

$$T = m\left(\frac{v^2}{R} + g \cos \theta\right)$$

Lowest Point: $\theta = 0^\circ$

$$F_n = 0; a_n = 0$$

Acceleration is only radial

$$T = m\left(\frac{v^2}{R} + g\right)$$

Highest Point: $\theta = \pi$

$$T = m\left(\frac{v^2}{R} - g\right)$$

Let $v = v_c$ when $T = 0$

$$0 = m\left(\frac{v_c^2}{R} - g\right)$$

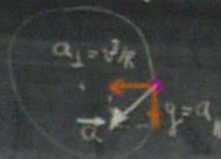
$$v_c = \sqrt{Rg}$$

lowest allowed speed!

$\theta = 90^\circ$

$$a_n = g$$

$$a_{\perp} = v^2/R$$



Example: loop-the-loop

$$m = 75 \text{ kg}$$

$$R = 10 \text{ m}$$

$v = 15 \text{ m/s}$ const.



Dynamics of Uniform Circular Motion

Speed: v , mass: m

Radius: R

$$a_{\perp} = \frac{v^2}{R} = \omega^2 R$$

$$F_c = m \frac{v^2}{R} = m \omega^2 R$$

If mass m moves in a circle with a speed v , the force F_c must be provided by some agent



Motion in a Vertical Circle

- Circular motion

- Non-uniform; speed increases/decreases

- v changes, energy/work

- \therefore have a_{\parallel} and a_{\perp}

$$F_{\parallel} = mg \sin \theta$$

$$F_{\perp} = T - mg \cos \theta$$

$$a_{\parallel} = \frac{F_{\parallel}}{m} = g \sin \theta$$

$$a_{\perp} = \frac{F_{\perp}}{m} = T - mg \cos \theta = \frac{mv^2}{R}$$



$$T = m\left(\frac{v^2}{R} + g \cos \theta\right)$$

Lowest Point: $\theta = 0^\circ$

$$F_{II} = 0; a_{II} = 0$$

Acceleration is only radial

$$T = m\left(\frac{v^2}{R} + g\right)$$

Highest Point: $\theta = \pi$

$$T = m\left(\frac{v^2}{R} - g\right)$$

Let $v = v_c$ when $T = 0$

$$0 = m\left(\frac{v_c^2}{R} - g\right)$$

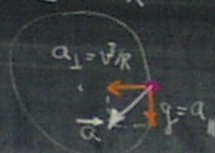
$$v_c = \sqrt{Rg}$$

lowest allowed speed!

$\theta = 90^\circ$

$$a_{II} = g$$

$$a_{\perp} = v^2/R$$



Example: loop-the-loop

$$m = 75 \text{ kg}$$

$$R = 10 \text{ m}$$

$$v = 15 \text{ m/s const.}$$



Bottom

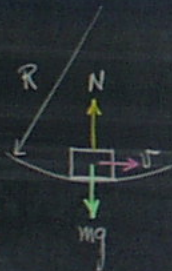
$$N - mg = m\frac{v^2}{R}$$

$$N = m\left(g + \frac{v^2}{R}\right)$$

$$= mg\left(1 + \frac{v^2}{gR}\right)$$

$$= mg\left(1 + \frac{1}{9.81} \times \frac{15^2}{10}\right)$$

$$= mg(3.29)$$



Top

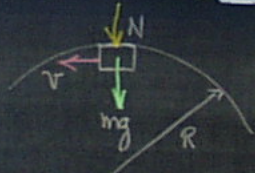
$$N + mg = m\frac{v^2}{R}$$

$$N = mg\left(\frac{v^2}{Rg} - 1\right)$$

$$= mg(2.29 - 1)$$

$$= 1.29 mg$$

$$\text{Suppose } \frac{v^2}{Rg} = 1 \Rightarrow v = \sqrt{Rg} = \sqrt{10 \times 9.81} = 9.9 \text{ m/s}$$



For $v = \sqrt{Rg}$

$N = 0$ Weight-less!

If $v < \sqrt{Rg}$?

What happens?

Work and Energy

Force \rightarrow acceleration
 \rightarrow velocity and position

Force \times Displacement = Work

Mass \times Velocity² = Kinetic Energy

Work + KE = Work-Energy Principle

\Rightarrow Analogous to Newton's 2nd Law.

Work-Energy Principle \rightarrow Cons. of Energy
Force \times Δ Time = Impulse \rightarrow Cons. of Momentum
Force \times Position = Torque \rightarrow Cons. of Ang. Mom.

Cons. Laws \leftrightarrow Symmetry

Mom. \leftrightarrow Inv. Transf. in Space

Energy \leftrightarrow Inv. Transf. in Time

Ang. Mom. \leftrightarrow Inv. to Rotations

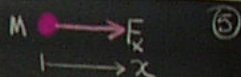
Work - 1D

- Constant Force

$$W = F_x \Delta x$$

\uparrow displacement
 \uparrow Force on particle
 \uparrow Work done

$$[W] = N \cdot m = \underline{\text{Joules}}$$



If:

$W > 0$ Force + Disp. same dir.

$W < 0$ Force + Disp. Opposite

$W < 0$ Work done by particle!!

Conical Pendulum

$$\Sigma F_x = T \sin \theta = \frac{mv^2}{R} \quad (1)$$

$$\Sigma F_y = T \cos \theta - mg = 0 \quad (2)$$

$$\tan \theta = v^2 / Rg$$

$$R = L \sin \theta$$

$$v = \frac{2\pi L \sin \theta}{T}$$

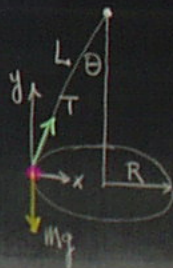
T = period of rotation

$$\text{Eq. (2)} \quad \cos \theta = \frac{g T^2}{4\pi^2 L}$$

$$v = 2\pi \sqrt{\frac{L}{g} \cos \theta}$$

$$L = 1.0 \text{ m}$$

$$R = 0.57 \text{ m}$$



if $v=0$, $W=0$

Ekstein Form

Lab Frame

Hamilton's eq.

$x \rightarrow 0$
 $x \rightarrow \infty$

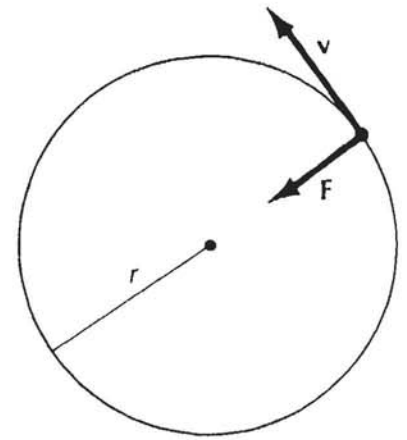
$x=b$ and the x -axis.

Dynamics of Uniform Circular Motion

13-16

For a particle moving with speed v in a circle of radius R , the centripetal acceleration is

$$a_c = \frac{v^2}{R}$$



Centripetal force for a particle in uniform circular motion.

If the particle has a mass m , the required force is given by

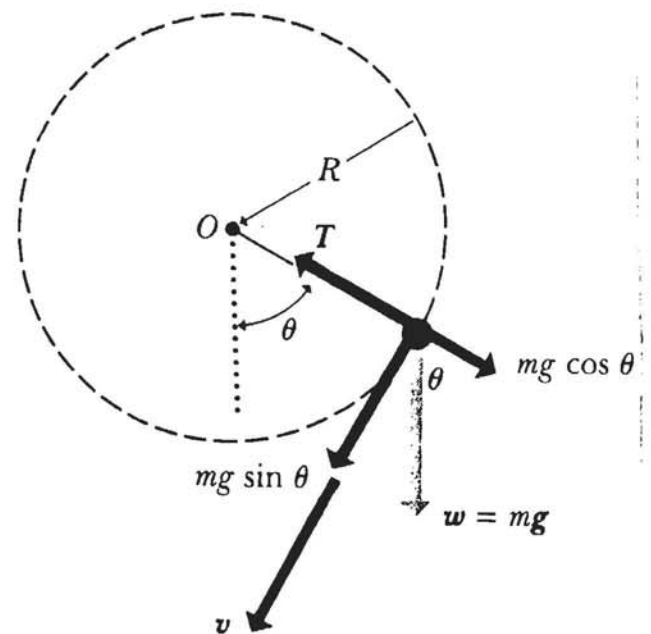
$$F_c = ma_c = \frac{mv^2}{R}$$

Whenever a particle moves in a circle with a speed v , this force must be provided by some external agent.

Motion in a Vertical Circle.

13-17

- Ball whirled in a vertical circle about point O.
- Motion circular but is not uniform.
- Speed increases on the way down, decreases on the way up.
- v changes continuously around the path.
- \therefore We must have a_{\perp} and a_{\parallel}
- Forces on ball are gravity and tension.



Forces on a body whirling in a vertical circle with center at O.

$$F_{\parallel} = mg \sin \theta \quad (1)$$

$$F_{\perp} = T - mg \cos \theta \quad (2)$$

The tangential acceleration:

$$a_{\parallel} = \frac{F_{\parallel}}{m} = g \sin \theta \quad (3)$$

$$a_{\perp} = \frac{F_{\perp}}{m} = \frac{T - mg \cos \theta}{m} = \frac{v^2}{R} \quad (4)$$

Solve (4) for $T = m \left(\frac{v^2}{R} + g \cos \theta \right)$

Lowest Point: $\theta = 0$

$F_{||} = 0$, $a_{||} = 0$, acceleration is purely radial.

$$T = m \left(\frac{v^2}{R} + g \right)$$

Highest Point: $\theta = 180^\circ$

- acceleration purely radial

$$T = m \left(\frac{v^2}{R} - g \right)$$

If speed equals a critical value v_c , tension vanishes, $T \equiv 0$.

$$0 = m \left(\frac{v_c^2}{R} - g \right)$$

$$v_c = \sqrt{Rg}.$$

The speed at any point can be determined from energy considerations given a value at some initial point.

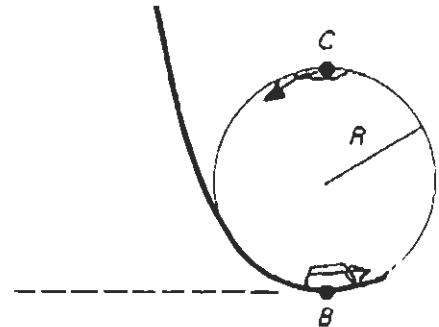
Example : Loop-the-loop

13-19

$$m = 75 \text{ kg}$$

$$R = 10 \text{ m}$$

$$v_0 = 15 \text{ m/s}$$



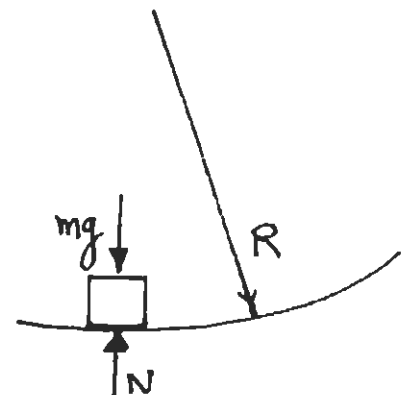
Man in a car executes motion in a vertical loop at a constant speed v_0 .

Calculate the apparent force the man feels pressing against him at the bottom and top of the loop. i.e. Calculate the apparent weight.

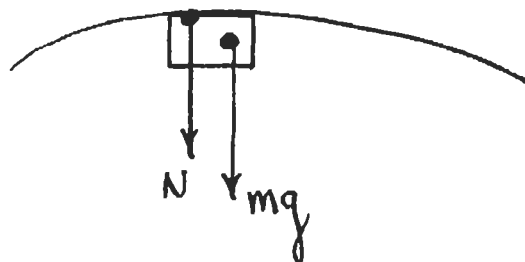
Bottom

$$N - mg = \frac{mv_0^2}{R} \quad (\text{circle})$$

↑ Normal force pushing on man (weight)



$$\begin{aligned} N &= m \left(g + \frac{v_0^2}{R} \right) = mg \left[1 + \frac{1}{g} \frac{v_0^2}{R} \right] \\ &= mg \left[1 + \frac{1}{9.81} \frac{15^2}{10} \right] = mg [1 + 2.29] \\ &= 3.29(mg) \end{aligned}$$

Top

$$N + mg = \frac{mv_0^2}{R}$$

$$\begin{aligned} N &= mg \left[\frac{v_0^2}{Rg} - 1 \right] \\ &= mg [2.29 - 1] \\ &= 1.29(mg) \end{aligned}$$

What happens if v_0 is decreased?

In particular what if

$$\begin{aligned} \frac{v_0^2}{Rg} = 1 &\Rightarrow v_0 = \sqrt{gR} = \sqrt{9.81 \times 10} \\ &= 9.90 \text{ m/s} \end{aligned}$$

$N \equiv 0$; No normal force.

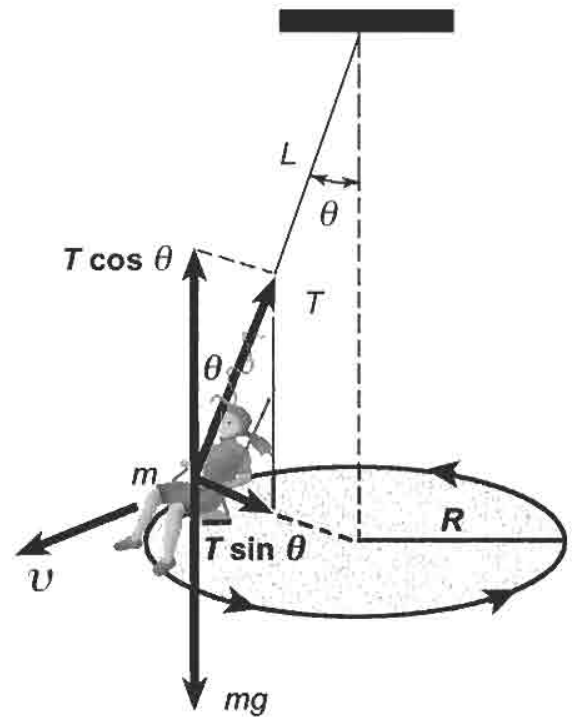
What if $v_0 < \sqrt{gR}$??

Conical Pendulum

Object at end of rope of length L . Total mass is m . Object swings in a horizontal circle of radius R at constant speed v .

Assume period of revolution is τ .

Find tension in rope T and angle θ .



A child's indoor swing.

$$\sum \vec{F} = m\vec{a}$$

$$T \sin \theta = m \frac{v^2}{R} \quad (1)$$

$$T \cos \theta - mg = 0 \quad (2)$$

$$\tan \theta = \frac{v^2}{Rg} \quad (3)$$

$$R = L \sin \theta$$

$$v = \frac{2\pi L \sin \theta}{\tau}$$

Then Eq. (3) becomes

$$\cos \theta = \frac{g \tau^2}{4\pi^2 L}$$

$$\text{or } \tau = 2\pi \sqrt{\frac{L}{g \cos \theta}}$$