

- ~~e) How long does it take for the cylinder to rotate through this angle? (This corresponds to the time it takes the Bismuth atom to cross the diameter of the cylinder).~~
- ~~d) What is the velocity of these Bismuth atoms?~~
- ~~e) Using the equipartition theorem  $v_{rms}^2 = 3RT/M$ , where the universal gas constant  $R = 8.31 \text{ J/mole} \cdot \text{K}$ , calculate the root mean square velocity of the Bismuth atoms. How does your answer compare to part d)?~~

### Problem 3: Burning Calories

A racing bicyclist, traveling at an average speed of 25 mph, has a metabolic rate of

$$dE / dt = 1.36 \times 10^3 \text{ W} .$$

The average power output is

$$dW / dt = 3.0 \times 10^2 \text{ W} .$$

The person cycles for 4 hours and covers 100 miles. The person has mass  $m = 60 \text{ kg}$  .

- a) What is the average force that the person applies?
- b) What is the ratio of power output to catabolic rate,  $\varepsilon = \frac{dW / dt}{dE / dt}$  ?
- c) How many kilocalories did the person burn up?
- d) What rate did the cyclist generate heat?
- e) What happens to this heat?

### ~~Problem 4: Experiment Mechanical Equivalent of Heat~~

~~*This problem will be part of the experiment write-up (last page) and given out in class the day of the experiment.*~~

### ~~Problem 5: Hand in the In-class problem Friday Dec 3 with your problem set.~~

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### Answer:

a) The cyclist applies an average force by pedaling. Suppose the bicyclist pedals at an angular velocity  $\omega_0$  . Then the power output of the cyclist is

$$P = \tau \omega_0 = 3.0 \times 10^2 \text{ W} .$$

Now assume that the cyclist applies an average force  $F_{ave}$  and the radius of the crankshaft is  $r_0$  . Then the cyclist applies a torque

$$\tau = F_{ave} r_0 .$$

The crankshaft on my bicycle is  $r_0 \cong 0.17 \text{ m}$  .

So the average force exerted by the cyclist is

$$F_{ave} = P / r_0 \omega_0$$

The angular velocity of the cyclist legs and the angular velocity of the wheel depends on the gear ratio. Let's assume that at this gear ration at 25 mph is about 5. The angular velocity  $\omega_1$  of the bicycle is given by

$$\omega_1 = v_{cm} / r_1$$

where  $r_1$  is the radius of the wheel. The radius of the wheel on bicycle is  $r_1 \cong 0.342 \text{ m}$ .

So the velocity of the bicycle is approximately

$$v_{cm} = (25 \text{ mph})(0.45 \text{ m} \cdot \text{s}^{-1} / \text{mph}) = 11.3 \text{ m} \cdot \text{s}^{-1}.$$

So the angular velocity is

$$\omega_1 = v_{cm} / r_1 = (11.3 \text{ m} \cdot \text{s}^{-1}) / (0.345 \text{ m}) = 3.3 \times 10^1 \text{ rad} \cdot \text{s}^{-1}.$$

The ratio of angular velocities is then

$$\omega_1 = (\text{gear ratio})\omega_0.$$

So the spin rate of the cyclist is

$$\omega_0 = \omega_1 / (\text{gear ratio}) = (3.3 \times 10^1 \text{ rad} \cdot \text{s}^{-1}) / 5 = 6.5 \text{ rad} \cdot \text{s}^{-1}.$$

Note that this corresponds to a frequency of 1 cycle per second,

$$f = \omega_0 / 2\pi = (6.5 \text{ rad} \cdot \text{s}^{-1}) / 2\pi \cong 1 \text{ Hz}.$$

Thus the average force exerted by the bicyclist is

$$F_{ave} = P / r_0 \omega_0 = (3.0 \times 10^2 \text{ W}) / (0.17 \text{ m})(6.5 \text{ rad} \cdot \text{s}^{-1}) = 2.7 \times 10^2 \text{ N}.$$

Note that cyclist pedal with two legs and they only apply a force per leg over an angle  $\theta$ .

So the effective force on each leg is the average force divided by two divided by the fraction of the circle the force is applied angle  $\theta / 2\pi$

$$F_{eff} = \frac{F_{ave}}{2} / \frac{\theta}{2\pi} = \frac{F_{ave} 2\pi}{2\theta}.$$

Suppose I apply a force for only 60 degrees,  $\theta = \pi / 3$ , then the effective force on each leg is

$$F_{eff} = \frac{F_{ave} 2\pi}{2\theta} = \frac{(2.7 \times 10^2 \text{ N})(2\pi)}{(2)(\pi/3)} = 8.1 \times 10^2 \text{ N}.$$

b)

$$\epsilon \approx 0.22$$

c) The amount of energy obtained through the metabolism is

$$E = \frac{dE}{dt} T \approx 1.96 \times 10^7 \text{ J} \approx 4.7 \times 10^3 \text{ kilocalories}$$

d) The cyclist generates heat at a rate

$$\frac{dQ}{dt} = \frac{dE}{dt} - \frac{dW}{dt} = 1060 \text{ W}$$

e) This heat increases the cyclist's temperature and is also exchanged with the environment (air).