

**Problem 1: (~~Second Law Applications: synchronous satellite~~)**

~~A synchronous satellite goes around the earth once every 24 h, so that its position appears stationary with respect to a ground station. The mass of the earth is  $m_e = 5.98 \times 10^{24} \text{ kg}$ . The mean radius of the earth is  $r_e = 6.37 \times 10^6 \text{ m}$ . The universal constant of gravitation is  $G = 6.67 \times 10^{-11} \text{ N} \cdot \text{m}^2 / \text{kg}^2$ . Your goal is to find the radius of the orbit of a synchronous satellite that circles the earth.~~

- ~~a) Describe what motion models this problem. Write down a vector description of the acceleration. Clearly indicate your choice of unit vectors and your reasoning for choosing them.~~
- ~~b) What is the radius of the orbit of a synchronous satellite that circles the earth? Approximately how many earth radii is this distance?~~

**Problem 2: (Circular motion: banked turn)**

A car of mass  $m$  is going around a circular turn of radius  $R$  which is banked at an angle  $\theta$  with respect to the ground. Assume there is a coefficient of static friction  $\mu_s$  between the wheels and the road. Let  $g$  be the magnitude of the acceleration due to gravity. You may neglect kinetic friction. In each part below show your force diagrams.



- a) Describe the motion of the car if the car is traveling very slowly or extremely fast. Is it possible for the car to travel at speeds such that it can undergo circular motion? If so, what coordinate system best suits this problem. In particular, describe the acceleration vector in terms of your coordinate system unit vectors.
- b) Derive an expression for the minimum velocity necessary to keep the car moving in a circle without slipping down the embanked turn. Express your answer in terms of the given quantities.
- c) Derive an expression for the maximum velocity necessary to keep the car moving in a circle without slipping up the embanked turn. Express your answer in terms of the given quantities.
- d) Derive an expression for the velocity necessary to keep the car moving in a circle without slipping up or down the embanked turn such that the static friction force vanishes. Express your answer in terms of the given quantities.

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**Answer:**

a) If the car is going very slowly and  $\tan(\theta) > \mu_s$ , the car will turn along the bank and slide down. Otherwise it can undergo circular motion. If the car is going very fast and  $\cot(\theta) > \mu_s$ , the car will turn along the bank and slide up. There always is a range of speeds for which the car can undergo circular motion. In this case the motion is restricted to a plane and the position of the car will be given by  $\vec{R}$  the vector pointing from the center of rotation to the car..

b) In this case friction is directed upwards along the incline and equal to its maximum value  $\vec{f}_f = \mu_s |\vec{N}|$  where  $\vec{N}$  is the normal force exerted by the bank on the car. Newton's second law in the direction of  $\vec{N}$  then gives  $|\vec{N}| - mg \cos(\theta) = m \frac{v^2}{R} \sin(\theta)$ .

Along the incline instead it gives:  $mg \sin(\theta) - m\mu_s(g \cos(\theta) + \frac{v^2}{R} \sin(\theta)) = m \frac{v^2}{R} \cos(\theta)$  from which

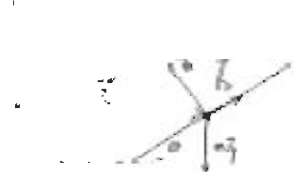
$$v = (Rg(\frac{\tan(\theta) - \mu_s}{1 + \mu_s \tan(\theta)}))^{\frac{1}{2}} = (Rg \tan(\theta - \alpha))^{\frac{1}{2}} \text{ where } \mu_s \equiv \tan(\alpha) .$$

c) This case simply corresponds to changing the direction of the friction force:  $mg \sin(\theta) + m\mu_s(g \cos(\theta) + \frac{v^2}{R} \sin(\theta)) = m \frac{v^2}{R} \cos(\theta)$  from which  $v = (Rg(\frac{\tan(\theta) + \mu_s}{1 - \mu_s \tan(\theta)}))^{\frac{1}{2}} = (Rg \tan(\theta + \alpha))^{\frac{1}{2}}$

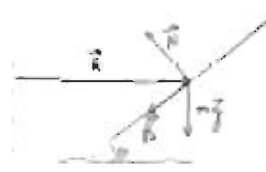
d) This case corresponds to  $\mu_s = 0 = \alpha$  so that:

$$v = (Rg \tan(\theta))^{\frac{1}{2}}$$

case 1



case 2



case 3

