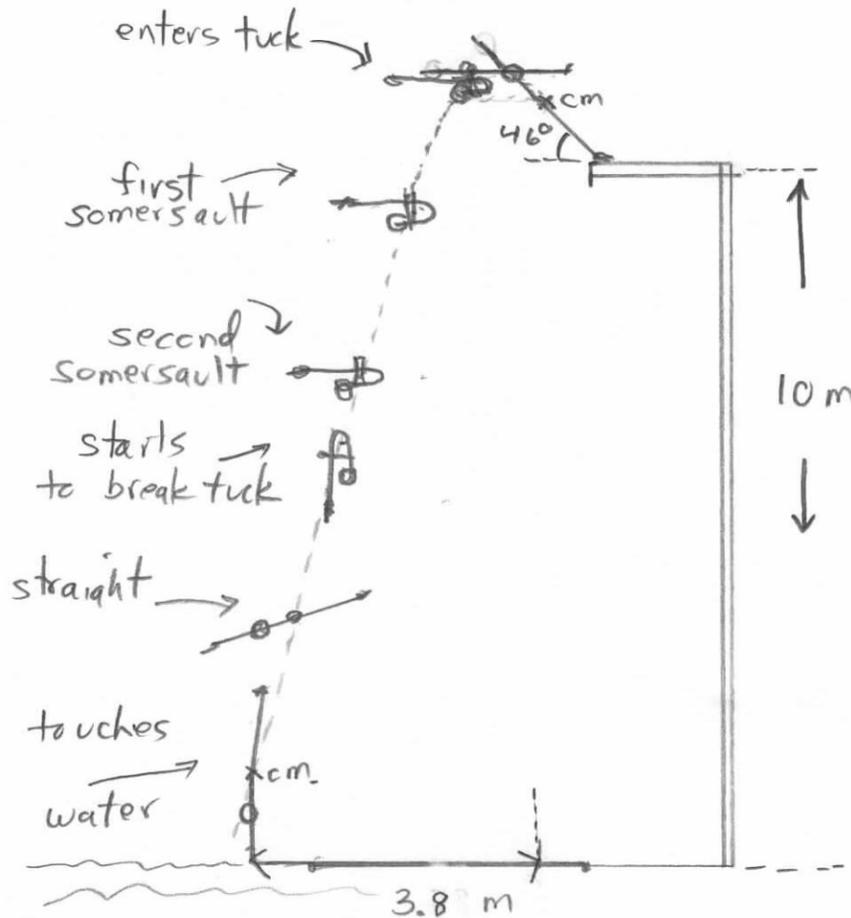


Problem 5: Platform Diving

In the 2002 World Cup Trials, Kyle Prandi set up a diving record with a back $3\frac{1}{2}$ somersault pike from the 10 m board. He pushed off from the board at an angle of $\theta_0 = 46^\circ$ with an initial speed $v_0 = 3.3\text{m/s}$. You may assume that his body was completely straight with his arms stretched above his head when he jumped.



He took .33 seconds to enter into a tuck after completing $\frac{1}{2}$ a rotation. At the .49 second mark he returned to his starting height. Once in a full tuck, he completed 2 revolutions at the 1.1 second mark. At that point he began to straighten out which he finished at the 1.47 second mark after making $\frac{1}{4}$ rotation. He made 1 more rotation, when his fingers touched the water 1.65 seconds after he left the platform. When he touched the surface his legs were bent but his center of mass was 1.3 m above the surface of the water. Kyle is 1.7 m long and when his arms are straight out above his head, his length is 2.2 m. His mass is 63 kg. His center of mass is 0.9 m above his soles. You may see the jump at

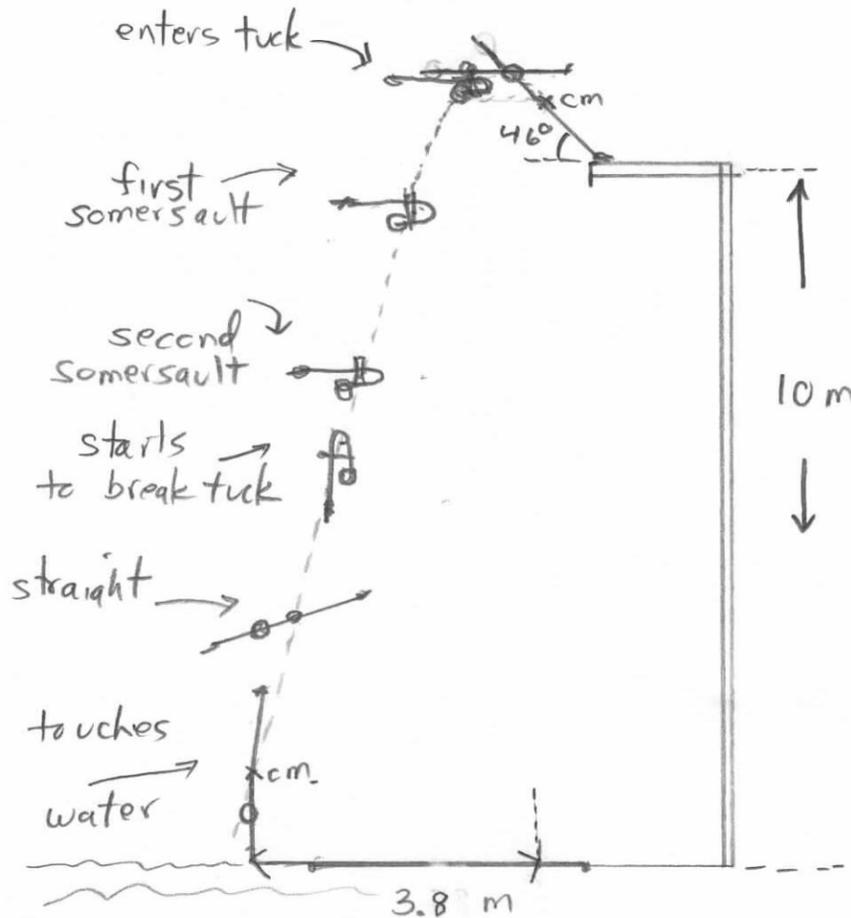
http://www.usadiving.org/USD_03redesign/media/video.htm

Somersaults	Dive details	time	Vertical distance starting point
zero	start	0	0
.25	Enters full pike	.33	
.75	Returns to starting height	.49	
1.25	Completes first turn in full pike	.73	
2.25	Completes second turn in full pike	1.1	
2.5	Starts to straighten	1.2	
3.4	Completely straight	1.47	
3.5	Fingers touch water	1.65	

- a) Based on the above information, make a graph of his angular velocity as a function of time. Indicate any assumptions that you have made for the various stages of his motion.
- b) Based on the initial conditions, calculate his vertical distance from his starting point at the various times indicated in the table.
- c) Explain whether or not you think his final angular velocity is equal to his initial angular velocity?
- d) Let I_0 denote his moment of inertia about his center of mass just after he left the board. Let I_1 denote his moment of inertia about his center of mass when he is in a full tuck. Let After he pulled his body into a tuck, by what fraction, $(I_1 - I_0)/I_0$, did his moment of inertia change?
- e) Suppose when he goes into a tuck, he has reduced his length by a factor of 2. Does the ratio, I_1 / I_0 , agree reasonably with your angular velocity data? Explain your answer.

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- Based on the initial conditions, calculate his vertical distance from his starting point at the various times indicated in the table.
- Explain whether or not you think his final angular velocity is equal to his initial angular velocity?
- Let I_0 denote his moment of inertia about his center of mass just after he left the board. Let I_1 denote his moment of inertia about his center of mass when he is in a full tuck. Let After he pulled his body into a tuck, by what fraction, $(I_1 - I_0)/I_0$, did his moment of inertia change?
- Suppose when he goes into a tuck, he has reduced his length by a factor of 2. Does the ratio, I_1/I_0 , agree reasonably with your angular velocity data? Explain your answer.

Solution:

a) We are given the sequence t_i of time marks and corresponding number of rotations n_i . To calculating the angular velocity at time t , we divide the difference between the number of rotations at this time and the number of rotations at the previous time by the corresponding difference in time and multiply the result by 2π , i.e. we approximate

$$\omega(t_i) \approx 2\pi \frac{n_i - n_{i-1}}{t_i - t_{i-1}}.$$

In addition, since the angular momentum about the diver's center of mass is conserved, and his moment of inertia at the end is equal to his initial moment of inertia, we take his initial angular velocity to be equal to the final angular velocity. The resulting points are presented in the table below and plotted on Fig. 3

t, s	0	0.33	0.49	0.73	1.1	1.2	1.47	1.65
# of somersaults	0	0.25	0.75	1.25	2.25	2.5	3.4	3.5
$\omega, 1/s$	3.5	4.8	19.6	13.1	17.	15.7	20.9	3.5
Height, m	10.6	10.9	10.6	9.8	7.3	6.4	3.5	1.2

Note that the fluctuations in the angular velocity are likely to be due to inaccuracy in measuring the rotational angle and the associated time. We expect the diver's angular velocity to be constant while in the tuck.

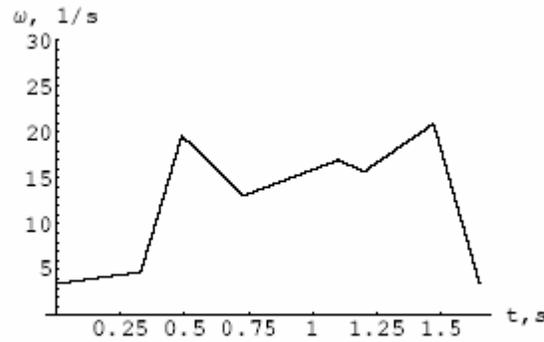


Figure 3: Angular velocity vs. time

b) Consider the motion of the center of mass. The vertical position is

$$y(t) = y_0 + v_{0y}t - \frac{gt^2}{2}$$

where $v_{0y} = v_0 \sin 46^\circ \approx 2.4$ m/s and $y_0 = 10 + 0.9 \sin 46^\circ = 10.6$ m. The values calculated using this formula are presented in the table above.

c) His final angular velocity must be equal to his initial angular velocity because he is straighten out both in the beginning and in the end, so his moment of inertia is the same in both cases. In addition, the angular momentum about his center of mass $L_{cm} = I_{cm}\omega$ is conserved, so the angular velocities must be the same. In fact, we have already used this fact to make the plot of angular velocity vs. time.

d) Let us denote by ω_0 his angular velocity when he leaves the ground and by ω_1 his angular velocity while in the tuck. To find ω_1 , we average ω over the first two and the last points on the graph and to find ω_0 we average ω over the middle portion of the graph. Thus $\omega_0 \approx 3.9$ 1/s and $\omega_1 \approx 17.3$ 1/s. Using conservation of angular momentum, $L_1 = I_1\omega_1 = L_0 = \omega_0 I_0$, we find

$$\frac{I_1 - I_0}{I_0} = \frac{\omega_0}{\omega_1} - 1 \approx -0.8$$

e) We expect $I \propto l^2$ where l is the diver's length. We find

$$\sqrt{\frac{I_0}{I_1}} = \sqrt{\frac{\omega_1}{\omega_0}} \approx 2.1$$

which is in excellent agreement with the fact that his length has changed by a factor of 2.