

### ↻ Last Lecture

- ↻ Intro to angular motion

### ↻ Today

- ↻ Statics and dynamics of rotational motion

### ↻ Important Concepts

- ↻ Equations for angular motion are mostly identical to those for linear motion with the names of the variables changed.
- ↻ Location where forces are applied is now important.
- ↻ Rotational inertia or moment of inertia (rotational equivalent of mass) depends on how the material is distributed relative to the axis.

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## Important Reminders

- ↻ Lectures will be M 11-12, T&W 10-12, F 11-12.
- ↻ Check schedule on web for new times and rooms for some recitations (all are still on Thursday).
- ↻ Switching of recitations will be permitted if you have a conflict with another IAP activity.
- ↻ Contact your tutor about session scheduling
  - ↻ Students working with Stephane Essame reassigned.
- ↻ Mastering Physics due this Wednesday at 10pm.
- ↻ Pset due this Friday at 11am.

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## Complete Description of Motion

- ↻ For an extended object, **all** of the equations learned last fall apply **exactly** without approximations to the motion of the center of mass.
- ↻ This is true whether or not an object is also rotating.
- ↻ The two motions (linear position of the center of mass and rotation around the center of mass) can be considered separately, **except** for kinetic energy where everything gets lumped into one equation.

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## Kinematics Variables

- |                    |                                 |
|--------------------|---------------------------------|
| ↻ Position $x$     | ↻ Angle $\theta$                |
| ↻ Velocity $v$     | ↻ Angular velocity $\omega$     |
| ↻ Acceleration $a$ | ↻ Angular acceleration $\alpha$ |
| ↻ Force $F$        | ↻ Torque $\tau$                 |
| ↻ Mass $M$         | ↻ Moment of Inertia $I$         |
| ↻ Momentum $p$     | ↻ Angular Momentum $L$          |

$$\omega = \frac{d\theta}{dt} \quad \alpha = \frac{d\omega}{dt} = \frac{d^2\theta}{dt^2}$$

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## Vector Nature Angular Motion

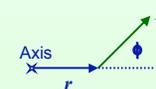
- ↻ The vector for any angular quantity ( $\theta$ ,  $\omega$ ,  $\alpha$ ,  $\tau$ ,  $J$ ) points along the axis with the direction given by a right-hand-rule.
  - ↻ Fingers curl in direction of  $\theta$ ,  $\omega$ ,  $\alpha$ ,  $\tau$ ,  $J$ , thumb points in the direction of the vector
- ↻ For most problems, all variables can be considered either clockwise (CW) or counter-clockwise (CCW).

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## Torque

- ↻ How do you make something rotate? Very intuitive!
  - ↻ Larger force clearly gives more "twist".
  - ↻ Force needs to be in the right direction (perpendicular to a line to the axis is ideal).
  - ↻ The "twist" is bigger if the force is applied farther away from the axis (bigger lever arm).
- ↻ In math-speak:  $\vec{\tau} = \vec{r} \times \vec{F}$   $|\tau| = |r||F|\sin(\phi)$



Torque is out of the page

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## More Ways to Think of Torque

- Magnitude of the force times the component of the distance perpendicular to the force (aka lever arm).
- Magnitude of the radial distance times the component of the force perpendicular to the radius.
- Direction from Right-Hand-Rule for cross-products and can also be thought of as clockwise (CW) or counter-clockwise (CCW).
- For torque, gravity acts at the center of mass.

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## Conditions for Equilibrium

- Same as before:  $\Sigma \vec{F} = 0$ 
  - It's totally irrelevant where the forces are applied to an object, only their direction and magnitude matters.
  - This gives one independent equation per dimension.
- Additional condition:  $\Sigma \vec{\tau} = 0$ 
  - This is true for **any** axis. However, if all of the forces are in the same plane (the only type of problem we will consider in this class), you only get **one** additional independent equation by considering rotation.

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## Equations for Dynamics

- Same as before:  $\Sigma \vec{F} = M\vec{a}$ 
  - Only the direction and magnitude of the forces matter.
  - This gives one independent equation per dimension.
- Additional condition:  $\Sigma \vec{\tau} = I\vec{\alpha}$ 
  - This is true for **any fixed** axis (for example, a pulley).
  - In addition, this equation holds for an axis through the center of mass, even if the object moves or accelerates.
  - As for statics, if all of the forces are in the same plane, you only get **one** additional independent equation by considering rotation.

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## Inertia in Rotation

- Depends linearly on the total mass
- Depends on how the mass is distributed. Mass farther from the axis is harder to rotate.
- The same object could have a different moment of inertia depending on the choice of axis.
- In the equation:  $\Sigma \vec{\tau} = I\vec{\alpha}$  all three quantities need to be calculated using the same axis (either a fixed axle or the center of mass).

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## Moment of Inertia

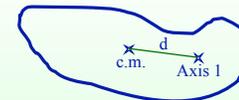
- Most easily derived by considering Kinetic Energy (to be discussed next week).
- $I = \Sigma m_i r_i^2 = \int r^2 dm$
- Some simple cases are given in the textbook on page 342, you should be able to derive those below except for the sphere. Will be on formula sheet.
  - Hoop (all mass at same radius)  $I = MR^2$
  - Solid cylinder or disk  $I = (1/2)MR^2$
  - Rod around end  $I = (1/3)ML^2$
  - Rod around center  $I = (1/12)ML^2$
  - Sphere  $I = (2/5)MR^2$

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## Parallel Axis Theorem

- Very simple way to find moment of inertia for a large number of strange axis locations.



- $I_1 = I_{c.m.} + Md^2$  where M is the total mass.

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