

- e) Calculate the impulse required at the second rocket burn (at aphelion).
- f) Compare the impulse under b) with the sum of the impulses under c) and e), and convince yourself that the latter procedure is more economical.
- g) Make a specific quantitative comparison for  $r = 20R$ .

#### 8.4 ~~Black hole in X-Ray Binary~~

~~An X-ray binary consists of 2 stars with masses  $m_1$  (the accreting compact object) and  $m_2$  (the donor). The orbits are circular with radii  $r_1$  and  $r_2$  centered on the center of mass.~~

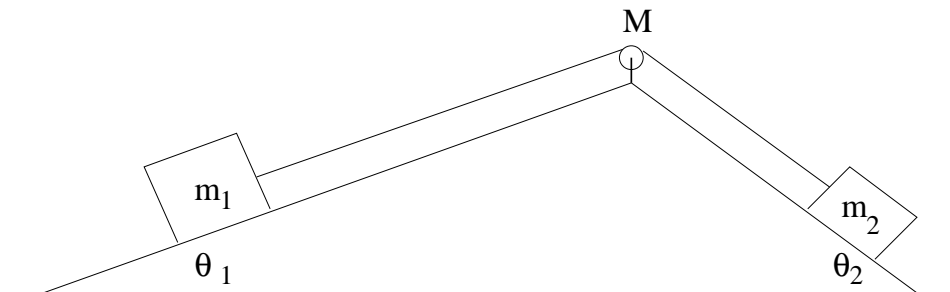
- a) ~~Derive the orbital period of the binary following the guidelines given in lectures.~~
- b) ~~In the case of Cyg X-1 (as discussed in lectures), the orbital period is 5.6 days. The donor star is a “supergiant” with a mass 30 times that of the sun. Doppler shift measurements indicate that the donor star has an orbital speed  $v_2$  of about 148 km/sec. Calculate  $r_2$ .~~
- c) ~~Calculate  $r_1$ . Your calculations will be greatly simplified if instead of  $r_1$  you set up your equations in terms of  $r_1/r_2$ . Once you have solved for  $r_1/r_2$ , you have found  $r_1$  as you already know  $r_2$  (see part b). You will find a third order equation in  $r_1/r_2$ . Only one solution is real; the other two are imaginary. There are various ways to find an approximation for  $r_1/r_2$ . You can find the solution by trial and error using your calculator, or you can plot the function.~~
- d) ~~Now calculate the mass  $m_1$  of the accreting compact object.~~  
~~As discussed in lectures, since this turns out to be substantially larger than 3 times the mass of the sun, it is strongly believed to be a black hole.~~

#### 8.5 ~~Rolling and Slipping Hoop~~ page 352, problem 45

#### 8.6 ~~Physical Pendulum~~ page 409, problem 34

#### 8.7 Two Blocks, Two Slopes and a Pulley

Two blocks  $m_1$  and  $m_2$  are connected by a light string (with negligible mass) passing over a pulley with mass  $M$  and radius  $R$  as shown. The pulley is a solid uniform disk, and the friction between it and the rope is such that when the blocks move the rope turns the pulley without slipping. The kinetic friction coefficient between the blocks and the slope is  $\mu$ . Find the acceleration of the blocks, the angular acceleration of the pulley, and the tension in each part of the rope. Assume that  $m_2$  is much larger than  $m_1$  (the blocks go from left to right), and that the bearings of the pulley rotate without friction.



#### 8.8 Doppler Effect I – page 462, problem 29

## Problem 8.7

We assume that the mass  $m_2$  is sufficiently larger than the mass  $m_1$  so that  $m_2$  accelerates down. This determines which direction the kinetic friction acts for each block. The force equation for  $m_2$  in the direction down the slope is

$$m_2 a = \sum F = m_2 g \sin \theta_2 - \underbrace{\mu m_2 g \cos \theta_2}_{\text{fric}} - T_2 \quad \Rightarrow$$

$$T_2 = m_2 g \sin \theta_2 - \mu m_2 g \cos \theta_2 - m_2 a \quad (1)$$

where  $a$  is the acceleration of  $m_2$  directed *down* the slope and  $T_2$  is the tension for that portion of the string. The force equation for  $m_1$  in the direction up the slope is

$$m_1 a = \sum F = T_1 - m_1 g \sin \theta_1 - \underbrace{\mu m_1 g \cos \theta_1}_{\text{fric}} \quad \Rightarrow$$

$$T_1 = m_1 a + m_1 g \sin \theta_1 + \mu m_1 g \cos \theta_1 \quad (2)$$

where  $a$  is also the acceleration of  $m_1$  directed *up* the slope (assuming the string does not stretch) and  $T_1$  is the tension for that portion of the string. The torque equation for the pulley is

$$I\alpha = \sum \tau$$

$$\frac{1}{2}MR^2\alpha = RT_2 - RT_1 \quad \Rightarrow$$

$$\alpha = \frac{2(T_2 - T_1)}{MR} \quad (3)$$

where  $\alpha$  is the angular acceleration clockwise in the picture and  $I = \frac{1}{2}MR^2$  is the moment of inertia about the axle. (Please see Table 12.1 on page 309.) The coefficient of  $RT_2$  is positive because  $T_2$  acts to increase  $\alpha$ ; the coefficient of  $RT_1$  is negative because  $T_1$  acts to decrease  $\alpha$ . The constraint of no slipping between the pulley and the string requires that

$$a = R\alpha \quad (4)$$

Substituting Equation (4) into Equation (3) gives

$$a = \frac{2(T_2 - T_1)}{M} \quad (5)$$

Substituting Equations (??) and (??) into Equation (??) and solving for  $a$  gives

$$a = \frac{m_2 g (\sin \theta_2 - \mu \cos \theta_2) - m_1 g (\sin \theta_1 + \mu \cos \theta_1)}{m_1 + m_2 + \frac{1}{2}M} \quad (6)$$

Substituting Equation (??) into Equation (??) gives

$$T_2 = m_2 g \sin \theta_2 - \mu m_2 g \cos \theta_2 - m_2 \cdot \frac{m_2 g (\sin \theta_2 - \mu \cos \theta_2) - m_1 g (\sin \theta_1 + \mu \cos \theta_1)}{m_1 + m_2 + \frac{1}{2}M}$$

Substituting Equation (??) into Equation (??) gives

$$T_1 = m_1 \cdot \frac{m_2 g (\sin \theta_2 - \mu \cos \theta_2) - m_1 g (\sin \theta_1 + \mu \cos \theta_1)}{m_1 + m_2 + \frac{1}{2}M} + m_1 g \sin \theta_1 + \mu m_1 g \cos \theta_1$$

Substituting Equation (??) into Equation (??) gives

$$\alpha = \frac{a}{R} = \frac{m_2 g (\sin \theta_2 - \mu \cos \theta_2) - m_1 g (\sin \theta_1 + \mu \cos \theta_1)}{R \left( m_1 + m_2 + \frac{1}{2}M \right)}$$

### **Problem 8.8** ~~(Ohanian, page 462, problem 29)~~

~~The trumpeters on the train act as a moving emitter with speed  $V_E = 60 \text{ km/h} = 16.67 \text{ m/s}$  and with frequency  $\nu = 329.7 \text{ Hz}$ . The frequency heard by a listener on the ground (rest frame of the air) is given by Equation (13) on page 449 as~~

$$\nu' = \nu \left( \frac{1}{1 \pm \frac{V_E}{v_s}} \right)$$

~~where the  $-$  sign corresponds to an approaching emitter and the  $+$  sign corresponds to a receding emitter and  $v_s = 331 \text{ m/s}$  is assumed to be the speed of sound in air. While the train is approaching, the frequency heard is~~

$$\nu' \approx 347.2 \text{ Hz}$$

~~which is closest to the musical note E. (See Table 17.2 on page 441.) While the train is receding, the frequency heard is~~

$$\nu' \approx 313.9 \text{ Hz}$$

~~which is closest to the musical note D#. (See Table 17.2 on page 441.)~~