinematics Circular Motion: arc length: $s = R\theta$; angular velocity: $\omega = d\theta/dt$
 tangential velocity: $v = R\omega$; angular acceleration: $\alpha = d\omega/dt = d^2\theta/dt^2$; tangential acceleration $a_\theta = R\alpha$.

Period: $T = \frac{2\pi R}{v} = \frac{2\pi}{\omega}$; **frequency:** $f = \frac{1}{T} = \frac{\omega}{2\pi}$,

Radial Acceleration: $|a_r| = R\omega^2$; $|a_r| = \frac{v^2}{R}$; $|a_r| = 4\pi^2 R f^2$; $|a_r| = \frac{4\pi^2 R}{T^2}$

Center of Mass: $\vec{R}_{cm} = \sum_{i=1}^{i=N} m_i \vec{r}_i / m^{total} \rightarrow \int_{body} dm \vec{r} / m^{total}$;

Velocity of Center of Mass: $\vec{V}_{cm} = \sum_{i=1}^{i=N} m_i \vec{v}_i / m^{total} \rightarrow \int_{body} dm \vec{v} / m^{total}$

Torque: $\vec{\tau}_S = \vec{r}_{S,P} \times \vec{F}_P$ $|\vec{\tau}_S| = |\vec{r}_{S,P}| |\vec{F}_P| \sin \theta = r_{\perp} F = r F_{\perp}$

Static Equilibrium: $\vec{F}_{total} = \vec{F}_1 + \vec{F}_2 + \dots = \vec{0}$; $\vec{\tau}_S^{total} = \vec{\tau}_{S,1} + \vec{\tau}_{S,2} + \dots = \vec{0}$.

Kinetic Energy: $K = \frac{1}{2}mv^2$; $\Delta K = \frac{1}{2}mv_f^2 - \frac{1}{2}mv_0^2$

Work: $W = \int_{r_0}^{r_f} \vec{F} \cdot d\vec{r}$; **Work- Kinetic Energy:** $W^{total} = \Delta K$

Power: $P = \vec{F} \cdot \vec{v} = dK/dt$

Potential Energy: $\Delta U = -W_{conservative} = -\int_A^B \vec{F}_c \cdot d\vec{r}$

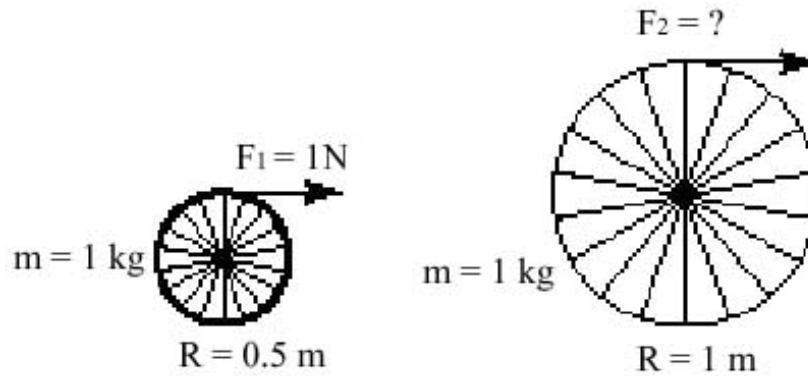
Potential Energy Functions with Zero Points:

Constant Gravity: $U(y) = mgy$ with $U(y_0 = 0) = 0$.

Inverse Square Gravity: $U_{gravity}(r) = -\frac{Gm_1 m_2}{r}$ with $U_{gravity}(r_0 = \infty) = 0$.

Hooke's Law: $U_{spring}(x) = \frac{1}{2}kx^2$ with $U_{spring}(x = 0) = 0$.

Work- Mechanical Energy: $W_{nc} = \Delta K + \Delta U^{total} = \Delta E_{mech} = (K_f + U_f^{total}) - (K_0 + U_0^{total})$



Problem 1d) (5 points) Angular Momentum and Rotational Energy

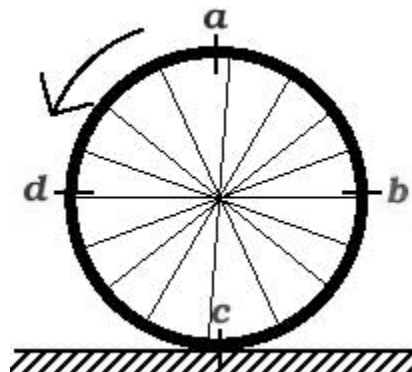
A figure skater stands on one spot on the ice (assumed frictionless) and spins around with her arms extended. When she pulls in her arms, she reduces her moment of inertia about the spin axis passing through her center of mass and her angular speed increases so that her angular momentum is conserved. Compared to her initial rotational kinetic energy, her rotational kinetic energy after she has pulled in her arms must be

1. the same.
2. larger because she's rotating faster.
3. smaller because her moment of inertia is smaller.

Explain your reasoning.

~~Problem 1c) (5 points) Rolling without slipping~~

~~A bicycle tire rolls without slipping across the ground (moving to the left in the figure below). Describe the direction and magnitude of the velocity at each of the four points shown in the figure as seen from an observer at rest on the ground.~~



Problem 1d) (5 points) Angular Momentum and Rotational Energy

A figure skater stands on one spot on the ice (assumed frictionless) and spins around with her arms extended. When she pulls in her arms, she reduces her moment of inertia about the spin axis passing through her center of mass and her angular speed increases so that her angular momentum is conserved. Compared to her initial rotational kinetic energy, her rotational kinetic energy after she has pulled in her arms must be

1. the same.
2. larger because she's rotating faster.
3. smaller because her moment of inertia is smaller.

Explain your reasoning.

Angular momentum is conserved

$$\text{so } L_i = L_f \Rightarrow I_i \omega_i = I_f \omega_f \Rightarrow \omega_f = \frac{I_i \omega_i}{I_f}$$

Rotational kinetic energy is not conserved

$$K_f = \frac{1}{2} I_f \omega_f^2 = \frac{1}{2} \left(\frac{I_i \omega_i}{I_f} \right)^2 = \left(\frac{1}{2} I_i \omega_i^2 \right) \frac{I_i}{I_f} = K_i \frac{I_i}{I_f}$$

since pulling arms in decreases

moment of inertia, $I_f < I_i$

the rotational kinetic energy increases.

note. $\frac{I_i}{I_f} = \frac{\omega_f}{\omega_i}$ so

$$K_f = \frac{\omega_f}{\omega_i} K_i \quad \text{and} \quad \omega_f > \omega_i \quad \text{so} \quad K_f > K_i$$

kinetic energy increases because she is rotating faster