

# Inertial Mass

- inertial mass is a ‘quantity of matter’
- standard body with mass  $m_s$  and SI units [kg]
- the mass of all other bodies will be determined relative to the mass of our standard body.
- apply the same action (force) to the standard body and an unknown body
- Define the unknown mass in terms of ratio of accelerations



$$\frac{m_u}{m_s} = \frac{a_s}{a_u}$$

# Standard Kilogram

- Cylindrical alloy of 90 % platinum and 10 % iridium

$$\rho = 21.56 \text{ g} \cdot \text{cm}^{-3}$$

- Density and volume of the standard kilogram,

$$V = m / \rho \cong 1000 \text{ g} / 21.56 \text{ g} \cdot \text{cm}^{-3} \cong 46.38 \text{ cm}^3$$

- Constant volume for a cylinder

$$V = \pi r^2 h$$

- The surface area

$$A = 2\pi r^2 + 2\pi r h = 2\pi r^2 + \frac{2V}{r}$$

- minimize the area with respect to the radius

$$\frac{dA}{dr} = 4\pi r - \frac{2V}{r^2} = 0$$

$$r = h / 2$$

- radius is one half the height,

$$r = (V / 2\pi)^{1/3} \cong 1.95 \text{ cm}$$

# Definition: Momentum (Quantity of Motion)

- Momentum is a vector quantity  $\vec{\mathbf{p}} = m\vec{\mathbf{v}}$
- Magnitude: product of the mass with the magnitude of the velocity
- Direction: the direction of the velocity
- In the SI system of units, momentum has units

$$[\text{kg-m-s}^{-1}]$$

# Definition: Force

- Force is a vector quantity.  $\vec{\mathbf{F}} \equiv m\vec{\mathbf{a}}$
- Magnitude of the total force acting on the object is the product of the mass and the magnitude of the acceleration .
- Direction of the total force on a body is the direction of the acceleration of the body.
- SI units for force are **[kg-m-s<sup>-2</sup>]**
- Unit has been named the Newton **1 N = 1 kg-m-s<sup>-2</sup>**

# Superposition Principle

Apply two forces  $\vec{\mathbf{F}}_1$  and  $\vec{\mathbf{F}}_2$  on a single body

Total force is the vector sum of the two forces

$$\vec{\mathbf{F}}^{total} = \vec{\mathbf{F}}_1 + \vec{\mathbf{F}}_2$$

# Average Impulse

Apply an average force  $\vec{\mathbf{F}}$  for an interval of time  $\Delta t$

Average impulse is the product of the average force and the time interval

$$\vec{\mathbf{I}} = \vec{\mathbf{F}}\Delta t$$

Units for impulse are the same as for momentum

$$[\text{kg}\cdot\text{m}\cdot\text{s}^{-1}] = [\text{N}\cdot\text{s}]$$

# Impulse

When force is applied continuously over a time interval

$$\left[ t_0, t_f \right]$$

Impulse is the integral

$$\vec{\mathbf{I}} = \int_{t_o}^{t_f} \vec{\mathbf{F}} dt$$

# Newton's Second Law

The change of momentum is equal to the applied average impulse

$$\vec{\mathbf{I}} = \vec{\mathbf{F}}\Delta t = \Delta\vec{\mathbf{p}}$$

For an instantaneous action of the total force, force is equal to the product of mass with acceleration.

$$\vec{\mathbf{F}}^{total} = \lim_{\Delta t \rightarrow 0} \frac{\Delta\vec{\mathbf{p}}}{\Delta t} \equiv \frac{d\vec{\mathbf{p}}}{dt}$$

When the mass remains constant in time,

$$\vec{\mathbf{F}}^{total} = m\vec{\mathbf{a}}$$