

$$F = C_1rv + C_2r^2v^2 \quad F = \frac{mMG}{r^2} \quad \mathbf{F} = \frac{d\mathbf{p}}{dt} = m\mathbf{a} \quad dW = \mathbf{F} \cdot d\mathbf{r} \quad a_{cent} = \frac{v^2}{r} = \omega^2r$$

$$U = \frac{-mMG}{r} \quad U = mgh \quad U = \frac{1}{2}kx^2 \quad K = \frac{1}{2}mv^2 \quad K = \frac{1}{2}I\omega^2$$

$$E_{\text{tot}} = K + U = \frac{1}{2}mv^2 - \frac{mMG}{r} = \frac{-mMG}{2a} \quad \mathbf{L} = \mathbf{r} \times \mathbf{p} \quad I = \sum_i m_i r_i^2$$

$$m_1r_1 = m_2r_2 \quad v = \omega r \quad T^2 = \frac{4\pi^2(r_1 + r_2)^3}{G(m_1 + m_2)} \quad \omega = \frac{d\theta}{dt} \quad \omega = \sqrt{k/m}$$

$$\boldsymbol{\tau} = \mathbf{r} \times \mathbf{F} = I\boldsymbol{\alpha} = \frac{d\mathbf{L}}{dt} \quad \alpha = \frac{d\omega}{dt} \quad T = \frac{2\pi}{\omega} \quad L = I\omega \quad \mathbf{I} = \int_0^{\Delta t} \mathbf{F} dt = \mathbf{p}_f - \mathbf{p}_i$$

$$\omega = \sqrt{g/l} \quad \omega_{\text{pr}} = \frac{\tau}{L_s} \quad T^2 = \frac{4\pi^2a^3}{GM}$$

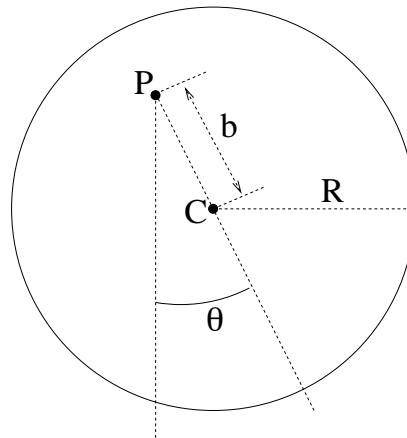
$$\text{Solid disk of mass } M \text{ and radius } R \text{ rotating about its cylindrical axis: } I = \frac{1}{2}MR^2$$

$$v_f - v_i = -u \ln \left(\frac{m_f}{m_i} \right) - gt \quad I = I_{\text{cm}} + Md^2 \quad I_z = I_x + I_y$$

$$f' = f \left(1 + \frac{v}{c} \cos \theta \right) \quad \lambda' = \lambda \left(1 - \frac{v}{c} \cos \theta \right)$$

Problem 3 (35 points)

A solid, uniform disk of mass M and radius R is oscillating about an axis through P . The axis is perpendicular to the plane of the disk. Friction at P is negligibly small and can be ignored. The distance from P to the center, C , of the disk is b (see figure). The gravitational acceleration is g .



- (7) When the displacement angle is θ , what then is the torque relative to point P ?
- (7) What is the moment of inertia for rotation about the axis through P ?
- (7) The torque causes an angular acceleration about the axis through P . Write down the equation of motion in terms of the angle θ and the angular acceleration.

As the disk oscillates, the maximum displacement angle, θ_{\max} , is very small, and the motion is a near perfect simple harmonic oscillation.

- (7) What is the period of oscillation?
- (7) As the disk oscillates, is there any force that the axis at P exerts on the disk? Explain your answer.

Problem 3 35 points

7 pts **a)** $\tau_p = |\vec{r}_p \times \vec{F}| = -bMg \sin \theta$

7 pts **b)** $I_p = I_c + Mb^2 = \frac{1}{2}MR^2 + Mb^2$

7 pts **c)** $\sum \tau_p = I_p \alpha$
 $-bMg \sin \theta = (\frac{1}{2}MR^2 + Mb^2)\ddot{\theta}$
 $\Rightarrow \ddot{\theta} + \frac{bg}{\frac{1}{2}R^2 + b^2} \sin \theta = 0$

7 pts **d)** Under the small angle approximation, $\sin \theta \approx \theta$, and the equation of motion is given by $\ddot{\theta} + \frac{bg}{\frac{1}{2}R^2 + b^2} \theta = 0$ which is simple harmonic with an angular frequency given by $\omega = \sqrt{\frac{bg}{\frac{1}{2}R^2 + b^2}}$. The period is given by $T = \frac{2\pi}{\omega} = 2\pi \sqrt{\frac{\frac{1}{2}R^2 + b^2}{bg}}$.

7 pts **e)** There must be a force at P or the CM would accelerate straight downwards.