

MASSACHUSETTS INSTITUTE OF TECHNOLOGY
Department of Physics

Physics 8.01 TEAL

Fall Term 2004

Exam 3: Equation Summary

Momentum:

$$\vec{p} = m\vec{v}, \quad \vec{F}_{ave} \Delta t = \Delta \vec{p}, \quad \vec{F}_{ext}^{total} = \frac{d\vec{p}^{total}}{dt}$$

$$\text{Impulse: } \vec{I} \equiv \int_{t=0}^{t=t_f} \vec{F}(t) dt = \Delta \vec{p}$$

$$\text{Torque: } \vec{\tau}_S = \vec{r}_{S,P} \times \vec{F}_P \quad |\vec{\tau}_S| = |\vec{r}_{S,P}| |\vec{F}_P| \sin \theta = r_{\perp} F = r F_{\perp}$$

Static Equilibrium:

$$\vec{F}_{total} = \vec{F}_1 + \vec{F}_2 + \dots = \vec{0}; \quad \vec{\tau}_S^{total} = \vec{\tau}_{S,1} + \vec{\tau}_{S,2} + \dots = \vec{0}.$$

$$\text{Rotational dynamics: } \vec{\tau}_S^{total} = \frac{d\vec{L}_S}{dt}$$

$$\text{Angular Velocity: } \vec{\omega} = (d\theta/dt) \hat{k}$$

$$\text{Angular Acceleration: } \vec{\alpha} = (d^2\theta/dt^2) \hat{k}$$

$$\text{Fixed Axis Rotation: } \vec{\tau}_S = I_S \vec{\alpha}$$

$$\tau_S^{total} = I_S \alpha = I_S \frac{d\omega}{dt}$$

$$\text{Moment of Inertia: } I_S = \int_{body} dm (r_{\perp})^2$$

$$\text{Angular Momentum: } \vec{L}_S = \vec{r}_{S,m} \times m\vec{v},$$

$$|\vec{L}_S| = |\vec{r}_{S,m}| |m\vec{v}| \sin \theta = r_{\perp} p = r p_{\perp}$$

Angular Impulse:

$$\vec{J}_S = \int_{t_0}^{t_f} \vec{\tau}_S dt = \Delta \vec{L}_S = \vec{L}_{S,f} - \vec{L}_{S,0}$$

Rotation and Translation:

$$\vec{L}_S^{total} = \vec{L}_S^{orbital} + \vec{L}_{cm}^{spin},$$

$$\vec{L}_S^{orbital} = \vec{r}_{S,cm} \times \vec{p}^{total},$$

$$\vec{L}_{cm}^{spin} = I_{cm} \vec{\omega}_{spin}$$

$$\vec{\tau}_S^{orbit} = \frac{d\vec{L}_S^{orbit}}{dt}, \quad \vec{\tau}_{cm}^{spin} = \frac{d\vec{L}_{cm}^{spin}}{dt}$$

Rotational Energy: $K_{cm} = \frac{1}{2} I_{cm} \omega_{cm}^2$

Rotational Power: $P_{rot} \equiv \frac{dW_{rot}}{dt} = \vec{\tau}_s \cdot \vec{\omega} = \tau_s \omega = \tau_s \frac{d\theta}{dt}$

One Dimensional Kinematics: $\vec{v} = d\vec{r} / dt$, $\vec{a} = d\vec{v} / dt$

$$v_x(t) - v_{x,0} = \int_{t'=0}^{t'=t} a_x(t') dt' \quad x(t) - x_0 = \int_{t'=0}^{t'=t} v_x(t') dt'$$

Constant Acceleration:

$$\begin{aligned} x(t) &= x_0 + v_{x,0}(t-t_0) + \frac{1}{2} a_x(t-t_0)^2 & v_x(t) &= v_{x,0} + a_x(t-t_0) \\ y(t) &= y_0 + v_{y,0}(t-t_0) + \frac{1}{2} a_y(t-t_0)^2 & v_y(t) &= v_{y,0} + a_y(t-t_0) \end{aligned}$$

where $x_0, v_{x,0}, y_0, v_{y,0}$ are the initial position and velocities components at $t = t_0$

Newton's Second Law: Force, Mass, Acceleration

$$\vec{F} \equiv m\vec{a} \quad \vec{F}^{total} = \vec{F}_1 + \vec{F}_2 \quad F_x^{total} = ma_x \quad F_y^{total} = ma_y \quad F_z^{total} = ma_z$$

Newton's Third Law: $\vec{F}_{1,2} = -\vec{F}_{2,1}$

Force Laws:

Universal Law of Gravity: $\vec{F}_{1,2} = -G \frac{m_1 m_2}{r_{1,2}^2} \hat{r}_{1,2}$, attractive

Gravity near surface of earth: $\vec{F}_{grav} = m_{grav} \vec{g}$, towards earth

Contact force: $\vec{F}_{contact} = \vec{N} + \vec{f}$, depends on applied forces

Static Friction: $0 \leq f_s \leq f_{s,max} = \mu_s N$ direction depends on applied forces

Kinetic Friction: $f_k = \mu_k N$ opposes motion

Hooke's Law: $F = k |\Delta x|$, restoring

Kinematics Circular Motion: arc length: $s = R\theta$; angular velocity: $\omega = d\theta/dt$
 tangential velocity: $v = R\omega$; angular acceleration: $\alpha = d\omega/dt = d^2\theta/dt^2$; tangential acceleration $a_\theta = R\alpha$.

Period: $T = \frac{2\pi R}{v} = \frac{2\pi}{\omega}$; **frequency:** $f = \frac{1}{T} = \frac{\omega}{2\pi}$,

Radial Acceleration: $|a_r| = R\omega^2$; $|a_r| = \frac{v^2}{R}$; $|a_r| = 4\pi^2 R f^2$; $|a_r| = \frac{4\pi^2 R}{T^2}$

Center of Mass: $\vec{R}_{cm} = \sum_{i=1}^{i=N} m_i \vec{r}_i / m^{total} \rightarrow \int_{body} dm \vec{r} / m^{total}$;

Velocity of Center of Mass: $\vec{V}_{cm} = \sum_{i=1}^{i=N} m_i \vec{v}_i / m^{total} \rightarrow \int_{body} dm \vec{v} / m^{total}$

Torque: $\vec{\tau}_S = \vec{r}_{S,P} \times \vec{F}_P$ $|\vec{\tau}_S| = |\vec{r}_{S,P}| |\vec{F}_P| \sin \theta = r_{\perp} F = r F_{\perp}$

Static Equilibrium: $\vec{F}_{total} = \vec{F}_1 + \vec{F}_2 + \dots = \vec{0}$; $\vec{\tau}_S^{total} = \vec{\tau}_{S,1} + \vec{\tau}_{S,2} + \dots = \vec{0}$.

Kinetic Energy: $K = \frac{1}{2}mv^2$; $\Delta K = \frac{1}{2}mv_f^2 - \frac{1}{2}mv_0^2$

Work: $W = \int_{r_0}^{r_f} \vec{F} \cdot d\vec{r}$; **Work- Kinetic Energy:** $W^{total} = \Delta K$

Power: $P = \vec{F} \cdot \vec{v} = dK/dt$

Potential Energy: $\Delta U = -W_{conservative} = -\int_A^B \vec{F}_c \cdot d\vec{r}$

Potential Energy Functions with Zero Points:

Constant Gravity: $U(y) = mgy$ with $U(y_0 = 0) = 0$.

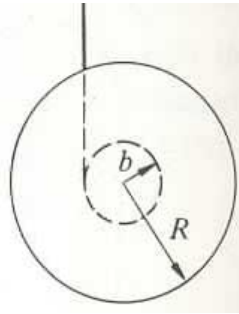
Inverse Square Gravity: $U_{gravity}(r) = -\frac{Gm_1 m_2}{r}$ with $U_{gravity}(r_0 = \infty) = 0$.

Hooke's Law: $U_{spring}(x) = \frac{1}{2}kx^2$ with $U_{spring}(x = 0) = 0$.

Work- Mechanical Energy: $W_{nc} = \Delta K + \Delta U^{total} = \Delta E_{mech} = (K_f + U_f^{total}) - (K_0 + U_0^{total})$

Problem 3: Torque, Rotation and Translation

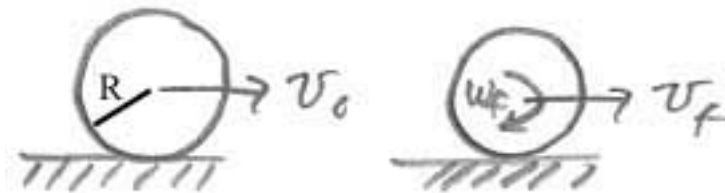
A Yo-Yo of mass m has an axle of radius b and a spool of radius R . Its moment of inertia can be taken to be $I = (1/2)mR^2$ and the thickness of the string can be neglected. The Yo-Yo is released from rest.



- a) What is the tension in the cord as the Yo-Yo descends?
- b) Use conservation of energy to find the angular velocity of the Yo-Yo when it reaches the bottom of the string.
- c) What happens to the Yo-Yo at the bottom of the string?

Problem 4: Rolling Cylinder

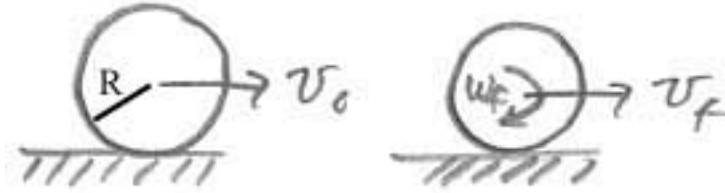
A solid cylinder of mass m and radius R is initially thrown along a wooden floor hallway with an initial speed v_0 and zero initial angular velocity $\omega_0 = 0$ as shown in the figure below.



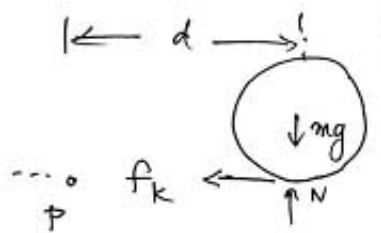
- a) What is the moment of inertia of the cylinder about the axis of rotation of the cylinder?
- b) What effects does friction have on the cylinder before it reaches its final speed? Is the friction static or kinetic? Which direction does it point?
- c) What is the angular speed, ω_f , of the cylinder when it just starts to roll without slipping? Express your answer in terms of m , R , and v_0 .

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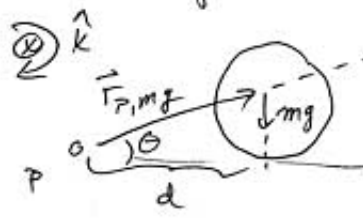


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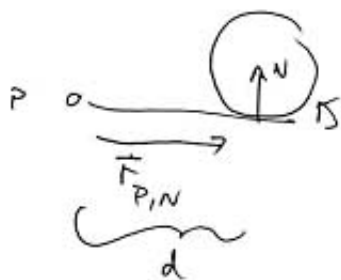


$$\vec{\tau}_P = \vec{r}_{P,f_k} \times \vec{f}_k + \vec{r}_{P,mg} \times \vec{mg} + \vec{r}_{P,N} \times \vec{N}$$

$\vec{\tau}_{P,f_k} = \vec{r}_{P,f_k} \times \vec{f}_k = 0$ because \vec{r}_{P,f_k} and \vec{f}_k are antiparallel.



$$\vec{\tau}_{P,mg} = \vec{r}_{P,mg} \times \vec{mg} = r_{P,mg} \sin \theta \, mg \, \hat{k} = d \, mg \, \hat{k}$$



$$\vec{\tau}_{P,N} = \vec{r}_{P,N} \times \vec{N} = -d N \, \hat{k}$$

in the vertical direction

$$N - mg = 0$$

Thus $\vec{\tau}_{P,mg} + \vec{\tau}_{P,N} = (dmg - dN) \hat{k} = \vec{0}$

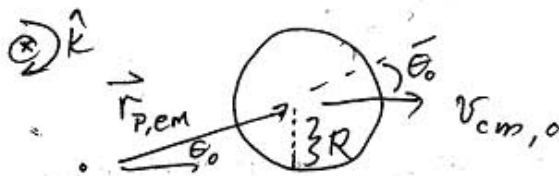
So $\vec{\tau}_P^{\text{total}} = 0 \Rightarrow \vec{L}_{P,0} = \vec{L}_{P,A}$

Angular momentum about any point lying on the line of contact is constant throughout motion.

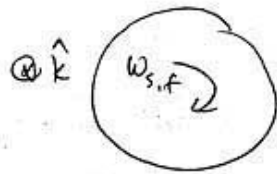
$$\underbrace{\vec{L}_P^{\text{total}}}_{\text{eng. mom. about } P} = \underbrace{\vec{L}_{P,cm}^{\text{orbital}}}_{\text{orbital eng. mom. about } P} + \underbrace{\vec{L}_{cm}^{\text{spin}}}_{\text{spin eng. mom. about cm}}$$

$$\vec{L}_P^{\text{total}} = \underbrace{\vec{r}_{P,cm}}_{\text{treat body as a point particle located at cm}} \times m \vec{v}_{cm} + \vec{I}_{cm} \vec{\omega}_{cm}$$

$$\vec{L}_{P,0}^{\text{orbital}} = \vec{r}_{P,cm} \times m \vec{v}_{cm,0} = \underbrace{r_{P,cm} \sin \theta_0}_{= R} m v_{cm,0} \hat{k}$$

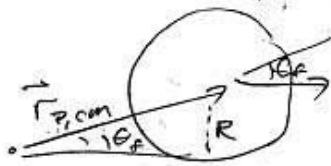


$$\vec{L}_{P,0}^{\text{total}} = R m v_{cm,0} \hat{k}$$



$$\begin{aligned} \vec{L}_{cm,f}^{\text{spin}} &= I_{cm} \omega_{s,f} \hat{k} \\ &= \frac{1}{2} m R^2 \omega_{s,f} \hat{k} \end{aligned}$$

$\vec{\omega}_f$ points in $+\hat{k}$ direction.



$$\begin{aligned} \vec{L}_{P,f}^{\text{orbital}} &= R m v_{cm,f} \hat{k} \\ &\text{(same as above)} \end{aligned}$$

$$\begin{aligned} \vec{L}_{P,f}^{\text{total}} &= R m v_{cm,f} \hat{k} + \frac{1}{2} m R^2 \omega_{s,f} \hat{k} \\ \vec{L}_{P,0}^{\text{total}} &= \vec{L}_{P,f}^{\text{total}} \Rightarrow \end{aligned}$$

$$\hat{k}: R m v_{cm,0} = R m v_{cm,f} + \frac{1}{2} m R^2 \omega_{s,f} \quad (1)$$

rolling without slipping condition: (2)

$$v_{cm,f} = R \omega_{s,f}$$

substitute into eq (1) \Rightarrow

$$R m v_{cm,0} = R m v_{cm,f} + \frac{1}{2} m R^2 \frac{v_{cm,f}}{R}$$

\Rightarrow

$$R m v_{cm,0} = \frac{3}{2} R m v_{cm,f}$$

\Rightarrow

$$v_{cm,f} = \frac{2}{3} v_{cm,0}$$