

~~Nov 24~~

~~Hour One: Archimedes Principle~~

~~Reading: YF 14.1-14.3~~

~~Hour Two: Archimedes Principle PRS Contest~~

~~Nov 26 No Class~~

~~Problem Set 12: Due Fri Dec 3 at 4:00 pm.~~

Problem 1 : Bohr hydrogen atom

The Bohr hydrogen atom models the atom as an electron orbiting the proton under the influence of an electric force producing uniform circular motion with radius a_0 . The mass of the electron is $m_e = 9.1 \times 10^{-31} \text{ kg}$; the electric charge is $e = 1.6 \times 10^{-19} \text{ C}$; the Planck constant is $h = 6.63 \times 10^{-34} \text{ J-s}$, and the magnitude of the electric force is given by Coulomb's Law $|\vec{F}| = ke^2/r^2$ where $k = 9.0 \times 10^9 \text{ N-m}^2/\text{C}^2$. The angular momentum is quantized according to $L = nh/2\pi$ where $n = 1, 2, \dots$

- Write down the equation that arises from the application of Newton's 2nd Law to the electron.
- What is the angular momentum of the electron about the center of the atom?
- Using your results from parts a) and b) derive an equation for the radius a_0 of the atom as a function of n, e, h, m_e , and k .
- What is the energy E_n for the atom? Express your answer in terms of n, e, h, m_e , and k .
- A hydrogen atom emits a photon which arise from an energy transition from the $n = 3$ to $n = 1$ energy level. Calculate the frequency of the light emitted

$$f = -\Delta E/h = -(E_1 - E_3)/h.$$

- What is the wavelength of the emitted light from part e)?

~~**Problem 2: Experiment 11 Angular Momentum: Analysis**~~

~~**Part One: Rotor Moment of Inertia**~~

~~Enter the results from your experiment report into the table below.~~

MASSACHUSETTS INSTITUTE OF TECHNOLOGY
Department of Physics

Physics 8.01T

Fall Term 2004

Problem Set 11: Angular Momentum, Rotation and Translation Solutions

Problem 1 : Bohr hydrogen atom

The Bohr hydrogen atom models the atom as an electron orbiting the proton under the influence of an electric force producing uniform circular motion with radius a_0 . The mass of the electron is $m_e = 9.1 \times 10^{-31} \text{ kg}$; the electric charge is $e = 1.6 \times 10^{-19} \text{ C}$; the Planck constant is $h = 6.63 \times 10^{-34} \text{ J}\cdot\text{s}$, and the magnitude of the electric force is given by Coulomb's Law $|\vec{F}| = ke^2/r^2$ where $k = 9.0 \times 10^9 \text{ N}\cdot\text{m}^2/\text{C}^2$. The angular momentum is quantized according to $L = nh/2\pi$ where $n = 1, 2, \dots$

- a) Write down the equation that arises from the application of Newton's 2nd Law to the electron.

Since the electron is much less massive than the proton the reduced mass is the mass of the electron. The radial component of Newton's Second Law is then

$$-\frac{ke^2}{a_0^2} = -m_e \frac{v^2}{a_0}.$$

You can solve this equation for the velocity in terms of the radius of the electron orbit,

$$v = \sqrt{\frac{ke^2}{m_e a_0}}$$

- b) What is the angular momentum of the electron about the center of the atom?

$$L = n \frac{h}{2\pi} = m_e a_0 v$$

- c) Using your results from parts a) and b) derive an equation for the radius a_0 of the atom as a function of n, e, h, m_e , and k .

Using the result for the velocity of the orbit, the angular momentum equation becomes

$$n \frac{h}{2\pi} = m_e \sqrt{\frac{ke^2 a_0}{m_e}}.$$

Square this equation and solve for the radius a_0 of the atom

$$n^2 \frac{h^2}{4\pi^2} = m_e^2 \frac{ke^2 a_0}{m_e}$$

Thus

$$a_0 = \frac{n^2 h^2}{4\pi^2 m_e ke^2}$$

d) What is the energy E_n for the atom? Express your answer in terms of n, e, h, m_e , and k .

Using the value for the velocity of the orbit, the energy is given by

$$E = \frac{1}{2} m_e v^2 - \frac{ke^2}{a_0} = \frac{1}{2} m_e \frac{ke^2}{m_e a_0} - \frac{ke^2}{a_0} = -\frac{ke^2}{2a_0}.$$

Now use the value for the radius of the orbit

$$E = -\frac{1}{2} \frac{ke^2}{a_0} = -\frac{ke^2}{2n^2 h^2 / 4\pi^2 m_e ke^2} = -\frac{2\pi^2 m_e k^2 e^4}{n^2 h^2}.$$

Set

$$A = \frac{2\pi^2 m_e k^2 e^4}{h^2} = 2\pi^2 \frac{(9.11 \times 10^{-31} \text{ kg})(8.99 \times 10^9 \text{ N} \cdot \text{m}^2 \text{C}^{-2})^2 (1.60 \times 10^{-19} \text{ C})^4}{(6.63 \times 10^{-34} \text{ J} \cdot \text{sec})^2}$$

$$A = (2.17 \times 10^{-18} \text{ J}) \frac{1 \text{ ev}}{1.60 \times 10^{-19} \text{ J}} = 13.6 \text{ ev}.$$

Then the energy is

$$E_n = -\frac{ke^2}{2n^2 h^2 / 4\pi^2 \mu ke^2} = -\frac{2\pi^2 \mu k^2 e^4}{h^2} \frac{1}{n^2} = -\frac{A}{n^2}$$

When $n = 1$, the constant $A = 13.6 \text{ ev} = -E_1$ is minus the ground state energy.

- e) A hydrogen atom emits a photon which arise from an energy transition from the $n = 3$ to $n = 1$ energy level. Calculate the frequency of the light emitted

$$f = -\Delta E/h = -(E_1 - E_3)/h.$$

Using our result for the energy levels, we have that

$$f = \frac{A}{h} \left(1 - \frac{1}{9} \right) = \frac{(8)(13.6 \text{ eV})(1.6 \times 10^{-19} \text{ J} \cdot \text{eV}^{-1})}{(9)(6.63 \times 10^{-34} \text{ J} \cdot \text{s})} = 3.46 \times 10^{15} \text{ J}$$

- f) What is the wavelength of the emitted light from part e)?

The wavelength of the emitted light λ is related to the frequency of light, f , and the speed of light $c = 3.00 \times 10^8 \text{ m} \cdot \text{sec}^{-1}$ by

$$f\lambda = c$$

Thus the inverse wavelength of the light is given by

$$\frac{1}{\lambda} = \frac{f}{c} = \frac{A}{hc} \left(\frac{1}{n_f^2} - \frac{1}{n_i^2} \right) = R \left(\frac{1}{n_f^2} - \frac{1}{n_i^2} \right)$$

where the Rydberg constant,

$$R = \frac{A}{hc} = \frac{2\pi^2 m_e k^2 e^4}{h^3 c} = 1.09 \times 10^7 \text{ m}^{-1}.$$

So the wavelength is

$$\lambda_{3,2} = \frac{1}{R} \left(\frac{n_i^2 n_f^2}{n_i^2 - n_f^2} \right)$$

When the electron in the energy level $n = 3$ drops to the energy level $n = 1$. The wavelength is then

$$\lambda_{3,2} = \frac{9}{8R} = \frac{9}{8(1.09 \times 10^7 \text{ m}^{-1})} = 1.03 \times 10^{-7} \text{ m},$$