

~~It is strongly recommended that you read about a subject before it is covered in lectures.~~

<del>Lecture Date</del>	<del>Material Covered</del>	<del>Reading</del>
<del>#25 Wed 11/10</del>	<del>Static Equilibrium - Stability - Rope Walker</del>	<del>Page 354 - 364</del>
<del>#26 Fri 11/12</del>	<del>Elasticity - Young's Modulus</del>	<del>Page 365 - 369</del>
<del>#27 Mon 11/15</del>	<del>Fluid Mechanics - Pascal's Principle - Hydrostatics Atmospheric Pressure Over Pressure in Lungs and Tires</del>	<del>Page 466 - 478</del>
<del>#28 Wed 11/17</del>	<del>Hydrostatics - Archimedes' Principle - Fluid Dynamics What makes your Boat Float? - Bernoulli's Equation</del>	<del>Page 478 - 483</del>

~~Due Wednesday, Nov 17, before 4 PM in 4-339B.~~

~~This is not an easy assignment. You have 9 days; START EARLY!~~

### ~~8.1 Conservation Laws~~

~~Imagine a spherical, non-rotating planet of mass  $M$  and radius  $R$  that has no atmosphere. A satellite is fired from the surface of the planet with speed  $v_0$  and at  $20^\circ$  to the local vertical. In its subsequent orbit the satellite reaches a maximum distance of  $5R$  from the center of the planet. Calculate  $v_0$ .~~

### ~~8.2 Much Ado About a Ham Sandwich~~

~~You can find this problem in the Lecture Supplement (on the Home Page) of Nov. 3. This supplement gives you all the necessary background as discussed in lectures.~~

### 8.3 Going to the Sun

A spacecraft of mass  $m$  is first brought into an orbit around the earth. The earth (together with the spacecraft) orbits the sun in a near circular orbit with radius  $R$  ( $R$  is the mean distance between the earth and the sun; it is about 150 million km).

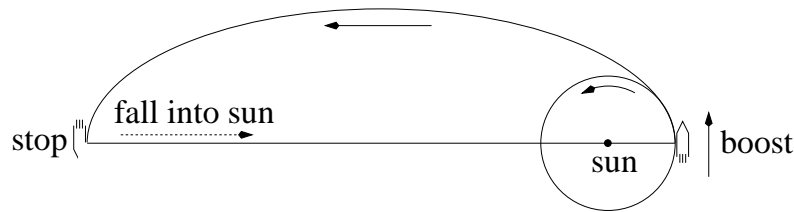
- a) What is the speed of the earth in its orbit around the sun?

We want the spacecraft to fall into the sun. One way to do this is to fire the rocket in a direction *opposite to the earth's orbital motion* to reduce the spacecraft's speed to zero (relative to the sun).

- b) What is the total impulse that would have to be given by the rocket to the spacecraft to accomplish this? You may ignore the effect of the earth's gravitation as well as the orbital speed of the spacecraft around the earth as the latter is much smaller (how much smaller?) than the speed of the earth around the sun. Thus, you may assume that the spacecraft, before the rocket is fired, has the same speed in its orbit around the sun as the earth.

We will now show that there is a more economical way of doing this (i.e., a much smaller rocket can do the job). By means of a brief rocket burn the spacecraft is first put into an elliptical orbit around the sun; the boost is provided tangentially to the earth's circular orbit around the sun (see figure). The aphelion of the new orbit is at a distance  $r$  from the sun. At aphelion the spacecraft is given a backward impulse to reduce its speed to zero (relative to the sun) so that it will subsequently fall into the sun.

- c) Calculate the impulse required at the first rocket burn (the boost).  
d) What is the speed of the spacecraft at aphelion?



- Calculate the impulse required at the second rocket burn (at aphelion).
- Compare the impulse under b) with the sum of the impulses under c) and e), and convince yourself that the latter procedure is more economical.
- Make a specific quantitative comparison for  $r = 20R$ .

#### ~~8.4 Black hole in X-Ray Binary~~

~~An X-ray binary consists of 2 stars with masses  $m_1$  (the accreting compact object) and  $m_2$  (the donor). The orbits are circular with radii  $r_1$  and  $r_2$  centered on the center of mass.~~

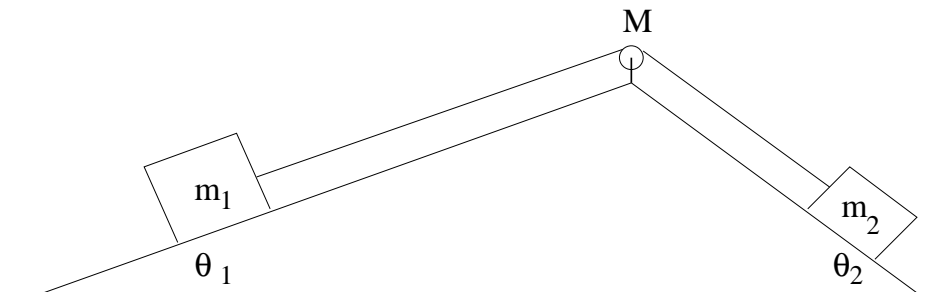
- ~~Derive the orbital period of the binary following the guidelines given in lectures.~~
- ~~In the case of Cyg X-1 (as discussed in lectures), the orbital period is 5.6 days. The donor star is a “supergiant” with a mass 30 times that of the sun. Doppler shift measurements indicate that the donor star has an orbital speed  $v_2$  of about 148 km/sec. Calculate  $r_2$ .~~
- ~~Calculate  $r_1$ . Your calculations will be greatly simplified if instead of  $r_1$  you set up your equations in terms of  $r_1/r_2$ . Once you have solved for  $r_1/r_2$ , you have found  $r_1$  as you already know  $r_2$  (see part b). You will find a third order equation in  $r_1/r_2$ . Only one solution is real; the other two are imaginary. There are various ways to find an approximation for  $r_1/r_2$ . You can find the solution by trial and error using your calculator, or you can plot the function.~~
- ~~Now calculate the mass  $m_1$  of the accreting compact object. As discussed in lectures, since this turns out to be substantially larger than 3 times the mass of the sun, it is strongly believed to be a black hole.~~

#### ~~8.5 Rolling and Slipping Hoop~~ page 352, problem 45

#### ~~8.6 Physical Pendulum~~ page 409, problem 34

#### ~~8.7 Two Blocks, Two Slopes and a Pulley~~

~~Two blocks  $m_1$  and  $m_2$  are connected by a light string (with negligible mass) passing over a pulley with mass  $M$  and radius  $R$  as shown. The pulley is a solid uniform disk, and the friction between it and the rope is such that when the blocks move the rope turns the pulley without slipping. The kinetic friction coefficient between the blocks and the slope is  $\mu$ . Find the acceleration of the blocks, the angular acceleration of the pulley, and the tension in each part of the rope. Assume that  $m_2$  is much larger than  $m_1$  (the blocks go from left to right), and that the bearings of the pulley rotate without friction.~~



#### ~~8.8 Doppler Effect I~~ – page 462, problem 29

### **Problem 8.3**

(a) The speed of the earth in a circular orbit, with a radius  $R = 1.5 \times 10^{11}$  m, about the sun, with a mass  $M = 1.99 \times 10^{30}$  kg, is given by Equation (12) on page 216 as

$$v_0 = \sqrt{\frac{GM}{R}} \approx 2.98 \times 10^4 \text{ m/s}$$

(b) The impulse is given by the change in momentum. The final momentum is zero; therefore the impulse is given by

$$I_0 = \Delta p = mv_0$$

(c) As discussed in Problem 1, mechanical energy and angular momentum are conserved after the rocket is finished firing. Consider the initial and final points, which are respectively the perihelion and aphelion of the elliptical orbit. Conservation of angular momentum gives

$$mv_1 R = mv_2 r \quad \implies \quad v_2 = \frac{R}{r} \cdot v_1$$

where  $R$  is the distance from the sun to the perihelion and  $v_1$  is the velocity there and  $r$  is the distance from the sun to the aphelion and  $v_2$  is the velocity there. Conservation of mechanical energy gives

$$-\frac{GMm}{R} + \frac{1}{2}mv_1^2 = -\frac{GMm}{r} + \frac{1}{2}mv_2^2 = -\frac{GMm}{r} + \frac{1}{2}m\left(\frac{R}{r} \cdot v_1\right)^2$$

Solving the above for  $v_1$  gives

$$v_1 = \sqrt{\frac{2GM\left(1 - \frac{R}{r}\right)}{R\left(1 - \left(\frac{R}{r}\right)^2\right)}} = v_0 \cdot \sqrt{\frac{2}{1 + \frac{R}{r}}}$$

The impulse is given by the change in momentum.

$$I_1 = \Delta p = mv_1 - mv_0 = I_0 \left( \sqrt{\frac{2}{1 + \frac{R}{r}}} - 1 \right)$$

(d) Using the result above following from angular momentum conservation, the final velocity is

$$v_2 = v_1 \cdot \frac{R}{r} = v_0 \cdot \sqrt{\frac{2}{1 + \frac{R}{r}}} \cdot \frac{R}{r}$$

(e) The impulse is given by the change in momentum. The final momentum is zero; therefore the impulse is given by

$$I_2 = \Delta p = mv_2 = I_0 \cdot \sqrt{\frac{2}{1 + \frac{R}{r}}} \cdot \frac{R}{r}$$

(f) The sum of each impulse is

$$\begin{aligned} I = I_1 + I_2 = \Delta p = mv_1 - mv_0 &= I_0 \left( \sqrt{\frac{2}{1 + \frac{R}{r}}} - 1 \right) + I_0 \cdot \sqrt{\frac{2}{1 + \frac{R}{r}}} \cdot \frac{R}{r} \\ &= I_0 \left( \sqrt{\frac{2}{1 + \frac{R}{r}}} \cdot \left( 1 + \frac{R}{r} \right) - 1 \right) = I_0 \left( \sqrt{2 \left( 1 + \frac{R}{r} \right)} - 1 \right) \end{aligned}$$

Therefore the difference is

$$I_0 - I = I_0 \cdot \left( 2 - \sqrt{2 \left( 1 + \frac{R}{r} \right)} \right) \geq 0$$

since  $r \geq R$ .

(g) For  $r = 20 \cdot R$  the above equation gives

$$I_0 - I \approx 0.55 \cdot I_0$$

The quantity  $I_0$  is defined above as

$$I_0 = mv_0 \approx 2.98 \times 10^4 \cdot m$$

Therefore, in terms of the mass of the spacecraft,  $m$ , the difference in the impulses is

$$I_0 - I \approx 1.6 \times 10^4 \cdot m$$

## **Problem 8.4**

~~(a)~~

$$\frac{\cancel{G}m_1m_2}{(\cancel{r_1} + r_2)^2} = \frac{\cancel{m_1}v_1^2}{r_1} = \frac{m_2v_2^2}{r_2}$$

$$v_1 = \frac{2\pi r_1}{\cancel{T}} \quad v_2 = \frac{2\pi r_2}{\cancel{T}}$$