

Your Name _____ Section _____

HOMEWORK #12 - 8.01 MIT - Prof. Kowalski

Due 4:00PM Wednesday Nov. 26, 2003

Topics: Universal Gravity and Harmonic Oscillators

Any following problems designated with a bold number indicate problems from Young and Freedman 11th edition.

1. Whither the Moon?

Consider the earth-moon system. The earth has $M_e = 5.98 \times 10^{24}$ kg, $R_e = 6.378 \times 10^6$ m, $I_e = 0.331 M_e R_e^2$, and $T_{\text{day}} = 86164$ sec (with respect to fixed stars). The moon has $M_m = 7.35 \times 10^{22}$ kg, $T_{\text{orbit}} = 27.3$ days (with respect to fixed stars, called the sidereal period as opposed to the lunar month), and is $D_{em} = 3.84 \times 10^8$ m from the center of the earth. Approximate the earth as lying at the center of mass of the system, and ignore the contribution of the moon's spin to its angular momentum. Also ignore any complications due to the presence of the sun.

The two bodies exert tidal forces on each other, which are strong enough to have slowed the moon's spin so that it always presents the same face to the earth. These tidal forces have the same effect on the earth: the drag of moving the earth through the ocean's tides reduces earth's spin angular momentum and dissipates energy.

- a. What is the effect of the tides: consider the variables of angular momentum about the center of the earth and rotational kinetic energy. What happens to each of these for the earth, the moon, and the earth-moon system?
- b. What configuration of the two bodies is stable against further change by the tidal forces? (i.e. describe the final configuration of this system?) Specify the final values for the length of the day, T_{day} and earth-moon separation, D_{em} . Note that you will find helpful the results of the CyberTutor problem on L(R) for a circular orbit.

2. **13.58**

3. **13.60**

4. **Happy Thanksgiving!**

Universal Gravity and Harmonic Oscillators.

1) Whither the Moon?

a) TIDES: The water (and to some extent the liquid core) of the earth bulge out toward and away from the moon to the fact that its gravitational attraction is stronger on the side of the earth toward the moon than on the center of the mass of the earth itself, so the water is pulled up relative the earth. On the far side of the earth (away from the moon) the moon's gravitational force is weaker and this water is left behind this accelerating earth, producing another tidal bulge. Therefore there are two high tides per day (in most places on the earth).

The height of the tide is scaled by $\sim \frac{a_{tidal}}{g}$.

$$a_{tidal} = \Delta g_{moon} = GM_m \left(\frac{1}{(D_{em} + R_e)^2} - \frac{1}{D_{em}^2} \right) \approx -\frac{2GM_m R_e}{D_{em}^3} \approx 10^{-7} g$$

Where $D_{em} = 3.84 \times 10^8$ m.

This produces tides about 1 m high (Since $R_e = 6.378 \times 10^6$ m) in mid-ocean but it can be magnified by over an order of magnitude by resonances with bays like the Irish Sea and the Bay of Fundy.

There are no torques on the earth moon system, so its total angular momentum is conserved:

As the earth rotates underneath the tidal bulges, energy is lost. The angular momentum of the earth decreases as well. Moreover, the tides are carried along by the earth rotation. This means that the bulge closer to the moon pulls the moon ahead in its orbit, increasing the angular momentum of the moon.

The initial angular momentum is:

$$L_i = I_e \omega_e + M_m D_{em}^2 \omega_m = M_m D_{em}^2 \omega_m \left(1 + \frac{I_e \omega_e}{M_m D_{em}^2 \omega_m}\right) = 1.203 M_m D_{em}^2 \omega_m$$

Note that the spin angular momentum of the earth is much smaller than that of the moon in its orbit.

b) FINAL CONFIGURATION:

The tidal dissipation will slow the earth until one of two things happens:

- 1) The moon is driven away from the still spinning earth.
- 2) The earth slows until one side faces the moon all the time. At this point the length of the day and the length of the lunar month will be the same.

The actual outcome is # 2 due to the much smaller angular momentum of the earth's spin relative to the moon's orbital angular momentum. The final angular momentum is easy to find as the earth-moon system all rotates at ω_f and so one simply needs the combined moment of inertia:

$$L_f = (I_e + M_m D_f^2) \omega_f$$

In fact one can approximate the final angular momentum by neglecting the final angular momentum of the earth to first approximation:

$$L_f \approx M_m D_f^2 \omega_f$$

From:

$$L_f = L_i \quad (\text{no torque on earth-moon system})$$

We get:

$$M_m D_f^2 \omega_f = 1.203 M_m D_{em}^2 \omega_m$$

From *Kepler's* law ♣ we know that:

$$\omega = \text{const. } D^{-\frac{3}{2}}$$

$$\therefore \text{const. } D_f^2 D_f^{-\frac{3}{2}} = \text{const. } D_{em}^2 D_{em}^{-\frac{3}{2}} \times 1.203 \Rightarrow \boxed{D_f = (1.203)^2 D_{em} = 1.45 D_{em}}$$

$$T_{final} = \left(\frac{D_f}{D_{em}}\right)^{\frac{3}{2}} T_{orbit} = 1.74 \times 27.3 \text{ days} = 47.5 \text{ days}$$

♣:

$$\text{Kepler: } T \sim a^{\frac{3}{2}} \quad (a \equiv \text{semi major axis} \sim D)$$

$$\omega \sim \frac{1}{T} \sim D^{-\frac{3}{2}} \quad D \equiv (\text{distance to moon})$$

$$\text{Scaling via Kepler's law} \Rightarrow T_f = \left(\frac{D_f}{d}\right)^{\frac{3}{2}} T_{orbit}$$